Heorhii Slukin
Department of Economic Cybernetics and Applied Economics,
V.N. Karazin Kharkiv National University, Kharkiv, Ukraine (corresponding author)
E-mail: slukingeorge@gmail.com
ORCID: https://orcid.org/0009-0005-8308-5533
ResearcherID: JXN-1340-2024
Vitalii Danich
Department of Economic Cybernetics and Applied Economics,
V.N. Karazin Kharkiv National University, Kharkiv, Ukraine
E-mail: danichvitaly@gmail.com
ORCID: https://orcid.org/0000-0001-6147-2309

Algebra of Social Phenomena

Abstract
The characteristics of modern societies are central to discussions of various types of policy (social, economic, etc.) and are often seen as predictors of socio-economic processes. Any society can be studied through the lens of social phenomena. Hence, it seems fair to assume that the better one understands the nature of social matter and its impact on various processes, the better one's decisions will be able to meet today's challenges. The *purpose* of this paper is to explain and present the new framework for measuring social phenomena in the form of an algebra based on the definition of social phenomenon, its components, rationality and mathematical logic. *Methodology*. Using the example of social cohesion, the paper examines the current fundamental methodological approaches to interpreting the definition and, consequently, measuring social phenomena. In this paper, the authors use an analogy with probability space and event algebra to develop their mathematical method. As a result, the article presents and explains the basic terms of the algebra of social phenomena that make up the new concept. *Practical implications*. The authors' approach to formalisation provides a tool for deriving the measure of a complex phenomenon from the measures of its components (simple phenomena), as demonstrated in the example. Thus, this study enriches the range of methods suitable for deepening one's knowledge of the social characteristics of the society under study. *Value/Originality*. The concept of algebra as a methodology for measuring social phenomena and establishing dependencies between them on the basis of mathematical logic has not yet been the subject of public discussion. Therefore, this publication is also an invitation to a scientific discussion.

Keywords
social phenomenon, algebra, socio-economic modelling, social cohesion, mathematical formalisation, measurement

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1 Introduction

Nowadays, there is a growing debate about social cohesion, sustainability and other characteristics of society that are shaped and manifested through human interaction. The current trend in social phenomena research can be described as linking certain social phenomena to observed economic, political, educational, medical or other processes in order to discover a causal effect. This can be seen in recent studies that found overcrowding to be a strong predictor of low educational attainment, lower learning outcomes, health problems, neurological and lack of vocational skills, leading to worsening socio-economic status (Khera, Yusuf & Al Mughairbi, 2023). The strong positive relationship between social cohesion and volunteering has also been identified recently (Horsham, Abrams, Davies, Lalot, 2023). The review also signals a deeper analysis of different forms of social capital, as they may respond differently to policies aimed at promoting volunteering through the development of social cohesion.

Defining and measuring social phenomena is a challenging task that opens up new possibilities for understanding their impact on political and economic processes and thus improving the accuracy of forecasts. For this reason, different models and methodologies have been developed to deal with social cohesion, poverty, happiness, inequality,

national identity and other phenomena. In other words, it is only rational to formalise the object for research purposes. The formalisation of social phenomena in the form of an algebra is the new discussion proposed by this paper.

The study includes the following steps in order to analyse the sociological basis and present a new formal framework:

1. Section 2 discusses the definition of a social phenomenon, the main problems of its measurement and the main modern methodological approaches to this process.

2. Section 3 introduces the algebra of social phenomena, which includes such concepts as the measure of a phenomenon, the space of measures of phenomena, the universal phenomenon, the negation of a phenomenon, and the simple and complex phenomenon. For the purpose of illustrating the concept, an example of deriving a measure of a complex phenomenon from measures of simple phenomena is given.

3. Possible directions for further development of the ideas described in this paper are discussed in Section 4.

4. Lastly, in Section 5, the paper concludes to facilitate discussion and lay the groundwork for further research.

2 The Complex Problem of Researching Social Phenomena

A social phenomenon is the main object of research in sociology and other social sciences, including socio-economic sciences. In order to work properly with this object, it is necessary to have a clear and generally accepted definition of the concept.

