CHAPTER «PHYSICAL AND MATHEMATICAL SCIENCES»

ANALYTICAL AND PRACTICAL DEVELOPMENT OF VARIANT OF MATHEMATICAL THEORY OF SHELLS OF SMALL CURVATURE OF ARBITRARY THICKNESS

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Abstract. The subject of theoretical and practical research is a variant of mathematical theory (MT) of transversely isotropic homogeneous shells of small curvature of arbitrary thickness and methods for solving the systems of differential equations (DE) obtained on their basis with high-order partial derivatives. The object of the study is the analytical and numerical dependences of the stress-strain state (SSS) of these shells on the mechanical-geometric parameters (MGP), the type of loading and various approximations of the MT variant in the problems of static. The purpose of this work is to summarize and extend the theoretical and numerical studies of the constructed variant MT of transversely isotropic shells of small curvature of arbitrary constant thickness at transverse static loading. The developed version of the MT takes into account all the components of the SSS shell and considers them as functions of three variable coordinates. It is based on the method of decomposition of displacements, stresses and strains in infinite mathematical series with a transverse coordinate using Legendre polynomials. The three-dimensional problem of shell elasticity theory is reduced to two-dimensional by means of the Reisner variational principle. Three-dimensional theory of elasticity DE is used to represent the components of the transverse stresses in the form of a series of Leandra

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polynomials. The boundary conditions on the front surfaces are met exactly. Representation of SSS components in the form of infinite series makes it possible to determine with high accuracy the internal SSS, which is independent of boundary effects, as well as of boundary effects – vortex and potential. Basic dependencies, DR equilibria, boundary conditions on the lateral surface are shown in general form. On this basis, dependencies and equations are obtained for different approximations, when the partial sums of the series take into account different numbers of additions. The DE system is obtained in displacements, reduced in general to a non-homogeneous DE system, which is then transformed into a defining system of equations for new functions. Different approximations are considered as special cases. The analysis and research of the obtained DE systems is performed in general form and on the basis of different approximations. Selected equations describing the boundary effect (BE) of vortex. Internal SSS and potential BE are determined by an interdependent DE system. A common general method of algebraic and operator transformations of the obtained DE systems is developed and forms of common solutions are constructed. This makes it possible to obtain general solutions of equilibrium equations by splitting high-order differential equations to small-order equations. Numerical studies of internal SSS for a wide class of MGP shells have been carried out. It is established: 1) results obtained on the basis of low approximations, including the theory of Tymoshenko-Reisner, can differ significantly from the exact ones; 2) Tymoshenko-Reisner type theory satisfactorily describes the SSS of thin shells of small curvature with low susceptibility to transverse shear at smooth external loads (which slowly change over the shell region); 3) the approximation, which takes into account the first four additions in mathematical series for tangential displacements, describes with high accuracy the SSS of thin shells and shells of medium thickness at smooth loads over a wide range of MGP changes; 4) at non-smooth loads higher approximations should be used; 5) the internal SSS of the shells depends most significantly on the smoothness and locality of the transverse load, the curvature of the middle surface, the thickness, the susceptibility to transverse shear; the MT approximation accuracy is increased with decreasing thickness, curvature, susceptibility to transverse displacements, and increasings load smoothness.

1. Introduction

Plate and shell structural elements are widely used in power engineering, engineering, industrial and civil engineering, and other industries. Ensuring their reliable operation requires the involvement of high-precision theories and adequate implementation techniques that take into account all components of the SSS and margins and effects.

Problem solving based on classical plate and shell theories [19, p. 3; 20, p. 22], which are subjected to local loads, have openings, a sharp change in MGP, as well as at considerable thickness and in other cases, which lead to a large gradient of SSS change, give unsatisfactory results.

Nonclassical clarifying theories [1, p. 242; 2; 5, p. 49; 10, p. 3; 16; 17, p. 52; 22, p. 195; 25; 26, p.184; 28, p. 32; 29, p. 744], which are based on different physico-geometric hypotheses or different models of representation of deformation of these elements [1, p. 242; 13, p. 910], for a certain class of boundary value problems also cannot accurately describe the SSS of plates and shells, since finding the components of SSS with arbitrary accuracy is limited by accepted assumptions. This is due to the image of the SSS components in the form of a small number of additives (or parametric functions) [1, p. 242; 13, p. 910], which are accepted on the basis of certain physical considerations. The resulting DE systems tend to be of low order.

Solution of boundary value problems for linearly elastic plates and shells in three-dimensional formulation [9; 19, p. 3; 20, p. 22] is associated with great mathematical difficulties. These are complex three-dimensional boundary value problems of mathematical physics in which the components of the SSS are functions of three variable coordinates. Only in a limited number of cases can an analytical solution be found.

A study of recent publications shows that Tymoshenko-Reisner-type theory is used in solving problems for plates and shells [17, p. 52; 26, p. 184; 29, p. 744] or their refining variants [3; 5, p. 49; 10, p. 3; 14, p. 238; 22, p. 195]. The validity of using this type of theory to solve the relevant problems requires additional research.

Hence the relevance and purpose of the study, which is the need to build and develop new MT options and develop effective methods that would give a real opportunity to determine all components of the SSS plates and shells (functions of three variables) with high accuracy, taking into account the boundary effects.

There are different approaches to constructing theories that do not use assumptions (MT variants). SSS components are considered to be functions of three coordinates. They can be represented in the form of tensor series [12], power series [15, p. 475], mathematical series using Legendre polynomials [3; 4, p. 238 ; 6; 8, p. 77; 11; 21, p. 84; 23, p. 335; 24, p. 51; 30, p. 191; 31, p. 54]. Three-dimensional problems are reduced to two-dimensional by different methods: projection [6; 8, p. 77; 30, p. 191], variational [11; 21, p. 84; 23, p. 335; 24, p. 51; 31, p. 54]. There are other approaches described in [3; 7, p. 49; 11; 19, p. 3].

Reviews and development of plate and shell theories can be found, in particular in [3; 6; 9; 11; 19, p. 3; 20, p. 22].

The scientific novelty of the obtained results is the following:

1). A mathematical approach to the analytical solution of boundary value prob-lems of transversal isotropic shells of small curvature of arbitrary thickness under static transverse loading is developed. The approach is based on the use of three-dimensional equations of the theory of elasticity, decomposition of all components of the SSS (functions of three variables) into infinite mathematical series with transverse coordinate using Legendre polynomials. Three-dimensional problems of the theory of elasticity are reduced to two-dimensional on the basis of the Reisner variational principle [27, p. 90] and interdependent equations [32, p. 67; 33, p. 92; 34, p. 137; 35, p. 496]. The boundary conditions are fulfilled precisely on the upper and lower surfaces of the shell.

2) Generalized and developed variant of MT of small curvature shells with higher approximations, which is reduced to high order DE systems, which allows to determine SSS with great accuracy.

3) Formulated boundary value problems in the general case; a unified mathematical technique of differential transformations of a system of equations of high order to convenient determining systems of differential equations of lower order is developed. Forms of general solutions are found.

4) A class of problems for shells for determining internal SSS in different approximations at different loads, slow-change and fast-varying in the region is solved; established the qualitative impact of MGP on SSS; limits of application of approximate theories depending on MGP and load type; obtained new qualitative effects and important conclusions. The validity of the method of construction of the considered variant of MT is given in

[31, p. 54], which shows the high accuracy and effectiveness of this MT option based on comparing the results for SSS with other theories.

In this work, theoretical and practical studies of the constructed variant of MT of transversely isotropic shells of small curvature of arbitrary thickness at any transverse load, which were obtained in previous works, are generalized and extended [32, p. 67; 33, p. 92; 35, p. 496].

2. Formulation of the problem

The purpose of the problem is theoretical and applied research of the constructed variant MT of homogeneous elastic transversal-isotropic shells of small curvature of arbitrary thickness, development on the basis of it of methods of solving DE of boundary value problems with consideration of all components of SSS and BE and analysis of numerical results obtained.

We shall consider shells of small curvature of arbitrary constant thickness h in a rectangular Cartesian coordinate system x, y, z. The dimensions of the shell in plan $a \times b$. The z axis is pointing up (in the direction of the bulge).

On the upper and lower surfaces of the shell there is a static transverse load $q_1(x, y)$ and $q_2(x, y)$. The boundary conditions on the surfaces have the form:

$$\sigma_z(z = h/2) = -q_1(x, y)\,;\ \sigma_z(z = -h/2) = q_2(x, y)\,; \tag{1}$$

$$\sigma_{xz}(z = \pm h/2) = \sigma_{yz}(z = \pm h/2) = 0\,. \tag{2}$$

For convenience, the transverse loading on the surfaces is depicted in the form of an algebraic sum of symmetric $p(x, y)$ and skew-symmetric $q(x, y)$ components. Then the boundary conditions (1) are written as follows:

$$\sigma_z(z = \pm h/2) = (\mp q(x, y) - p(x, y))/2\,, \tag{3}$$

where $p(x, y) = q_1(x, y) - q_2(x, y)$, $q(x, y) = q_1(x, y) + q_2(x, y)$.

Conditions on the side surface of the shell can be static, kinematic or mixed.

All SSS components are considered functions of three variable coordinates. Tangential components of the displacements in expressions for the transverse angular deformations γ_{xz}, γ_{yz}, which are neglected in the theory of thin shells, are also taken into account. The deformations are defined as follows:

$$\varepsilon_x = \partial U / \partial x + k_1 W;\ \ \varepsilon_y = \partial V / \partial y + k_2 W;$$

$$\varepsilon_z = \partial W / \partial z;\ \ \gamma_{xy} = \partial U / \partial y + \partial V / \partial x;$$

$$\gamma_{xz} = \partial W / \partial x + \partial U / \partial z - k_1' U\,,\ (x, y;\ U \to V;\ k_1' \to k_2')\,,$$

$$(k_i = 1/R_i;\ \ k_i' = k_i,\ i = 1, 2)\,,$$

where R_1, R_2 is the principal radii of curvature of the middle surface of the shell.

Physical dependencies for a transversal isotropic shell whose isotropy surface is parallel to the median surface are as follows:

$$\varepsilon_x = (\sigma_x - \nu\sigma_y) / E - \nu'\sigma_z / E', \ (x, y) ; \varepsilon_z = [\sigma_z - \nu'(\sigma_x + \sigma_y)] / E';$$

$$\gamma_{yx} = \sigma_{yx} / G; \quad \gamma_{xz} = \sigma_{xz} / G'; \quad \gamma_{yz} = \sigma_{yz} / G', (G = E / (2(1+\nu))),$$

where E, E', G, G', ν, ν' – the generally accepted mechanical characteristics of the material.

3. Image of the SSS components by mathematical series

3.1. Movement components. We represent the components of displacements in the form of infinite Fourier-Legendre series:

$$U(x, y, z) = \sum_{k=0}^{\infty} P_k(2z / h)\, u_k(x, y); \ V(x, y, z) = \sum_{k=0}^{\infty} P_k(2z / h)\, v_k(x, y);$$
$$W(x, y, z) = \sum_{k=1}^{\infty} P_{k-1}(2z / h)\, w_k(x, y); \tag{4}$$

where u_k, v_k, w_k, are the unknown functions sought; they must continue to satisfy DE equilibria and boundary conditions. The representation of the SSS components as infinite rows by Legendre polynomials enables the solution to be obtained with any high precision. In practical calculations in mathematical series (4) a certain number of additions is taken. If the components of the displacement take into account the $u_0, v_0, u_1, v_1, w_1, ..., u_n, v_n, w_n$ components, then we call this approximation K01… n or K0-n.