The following definitions are given. In particular, a social phenomenon is one of the fundamental concepts of sociology and social philosophy, which means an element of social reality that has the fullness of social properties and characteristics; it is any manifestation of relations or interaction of people, or even a single event or case, implying that everything that manifests itself in social reality exists. Social phenomena are the phenomena of human interaction that take place in social space: directly in a contact group or indirectly through the involvement of individuals in communities, social organisations and institutions (Dvoretska, 2002).

It should be emphasised that a social phenomenon is everything that manifests itself, exists and is present in social reality. It should be noted that a single event or case can also be interpreted as a phenomenon, because it has happened, has taken place and has been perceived by people.

A social phenomenon as an object of study poses a number of problematic circumstances for the researcher. Human nature makes it difficult to predict the consequences of certain events, and it is also influenced by a large number of factors that require careful selection. In addition, the possibility of conducting experiments is limited for ethical reasons compared to research in the natural sciences.

Sociological research reveals the essence of phenomena, identifies and analyses social trends and patterns of development (Horodianenko, 2008). The methods of mathematical statistics are widely used in these studies. One of the main tasks of such research is to reveal factors that influence the characteristics of a phenomenon, determine its state, dynamics, influence certain phenomena and bring them to the desired state. The next step is to build mathematical models of the dependence of the characteristics of the phenomenon on these factors.

This problem can be solved in different ways. Correlation methods are most often used, although in most cases they are not supported by the "physics" of social processes and the nature of social relations.

It would also not be rational to reduce the assessment of a complex to a single indicator. The reason for this is the lack of context, which can lead to erroneous conclusions. A good example is the study on social cohesion and extremism in Toulouse Métropole (Berner, Bertrand, 2023). The paper notes that the GINI index in Reynerie (a suburb of Toulouse) is quite low, which means that income distribution is fairly even and signals a low risk of threats to social cohesion. However, it can be said that "[...] living in Reynerie, a neighbourhood with a high poverty rate and a low GINI index, means being relatively equal with one's neighbours but suffering inequalities with the rest of the population of the metropolis or the most affluent part (Toulouse Métropole 2022). This example shows that the use of the GINI index alone leads to false conclusions regarding the relationship between income and the emergence of radicalization." (Berner, Bertrand, 2023: 81)

The phenomenon of social cohesion is an excellent example. The phenomenon is dynamic, i.e., it is constantly changing, and its definitions vary considerably from one researcher to another. As the analysis of the sources has shown, scholars mention different components of cohesion, such as primary and secondary social ties (Cooley, 1909; Granovetter, 1973), emotional ties (Freud, 1921; Lott, Lott, 1966), the presence of shared attitudes, rules, behaviours, institutions (Green, Janmaat, 2011), a common "moral compass" (Larsen, 2013) or willingness to reconcile differences (Fonseca, Lukosch & Brazier, 2019), willingness to cooperate for the sake of survival and prosperity (Homans, 1958; Stanley, 2003), an environment that does not inhibit cohesion (Lewin, 1946), shared values and room for interpretation (Maxwell, 1996), levels of trust among members of society and/or between individuals and institutions (Lockwood, 1999), a sense of interdependence and interconnectedness

(Durkheim, 1897), and equality of income/wealth (Putnam, 1993).

Social cohesion can be described as a phenomenon of unity around a common goal or idea. The technical key to the formation of cohesion is the ability of a social group to form and use social ties. An example of an idea is buying services/goods from a certain company or supporting certain policies/politicians, and an example of a technical implementation is an information campaign.

Using social cohesion as an example, it is useful to draw attention to the variety of methods used to measure social phenomena. They can be summarised in three approaches:

1. Factor-based measurement. The purpose of the factor approach is to identify the factors of the phenomenon under study, as well as the level and nature of their influence. For this purpose, methods of analysing the subject area, conducting sociological research and various statistical methods of processing the collected data are used.

2. Assessment of a phenomenon by its consequences. This methodological approach assumes that it is possible to identify the features of a society in which a certain social phenomenon is fully present. By collecting and processing information about the identified indicators, an assessment of the degree of presence of the phenomenon itself is formed.

3. Analysis of social networks. To understand this area of research, it is important to remember that any social group (society on a larger scale) can be represented as a graph, where people are nodes and the connections between them are edges. Modelling network structures allows to study the peculiarities of information exchange and mutual influence of participants. This is where the analogy between cohesion and connectivity comes from.