3.2. Stresses components. The transverse stresses σ_{xz} (x, y, z), σ_{yz} (x, y, z) satisfy the boundary conditions (2) and are represented as follows:

$$\sigma_{xz}(x, y, z) = \sum_{k=1}^{\infty} \alpha_k(z)\, Q_{kx}(x, y); \ \sigma_{yz}(x, y, z) = \sum_{k=1}^{\infty} \alpha_k(z)\, Q_{ky}(x, y), \tag{5}$$

where

$$\alpha_k(z) = 3\,(P_{k-1} - P_{k+1}) / (h(2k+1)), \ (k = 1, 2, ...).$$

Functions Q_{kx}, Q_{ky} are unknown. They are determined by the Reisner variational principle:

$$Q_{kx}(x, y) = \sum_{i=1,3}^{\infty} h_{ki} \frac{\partial w_i}{\partial x} + \sum_{i=0,1}^{\infty} l_{kxi} u_i, (k = 1, 3, ...);$$
$$Q_{ky}(x, y) = \sum_{i=1,3}^{\infty} h_{ki} \frac{\partial w_i}{\partial y} + \sum_{i=0,1}^{\infty} l_{kxy} v_i, (k = 2, 4, ...), \tag{6}$$

where h and l with the lower indices – MGP shell, they are different for different approximations.

The normal stresses $\sigma_z(x,y,z)$ satisfy the boundary conditions (1), (3) and are defined as follows:

$$\sigma_z(x,y,z) = \sum_{k=0}^{\infty} \chi_k(z)\,\omega_k(x,y)\,, \tag{7}$$

where

$$\chi_0(z) = 1;\ \chi_1(z) = -3P_1 / 5 + P_3 / 10\,;$$

$$\chi_k(z) = -3(P_{k-2} / ((2k-1)(2k+1)) - 2P_k / ((2k-1)(2k+3)) +$$

$$+ P_{k+2} / ((2k+1)(2k+3)) / 2, \quad k \geq 2.$$

The functions $\omega_0(x,y), \omega_1(x,y)$ are determined from the boundary conditions (1), (3). The functions $\omega_k(x,y)$ ($x_{si} \to$) – are un-known. They are determined by the

Reisner variational principle:

$$\omega_0(x,y) = -p(x,y) / 2;\ \omega_1(x,y) = q(x,y);$$

$$\omega_k(x,y) = \sum_{i=2,3}^{\infty} q_{ki} w_i + \sum_{i=1,3}^{\infty} e_{ki}\phi_i + e_{kq} q\ ,\ (k = 3,5,...)\ ; (8)$$

$$\omega_k(x,y) = \sum_{i=1,2}^{\infty} q_{ki} w_i + \sum_{i=0,2}^{\infty} e_{ki}\phi_i + e_{kp} p,\ (k = 2,4,...);\ \phi_i = \partial u_i / \partial x + \partial v_i / \partial y\,,$$

where e_{ki}, e_{kq}, e_{kp} is MGP; they have different meanings for different approximations. $\omega_k(x,y)$ functions depend on the components in the movements with even and odd indexes. If (8) put k_1 and k_2 equal to zero, then we obtain the corresponding functions for the transversal isotropic plate.

Dependencies between stresses and components in displacements for the shell, taking into account the above relations (5) – (8), are represented by infinite mathematical series:

$$\begin{gathered} \sigma_{xz}(x,y,z) = \sum_{i=0}^{\infty} P_i\, t_{xi};\ \sigma_{yz}(x,y,z) = \sum_{i=0}^{\infty} P_i\, t_{yi};\sigma_z(x,y,z) = \sum_{i=0}^{\infty} P_i\, s_{zi}; \\ \sigma_x(x,y,z) = \sum_{i=0}^{\infty} P_i\, s_{xi};\ \sigma_y(x,y,z) = \sum_{i=0}^{\infty} P_i\, s_{yi};\ \sigma_{xy}(x,y,z) = \sum_{i=0}^{\infty} P_i\, t_{yxi}. \end{gathered} \tag{9}$$

In (9) $t_{xi},\ t_{yi}, s_{zi},\ s_{xi},\ s_{yi},\ t_{yxi}$ depends on the components in the displacements.

4. Differential equilibrium equations and boundary conditions

4.1. Differential equilibrium equations in the approximation K0-n. The DE equilibrium system and boundary conditions are derived from the Reisner variational equation. We are leading a system of DE equilibria and boundary conditions in the approximation K0-n.

System of DE of equilibrium transtropic shell of small curvature:

$$\sum_{k=0}^{n}(D_{ui,k}u_k + D_{vi,k}v_k) + \sum_{k=1}^{n} D_{wi,k}w_k = D_{ip}\,p(x,y) + D_{iq}\,q(x,y),\ \ (i = 1,2,...,3n+2), \quad (10)$$

where $D_{ui,k}$, $D_{vi,k}$, $D_{wi,k}$ is the differential operators not higher than the second order, D_{ip}, D_{iq} is the differential operators not higher than the first order. Equally marked operators in different approximations are different.

4.2. Boundary conditions in the K0-n approximation. To obtain the boundary conditions, we decompose the load X_v, Y_v, Z_v acting on part Γ_1 of the lateral surface Γ of the shell into series by Legendre polynomials in the transverse coordinate:

$$X_v(z,s) = \sum_{i=0}^{n} P_i(2z/h)x_{si}(s), \quad (11)$$

where functions x_{si} are defined on part S_1 of the contour S of the shell $(x, y \in S_1)$:

$$x_{si}(x,y) = \frac{2i+1}{h}\int X_v(x,y,z)P_i(2z/h)\,dz,\ (X_v \to Y_v \to Z_v,\ \ x_{si} \to y_{si} \to z_{si}).$$

Let us represent the components of the displacement $U_{\Gamma_2}(x,y,z)$, $V_{\Gamma_2}(x,y,z)$, $W_{\Gamma_2}(x,y,z)$ given on part Γ_2 of the lateral surface ($\Gamma = \Gamma_1 + \Gamma_2$) of the shell in the form of finite sums:

$$U_{\Gamma_2}(x,y,z) = \sum_{j=0}^{n} P_j(2z/h)\,u_{j\Gamma_2}(x,y);\ V_{\Gamma_2}(x,y,z) = \sum_{j=0}^{n} P_j(2z/h)\,v_{j\Gamma 2}(x,y);$$

$$W_{\Gamma_2}(x,y,z) = \sum_{j=1}^{n} P_j(2z/h)\,w_{j\Gamma_2}(x,y), \quad (12)$$

where the functions $u_{j\Gamma 2}(x,y)$, $v_{j\Gamma_2}(x,y)$, $w_{j\Gamma_2}(x,y)$ are defined on part S_2 of the contour S shell $(x, y \in S_2; S_1 + S_2 = S)$:

$$u_{j\Gamma_2}(x,y) = \frac{2j+1}{h}\int_z U_{\Gamma_2}(x,y,z)\,P_j(\frac{2z}{h})dz,$$

$$(u_{j\Gamma_2} \Rightarrow v_{j\Gamma_2}, U_{\Gamma_2} \to V_{\Gamma_2}), (j = 0,1,...,n); \quad (13))$$

$$w_{j\Gamma_2}(x,y) = \frac{2j-1}{h}\int_z W_{\Gamma_2}(x,y,z)\,P_{j-1}(\frac{2z}{h})dz,\ (j = 0,1,...,n).$$

The boundary conditions are obtained from the Reisner variation equation:

$$\int_{(s)}\{\sum_{j=0}^{n}\frac{h}{(2j+1)}((s_{xj}\,l_x+t_{yxj}\,l_y-x_{sj})\,\delta\,u_j+(t_{yxj}\,l_x+s_{yj}l_y-y_{s\,j})\,\delta\,v_j)+ \\ +\sum_{j=0}^{n-1}\frac{h}{(2j+1)}(t_{xj}\,l_x+t_{yj}\,l_y-z_{sj})\delta\,w_{j+1})\}ds=0, \tag{14}$$

where $l_x,\ l_y$ – is the guide cosines normal to the lateral surface.

Equation (14) with respect to (11) – (13) yields different boundary conditions in the approximation K0-n. Here are some of them.

1) Boundary conditions in displacements. Only the displacement components $U_\Gamma(x,y,z),\ V_\Gamma\,(x,y,z),\ W_\Gamma\,(x,y,z)$. are known on the side surface of the shell. Boundary conditions:

$$u_j(x,y)=u_{j\Gamma}(x,y);\ v_j(x,y)=v_{j\Gamma}(x,y),\ (j=0,1,...,n); \\ w_j(x,y)=w_{j\Gamma}(x,y),\ (j=1,...,n);\ x,y\in S. \tag{15}$$

2) Boundary conditions in stresses. Only the external load $X_v,\ Y_v,\ Z_v$ is set on the side surface Γ. Then we have the following boundary conditions:

$$s_{x\,j}(x,y)\,l_x+t_{y\,x\,j}(x,y)\,l_y=x_{s\,j}(x,y);t_{y\,x\,j}(x,y)\,l_x+s_{y\,j}(x,y)\,l_y=y_{s\,j}(x,y), \\ (j=0,1,...,n);\ t_{x\,j}(x,y)\,l_x+t_{y\,j}(x,y)\,l_y=z_{s\,j}(x,y),\ (j=0,1,...,n-1),x,y\in S, \tag{16}$$

where $s_{xj},\ s_{yj},\ t_{yxj},\ t_{xj},\ t_{yj}$ – functions from $u_j,\ v_j,\ w_j$.

5. Basic dependences and equations in the approximation K0-n

We assume that n is an odd integer. The components of dis-placements and stresses are expressed by partial sums of infinite series.

Movement components:

$$U(x,y,z)=\sum_{k=0}^{n}P_k(2z/h)\,u_k(x,y);\ V(x,y,z)=\sum_{k=0}^{n}P_k(2z/h)\,v_k(x,y); \\ W(x,y,z)=\sum_{k=1}^{n}P_{k-1}(2z/h)\,w_k(x,y). \tag{17}$$

Stresses components:

$$\sigma_{xz}(x,y,z)=\sum_{i=0}^{n+1}P_i\,t_{xi};\ \sigma_{yz}(x,y,z)=\sum_{i=0}^{n+1}P_i\,t_{yi};\sigma_z(x,y,z)=\sum_{i=0}^{n+2}P_i\,s_{zi}; \\ \sigma_x(x,y,z)=\sum_{i=0}^{n+2}P_i\,s_{xi};\ \sigma_y(x,y,z)=\sum_{i=0}^{n+2}P_i\,s_{yi};\ \sigma_{xy}(x,y,z)=\sum_{i=0}^{n}P_i\,t_{yxi}. \tag{18}$$

To approximate K0-5, functions $t_{xi},\ t_{yi},...,t_{yxi}$ are given in [35, p. 496].

Differential equilibrium equations. The DR equilibrium system has the form (10). It has order $(6n+4)$.

Boundary conditions. The boundary conditions are determined by equations (11) – (16).

The main dependencies and equations are obtained in the approximations K01, K0-3, K0-5 from (10) – (18) if we put $n=1,\ n=3,\ n=5$ in them. The order of systems of differential equations of equilibrium is $(6n+4)$.

It is shown analytically that the differential matrix in all approximations is symmetric:. Expressions for system operators in the approximation K0-5 are given in [32, p. 67].

6. Transformation of DE systems. Forms of general solutions

DE systems for the shell of small curvature of arbitrary thickness in all approximations are not divided into separate systems that describe symmetric and oblique deformation. If k_1' and k_2' are taken into account, these systems are not divided into the equation of vortex and potential BE with internal SSS.