However, none of these approaches presents a system of clear dependencies between the assessment (measure) of phenomena and the assessment (measure) of a complex phenomenon. This paper proposes an approach that is based on the nature of social relations, on the definition of components of social phenomena, rationality and mathematical logic.

3 Algebra of Social Phenomena as a Sigma Algebra

The authors' approach uses an analogy with the probability space and event algebra to model the dependence of the probability of a complex event on the probabilities of its components. Event probability models are built on the basis of the σ-algebra (sigma-algebra) of events and the probability measure. The concept of the space of elementary events is introduced (Kartashov, 2007).

As for phenomena, the situation is much more complicated. Firstly, there is no clear definition of the concept of an elementary phenomenon and, accordingly, of the set or space of elementary phenomena. Secondly, the concept of a phenomenon measure is more uncertain than a probabilistic measure based on the frequency of certain events. At the same time, there is a rather natural approach to the measure of a phenomenon as a share of support for this phenomenon among the subjects of a certain community.

The authors introduce the concept of the space of measures of phenomena, based on the concepts of σ-algebra and the measure of a phenomenon.

Regarding the definition in question:

(Ω, F, P) the space of the measure of phenomena, where

Ω is an arbitrary set of phenomena, which is further called the space of basic phenomena

F is the σ-algebra of subsets of Ω

P is a measure of phenomena, i.e., a sigma-additive finite measure, such that

$P(E) \le 1, \ \forall E \in F$.

A set F of subsets Ω is called a σ-algebra if it meets the following properties:

1. F contains a set of Ω.

2. If $E \in F$, then its complement $\Omega \backslash E \in F$

3. The union or intersection of a countable subfamily with F belongs to F, i.e.,

$$\bigcup_{i=1}^{\infty} E_i \in F \ , \ \bigcap_{i=1}^{\infty} E_i \in F$$

The measure of a phenomenon E $P(E)$ is the proportion of social actors S that support (consider positively) the phenomenon E. $S(E)$ denotes the subset of actors S that support the phenomenon E:

$$P(E) = \frac{\mu(S(E))}{\mu(S)} \quad (1)$$

where μ(S(E)) is the value of the set, i.e., the number of social actors who support (have a positive attitude) to the phenomenon.

μ(S) is the value of the set, i.e., the total number of social subjects.

Consider the operations of the σ-algebra and define the measure of the result of these operations:

$E_u = E_1 \cup E_2$ is a phenomenon that is a combination of two phenomena (either E_1 or E_2).

$E_i = E_1 \cap E_2$ is the intersection of two phenomena (phenomena E_1 and E_2).

The measure E_u is the ratio of the sum of the number of items in the set S(E_1) and the set S(E_2) minus the number of items that are at the intersection of S(E_1) and S(E_2), i.e.,

$$P(E_u) = \frac{\mu(E_1) + \mu(E_2) - \mu(E_i)}{\mu(S)} \quad (2)$$

$$P(E_i) = \frac{\mu(E_1 \cap E_2)}{\mu(S)} \quad (3)$$

The measure $P(E)$ can be called the measure of support P_p and similarly define the measure of negation P_d and the measure of neutrality P_n. In this case,

$P_p + P_d + P_n = 1$.

Introduce the concept of a universal phenomenon (a generally accepted phenomenon that includes all phenomena) – U. $P(U) = 1$. It is important to note that the universal phenomenon is an abstraction that is almost never encountered in practice.

The negation of the phenomenon E, $(\bar{E})$, is a phenomenon that is a complement to the phenomenon U.

$$\bar{E} = U / E,\ E + \bar{E} = U,\ P(U) = P(E) + P(\bar{E}) = 1 \quad (4)$$

Introduce the concept of a complex phenomenon as one that is formed by a combination of simple phenomena and operations (conjunction, disjunction) over them. Accordingly, the problem of finding the measure of a complex phenomenon from the measures of simple phenomena arises. The authors' approach allows to solve this problem.

E is the phenomenon under consideration.

$\bar{E}$ is a complementary phenomenon.