Additional studies show that for shells with parameters $R_{1,2}\ /\ a;\ a \ge 2;\ a \ge b$ in the equations can be put $k_1' = k_2' = 0$. Then, a system of vortex BE equations and a system describing the internal SSS with potential BE are singled out from these systems. Hereinafter we shall consider $k_1' = k_2' = 0$.

6.1. The approximation K0-n (n-odd). Consider the transformation of the system DR (10) in the general case in the approximation K0-n.

The vortex BE is described by a system of order 2n, which is divided into two separate systems. One order system (n + 1) determines the BE at skew symmetric deformation:

$$\sum_{j=1,3}^{n} H_{c\,i\,j}\psi_j = 0,\ \ (i = 1, 3, ..., n)\ . \tag{19}$$

The other system is of the order (n-1) and determines the BE in symmetric deformation:

$$\sum_{j=2,4}^{n-1} H_{s\,i\,j}\psi_j = 0,\ \ (i = 2, 4, ..., n-1);\ \ \psi_j = \partial u\ /\ \partial y - \partial v\ /\ \partial x\ . \tag{20}$$

In (19) and (20) $H_{c\,i\,j},\ H_{s\,i\,j}$ the differential operators are not higher than the second order, which depends on the MGP, except for the curves.

Therefore, the curvature of the shell does not affect the eddy BE, and the equations coincide with the equations for the plates.

After some algebraic and differential transformations of system (10), a 24-order DE system with partial derivatives with respect to functions $u_0, v_0, w_1, .., w_n$ is obtained, which determines the interdependent internal SSS and potential BE:

$$P_{i1}u_o + P_{i2}v_o + \sum_{k=3,4}^{n+2} P_{ik}w_{k-2} = P_{iq}q + P_{ip}p, \ (i = 1, 2, ..., n+2), \quad (21)$$

where P with indices are differential operators.

System (21) is not divided into independent systems of internal SSS and potential BE. This means that the curvatures of the shell affect the interdependence of the internal SSS and the potential BE. For the plates of the system of equations of internal SSS and potential BE are separated [31, p. 84; 32, p. 54].

The systems of solving equations (19) – (21) are reduced to more convenient systems of DE.

DE (19) is reduced to one DE (order $(n+1)$) with respect to the new function $\psi_c(x, y)$:

$$H_c \psi_c(x, y) = 0, \quad (22)$$

where H_c is the differential determinant of system (19):

$H_c = (\nabla^2 - a_1)(\nabla^2 - a_3)...(\nabla^2 - a_n)$, a_i $(i = 1, 3, ..., n)$ –MGP.

The general solution of DE (22) is defined as:

$$\psi_c(x, y) = \sum_{i=1,3}^{n} \psi_{ci}(x, y), \quad (23)$$

where $\psi_{ci}(x, y)$ are the general solutions of DE Helmholtz:

$$(\nabla^2 - a_i)\psi_{ci}(x, y) = 0, \ (i = 1, 3, ..., n). \quad (24)$$

The general solutions of system (19), taking into account (22) – (24), will be as follows:

$$\psi_j(x, y) = H_{c1j}\psi_c(x, y), \ (j = 1, 3, ..., n), \quad (25)$$

where H_{c1j} is the adjuncts of the differential determinant of system (19).

Similarly, system (20) is reduced to one DE of order with respect to the new function $\psi_s(x, y)$:

$$H_s \psi_s(x, y) = 0, \quad (26)$$

where H_s is the differential determinant of system (20):

$H_s = (\nabla^2 - b_2)(\nabla^2 - b_4)...(\nabla^2 - b_{n-1})$; b_i $(i = 2, 4, ..., n-1)$ –MGP.

Numerical studies show that in the approximations K01, K0-3, K0-5 the a_i, b_i parameters are positive numbers.

The general solution of DE (26) will be:

$$\psi_s(x, y) = \sum_{i=2,4}^{n-1} \psi_{si}(x, y), \qquad (27)$$

where $\psi_{si}(x, y)$ are the general solutions of DE Helmholtz:

$$h / a = 0,2; R_{1,2} / a = 5; \; G' / G = 0,1; E' / E = 0,1; \nu' = 0,03; \nu = 0,3; \; m = n = 1. \qquad (28)$$

The general solutions of system (20) taking into account (26) – (28) will take the form:

$$\psi_j(x, y) = H_{s1j}\, \psi_s(x, y), \; (j = 2, 4, ..., n-1), \qquad (29)$$

where H_{s1j} is the adjuncts of the differential determinant of the system (20).

The dependencies (25) and (29) are the forms of common solutions of DE vortex BE.

We reduce the system DE (21) to the system of relatively new required functions $D_i(x, y)$. To do this, we represent the components of the operator component of the displacement through these functions:

$$\begin{aligned} u_0(x, y) &= \sum_{i=1,2}^{n+2} P_{i1}^0 D_i(x, y); \; v_0(x, y) = \sum_{i=1,2}^{n+2} P_{i2}^0 D_i(x, y); \\ w_{k-2}(x, y) &= \sum_{i=1,2}^{n+2} P_{ik}^0 D_i(x, y), \; (k = 3, 4, ..., n+2), \end{aligned} \qquad (30)$$

where P_{ik}^0 is the adjuncts of system (21). To determine the required functions $D_i(x, y)$ we obtain a system of DE of order $4(n+1)$:

$$P\, D_i(x, y) = P_{iq} q(x, y) + P_{ip} p(x, y), \; (i = 1, 2, ..., n+2), \qquad (31)$$

in which P is the differential determinant of system (21).

The general solutions of system (31) will appear as:

$$D_1(x, y) = D_{10}(x, y) + D_{1,r}(x, y); \; D_i(x, y) = D_{i,r}(x, y), \; (i = 2, 3, ..., n+2), \; (32)$$

where $D_{10}(x, y)$ is the general solution of the homogeneous equation $PD_1(x, y) = 0$;

$D_{1,r}(x, y), ..., D_{(n+2),r}(x, y)$ – partial solutions of inhomogeneous DE (31).

Taking into account (30) and (32) we obtain:

$$u_0(x, y) = P_{11}^0 D_{10}(x, y) + \sum_{i=1}^{n+2} P_{i1}^0 D_{i,r}(x, y); \; v_0(x, y) = P_{12}^0 D_{10}(x, y) + \sum_{i=1}^{n+2} P_{i2}^0 D_{i,r}(x, y);$$

$$w_{k-2}(x,y) = P_{ik}^{0} D_{10}(x,y) + \sum_{i=1}^{n+2} P_{ik}^{0} D_{i,r}(x,y). \tag{33}$$

Formulas (33) determine the forms of the general solution of the system (21) of internal SSS and potential BE. Components $u_1, v_1, u_2, \dots,, u_n, v_n$ are determined from the system of equilibrium DE (10). The stress components are given by formulas (18).

The equilibrium system of DE (10) can be solved analytically by methods of double and single trigonometric series depending on the boundary conditions of the shell contour [33, p. 92], by the method of perturbation of the geometric parameters of the shell [34, p. 137], by methods of integral transformations. The perturbation method leads to the solution of the recurrent sequence of two DE systems, the right parts of which in each approximation to a small parameter depend on the solution in the previous approximation. The perturbation method can also be applied to the DE (31) determination system since the vortex equations BE (22) and (26) are solved directly. Integral conversion methods are also better applied to system (31) than to DE (10). This makes it much easier to find common solutions.

6.2. The approximations K01, K0-3, K0-5. We obtain all transformed equations from equations (10), (19) – (33) if we put in them $k = 1$ (in approximation K01), $k = 3$ (in approximation K0-3), $k = 5$ (approximation K0-5). The SSS components for these approximations will be determined by formulas (18) with the corresponding k. Expanded ratios and basic equations to approximate K0-5 are given in [32, p. 67].

7. Numerical results and their analysis

Based on the obtained systems of differential equations for different approximations (19) – (21), we investigate the SSS of transstropic shells ($a \times b \times h$) of small curvature at boundary conditions Navier from the effect of transverse loading

$$q(x,y) = q_{mn} \sin\frac{m\pi x}{a} \sin\frac{n\pi y}{b}, \quad p(x,y) = p_{mn} \sin\frac{m\pi x}{a} \sin\frac{n\pi y}{b}$$

(q_{mm}, p_{mn} –const) for different MGP $a / h, G'/ G, E'/ E, \nu', \nu, R / a, k'_{12}, m, n$ at $p_{mn} / q_{mn} = 0, p_{mn} / q_{mn} = 1$. The internal SSS components were found by classical theory and by the approximations K01, K0-3, K0-5, using the corresponding dependencies for these approximations. Based on the developed algorithms, we investigate the SSS of the square in the plane of the transtropic shells of different thickness at smooth and non-smooth loads for a wide class of MGP. This provided an in-depth analysis of the impact of MGP and the type of load on SSS, as well as the convergence of results and their accuracy, depending on the approximation of the MT variant. Hereafter $\tilde{\sigma}_x = \sigma_x / q$; $\tilde{W} = WE / (qh)$; $\tilde{z} = z / h$. SSS at skew-load is characterized by the parameter $p_{mn} / q_{mn} = 0$, and when loaded on the upper face plane – by the parame-ter $p_{mn} / q_{mn} = 1$; Δ with lower indices means a corresponding difference in percentages. The lines in the graphs correspond to: is the approximation of K0-5 (or K135); – K0-3 (or K13); – K01 (or K1); – Classical Theory (CT).

Figures 1,2 show the graphs of the dependencies of the SSS components at $R_{1,2} / a = 5$; $G' / G = 0,1$, which characterize the nonlinearity of the SSS and the difference between the results of the approximations. The following results are for $E' / E = 1$, $\nu' = \nu = 0,3$.

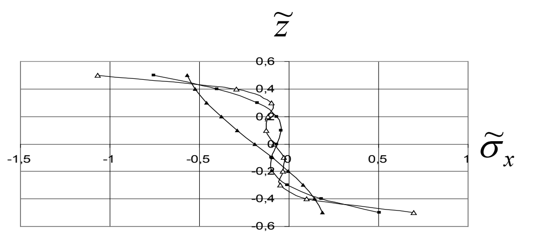

Figure 1. The stress component $\tilde{\sigma}_x$ $(h / a = 0,1; m = n = 9; p_{mn} / q_{mn} = 1)$

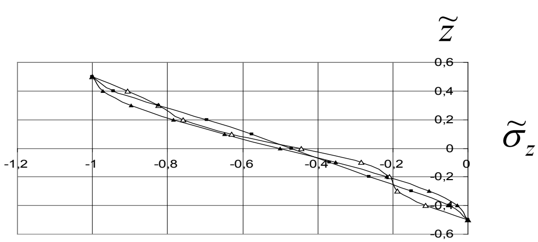

Figure 2. The stress component $\tilde{\sigma}_z$ $(h / a = 0,1;\ m = n = 9; p_{mn} / q_{mn} = 1)$.

Tables 1-4 characterize the components of the SSS depending on the approximation, the MGP, the type of load, and the consideration of the curvatures ($k'_{1,2} = k_{1,2}$) in the transverse shear deformations. In the table. 1, 2 Δ characterizes the effect of $k'_{1,2}$ curvature on SSS. A practical analysis of the series convergence for the SSS components is performed. Additional numerical studies show a high convergence of results with a skew-symmetric, slowly varying loading region even for thick isotropic ($h / a = 0,5;\ R_{1,2} / a = 5;\ \nu = 0,3$) shells.

Table 1

Components of the SSS of the transtropic shell

($h / a = 0,2; a = b; R_{1,2} / a = 5;\ G' / G = 0,1;\ m = n = 1; p_{mn} / q_{mn} = 1;$
$k'_{1,2} \neq 0 / k'_{1,2} = 0$)

z/h	K01	K0-3	K0-5	Δ %	Δ_{31}%	Δ_{53}%
	$\tilde{\sigma}_x$					
-0,5	3,479 3,444	4,844 4,832	4,841 4,832	0,19	28,2 -	0,06 -
W	$\tilde{W}$					
-0,5	-52,17 -51,91	-50,83 -50,59	-49,52 -49,28	0,48	2,64 -	2,65 -

Table 2

SSS components of the isotropic shell ($z / h = 0,5$;
$h / a = 0,5; a = b;\ R_{1,2} / a = 29 / 40;\ G' / G = 1;\ m = n = 1; p_{mn} / q_{mn} = 0;$
$k'_{1,2} \neq 0 / k'_{1,2} = 0$)

SSS	CT	K01	K0-3	K0-5	Δ %	Δ_{1k}%	Δ_{31}%	Δ_{53}%
$\tilde{\sigma}_x$	-0,7789	-0,9052 -0,9038	-0,9740 -0,9091	-0,9921 -0,9267	6,59	14,0 13,8	7,06 0,58	1,82 1,90
$\tilde{W}$	-0,3696	-0,8225 -0,6840	-0,7379 -0,5982	-0,7485 -0,6118	18,3	55,1 46,0	11,5 14,3	1,42 2,22

To determine the SSS for non-smooth loads on the shell region, higher approximations (K0-5 and higher) should be taken into account. For thick transtropic shells ($h / a \geq 0,5;\ R_{1,2} / a \leq 1$), the dependence of the transverse shear deformations on the curvatures must be taken into account, and symmetrical components in the displacement components should be taken into account in the skew-symmetrical loading (Table 3) (the difference is characterized by the magnitude Δ'). Transverse crimping can significantly

Table 3

Components of the SSS of the transtropic shell

$(h / a = 0,5;\ a = b;\ R_{1,2} / a = 29 / 40;\ G' / G = 0,1;\ m = n = 1;\ k'_{1,2} \neq 0;\ p_{mn} / q_{mn} = 0)$

$\frac{z}{h}$	K01	K013 K0-3 Δ' %	K0135 K0-5 Δ' %	K01	K013 K0-3 Δ' %	K0135 K0-5 Δ' %
	$\tilde{\sigma}_x$			$\tilde{W}$		
0,5	-0,874	-1,052 -0,939 12,0	-1,129 -1,013 11,5	-1,690	-1,726 -1,562 10,5	-1,719 -1,568 9,63

Table 4

Components of the SSS of the transtropic shell

$(z / h = -0,5;\ h / a = 0,1;\ a = b;\ R_{1,2} / a = 5;\ G' / G = 0,1;\ m = n = 9;$
$k'_{1,2} \neq 0;\ p_{mn} / q_{mn} = 0\ /\ p_{mn} / q_{mn} = 1)$

SSS	K01	K0-3	K0-5	Δ_{pq}%	Δ_{31}%	Δ_{53}%	Δ_{51}%
$\tilde{\sigma}_x$	0,3595 0,1868	0,6126 0,5042	0,8815 0,6982	26,3	41,3 63,0	30,5 27,8	59,2 73,2
$\tilde{W}$	-1,979 -1,977	-1,886 -1,659	-1,852 -1,586	16,8	4,93 19,2	1,84 4,60	3,94 24,7

affect the SSS not only for thick, but also for shells of medium thickness ($h / a = 0,1 \div 0,2$), especially for non-smooth loads on the shell region (Table 4). The magnitude of Δ_{pq} characterizes the difference between the results when the transverse compression is taken into account and not taken into account.

Tables 5-7 compare the components of the SSS for the shells with the exact components for the respective plates (Δ_{po} indicates the difference between the results for the plates and the shells with respect to the plates). The bold text in these tables shows MGP and loads, at which the shells of small curvature can be replaced by corresponding plates with an accuracy of up to 4%.

For isotropic shells, the discrepancy with the plates is less than 3.84%. This makes it possible to replace the calculation of shells in some cases with appropriate plates.

Table 5

$\tilde{\sigma}_x$ values for isotropic shells and plates

$(G'/G = 1;\ E'/E = 1;\ \nu' = \nu = 0,3;\ m = n = 1;\ k'_{1,2} \neq 0;\ p_{mn}/q_{mn} = 0\ /\ p_{mn}/q_{mn} = 1)$

$\frac{h}{a}$	$\frac{z}{h}$	Plate (The exact solution)	Shell $\frac{R_{1,2}}{a} = 10$	Δ_{po} %	Shell $\frac{R_{1,2}}{a} = 20$	Δ_{po} %	Shell $\frac{R_{1,2}}{a} = 40$	Δ_{po} %
$\frac{1}{3}$	0,5	-1,995 -2,125	-2,040 -2,128	2,26 0,14	-2,019 -2,127	1,20 0,09	**-2,007** **-2,126**	**0,60** **0,05**
	-0,5	1,995 1,865	**1,936** **1,766**	**2,96** 5,31	**1,966** **1,816**	**1,45** **2,63**	**1,981** **1,840**	**0,70** **1,34**
$\frac{1}{5}$	0,5	-5,148 -5,244	**-5,309** **-5,330**	**3,13** **1,64**	**-5,239** **-5,298**	1,77 1,03	-5,196 -5,274	0,93 0,57
	-0,5	5,148 5,052	4,913 4,749	4,56 6,00	**5,040** **4,909**	**2,10** **2,83**	**5,096** **4,983**	**1,01** **1,37**
$\frac{1}{10}$	0,5	-20,02 -20,10	**-20,82** **-20,75**	**4,00** **3,23**	**-20,54** **-20,55**	**2,60** **2,24**	**-20,29** **-20,33**	**1,35** **1,14**
	-0,5	20,02 19,94	18,00 17,79	10,1 10,8	19,10 18,95	4,60 4,96	**19,57** **19,45**	2,25 2,46

Table 6

$\tilde{\sigma}_x$ values for transtropic shells and plates

$(G'/G = 0,1;\ E'/E = 1;\ \nu' = \nu = 0,3;\ m = n = 1;\ k'_{1,2} \neq 0;\ p_{mn}/q_{mn} = 0\ /\ p_{mn}/q_{mn} = 1)$

$\frac{h}{a}$	$\frac{z}{h}$	Plate (The exact solution)	Shell $\frac{R_{1,2}}{a} = 10$	Δ_{po} %	Shell $\frac{R_{1,2}}{a} = 20$	Δ_{po} %	Shell $\frac{R_{1,2}}{a} = 40$	Δ_{po} %
$\frac{1}{3}$	0,5	-3,190 -3,299	-3,360 **-3,393**	5,33 **2,85**	-3,278 -3,349	2,76 1,52	**-3,226** **-3,316**	**1,13** **0,52**
	-0,5	3,190 3,081	2,871 2,702	10,0 12,3	3,031 2,890	4,98 6,20	**3,102** **2,978**	**2,76** **3,34**
$\frac{1}{5}$	0,5	-6,593 -6,685	-6,981 -6,973	5,89 4,31	**-6,823** **-6,866**	**3,49** **2,71**	-6,715 -6,783	1,85 1,47
	-0,5	6,593 6,502	5,921 5,748	10,2 11,6	6,285 6,150	4,67 5,41	**6,445** **6,332**	**2,24** **2,61**
$\frac{1}{10}$	0,5	-21,57 -21,65	-22,75 -22,67	5,47 4,71	**-22,41** **-22,41**	**3,89** **3,51**	**-22,05** **-22,09**	**2,23** **2,03**
	-0,5	21,57 21,49	18,62 18,41	13,7 14,3	20,28 20,12	5,98 6,38	**20,98** **20,86**	2,74 2,93

Table 7

$\tilde{W}$ values for transtropic shells and plates

($G'/G = 0,1;\ E'/E = 1;\ \nu' = \nu = 0,3;\ m = n = 1;\ k'_{1,2} \neq 0;\ p_{mn}/q_{mn} = 0\ /\ p_{mn}/q_{mn} = 1$)

$\frac{h}{a}$	$\frac{z}{h}$	Plate (The exact solution)	Shell $\frac{R_{1,2}}{a} = 10$	Δ_{po} %	Shell $\frac{R_{1,2}}{a} = 20$	Δ_{po} %	Shell $\frac{R_{1,2}}{a} = 40$	Δ_{po} %
$\frac{1}{3}$	**0,5**	**-15,56** **-15,79**	**-15,25** **-15,09**	**1,99** **4,43**	**-15,47** **-15,50**	**0,58** **1,84**	-15,54 -15,67	0,06 0,76
	0,5	-15,56 -15,34	-15,42 -14,80	0,90 3,52	-15,54 -15,13	0,13 1,37	-15,55 -15,26	0,06 0,52
$\frac{1}{5}$	0,5	-55,15 -55,38	-53,87 -53,29	2,32 3,77	-54,78 -54,60	0,67 1,41	-55,04 -55,06	0,20 0,58
	-0,5	**-55,15** **-54,92**	**-54,22** **-53,18**	**1,69** **3,17**	**-54,96** **-54,32**	**0,34** **1,09**	**-55,13** **-54,70**	**0,04** **0,40**
$\frac{1}{10}$	0,5	-432,8 -433,0	-414,7 -411,9	4,18 4,87	**-428,0** **-426,6**	**1,11** **1,48**	**-431,5** **-430,9**	**0,30** **0,48**
	-0,5	-432,8 -432,5	**-416,0** -412,7	**3,88** 4,58	**-428,6** **-426,8**	**0,97** **1,32**	**-431,8** **-430,8**	0,23 0,39

8. Conclusions

Based on studies of the MT variant of transversely isotropic shells of small curvature of arbitrary thickness, the following conclusions are obtained.

Derived DE systems and boundary conditions in general form and as special cases for different approximations. The obtained DE systems take into account the shear deformations of the transverse shear, which can significantly affect the SSS of the shells. The SSS of such shells in each approximation is determined by the solution of systems of interdependent differential equations with partial derivatives.

Forms of general solutions of the DE equilibrium system are constructed in arbitrary approximation. Approaches K01, K0-3, K05 are considered as special cases. Highlighted equations that describe vortex BE. Internal SSS and potential BE in the K0-n (n odd, $n \geq 3$) approximations are determined by the interdependent DE systems.

Analytical solutions of boundary value problems of MT variant in double trigonometric series are constructed. The boundary value problems for determining the internal SSS for a wide class of MGP in different approximations are solved. The convergence of results is generally improved by reducing the thickness, the shearability of the material in shear, and by reducing the curvature of the middle surface.

SSS in the scope of the BE, with non-smooth and local loads, should be determined by high MT approximations. The results obtained from low approximations may differ significantly from the exact ones. Tymoshenko-Reisner-type theories satisfactorily describe the SSS of thin, low-curved shells with low transverse shear at smooth loads. The K0-3 approximation describes the SSS of thin shells and medium-thickness shells with high accuracy over a wide range of MGP variations. At non-smooth loads of approximation K01, K0-3 can give unsatisfactory results not only for medium thickness shells, but also for thin shells. This indicates the need to use higher approximations.

The internal SSS of the shells depends essentially on the nature of the variability of the transverse load, the curvature of the middle surface, the thickness, and the susceptibility to transverse displacement. The accuracy of the approximations increases with decreasing thickness, curvature, lateral displacement, and increasing the transverse smoothness.

The constructed variant of MT of transversely isotropic shells of small curvature of arbitrary thickness makes it possible to solve different classes of boundary value problems with high accuracy based on the analysis of the convergence of numerical results.

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