U is a universal phenomenon that is always supported by everyone.

$$E = E_1 \cup E_2 \cup \ldots \cup E_n \text{ or } E = E_1 + E_2 + \ldots + E_n \quad (5)$$

$$E + \bar{E} = U \quad (6)$$

$$P(E) = P\left(U - \overline{\sum_{i=1}^{n} E_i}\right) = P(U) - P\left(\overline{\sum_{i=1}^{n} E_i}\right) \quad (7)$$

$$P(U) = 1 \quad (8)$$

$$P(E + \bar{E}) = P(E) + P(\bar{E}) \quad (9)$$

The phenomena E and $\bar{E}$ are incompatible.

$$\text{Then } P(E) = 1 - P\left(\overline{\sum_{i=1}^{n} E_i}\right) \quad (10)$$

The negation of the sum is equal to the product of the negations, i.e.,

$$\overline{\sum_{i=1}^{n} E_i} = \prod_{i=1}^{n} \overline{E_i} \quad (11)$$

Hence, it follows:

$$P(E) = 1 - P\left(\prod_{i=1}^{n} \overline{E_i}\right) \quad (12)$$

The measure of a product of phenomena is equal to the product of measures if the phenomena are independent. Thus,

$$P(E) = 1 - \prod_{i=1}^{n} P(\overline{E_i}) = 1 - \prod_{i=1}^{n} \left[1 - P(E_i)\right] \quad (13)$$

This formula allows to express the measure of a complex phenomenon through the measure of its components.

Here is an example of a complex phenomenon.

A is a phenomenon of support for candidate A in social group N.

n is the total number of subjects in group N.

Support is the proportion of group members who support a candidate.

a is the number of subjects who support candidate A.

$$P(A) = \frac{a}{n} \quad (14)$$

B is a phenomenon of support for candidate B in social group N.

$$P(B) = \frac{b}{n} \quad (15)$$

C is a phenomenon of support for candidate C in social group N.

$$P(C) = \frac{c}{n} \quad (16)$$

D is a phenomenon of support for candidate A or support for candidate B and candidate C.

$$D = A \cup B \cap C \text{ or } D = A + B * C$$

$$D = U - \overline{A \cup B \cap C} \quad (17)$$

Calculate the measure of the phenomenon D:

$$P(D) = 1 - P\left(\overline{A \cup B \cap C}\right) = 1 - P\left(\bar{A} \cap \overline{B \cap C}\right) =$$

$$= 1 - P(\bar{A}) * P\left(\overline{B \cap C}\right) = 1 - \left[1 - P(A)\right] * \left[1 - P(B \cap C)\right] =$$

$$= 1 - \left[1 - P(A)\right] * \left[1 - P(B) * P(C)\right] =$$

$$1 - 1 + P(B) * P(C) + P(A) - P(A) * P(B) * P(C) =$$

$$= P(A) + P(B) * P(C) - P(A) * P(B) * P(C) \quad (18)$$

Therefore, the measure of phenomenon D corresponds to the sum of the measure of phenomenon A and the intersection of the measures of phenomena B and C, except for the intersection of the measures of all three phenomena A, B and C. Hence, this example provides a formula for calculating the measure of a complex phenomenon through the measures of its components.

4 Discussions

Along with the logic and consistency of the formulas of the algebra of social phenomena, there are still questions that need to be answered. It is obvious that the totality of social phenomena includes many elements. An important task in the practical application of this approach is to identify the links between phenomena. The solution to this problem requires a thorough analysis of scientific literature and the study of international experience, which allows to identify patterns of social life. In this regard, it is advisable to analyse scientific works devoted to the study of social patterns.

The issue of correlating theoretical calculations with reality is also relevant, as it is necessary to check whether the hypotheses and forecasts put forward are true. Certainly, for the phenomenon of support, this can be done with the help of a sociological survey, in-depth interviews, and focus groups. However, the answer to the question of the completeness of this toolkit requires further research.

Finally, in formula (5), it makes sense to adjust the sum of the components of a social phenomenon using weighting factors. Accordingly, the measure of the phenomenon E can be represented as follows:

$$P(E) = q_1 * P(E_1) + q_2 * P(E_2) + \ldots + q_n * P(E_n) \quad (19)$$

Determining the importance of each component is a research task related to identifying the nature of the relationships between social phenomena.

5 Conclusions

The article analyses the definition of a social phenomenon and identifies the difference between the concepts of a phenomenon and an event. The features of studying social phenomena and the problems faced by researchers are outlined. Using the example of the complex phenomenon of social cohesion, this paper demonstrates the diversity of scientific approaches to defining its components, analyses and identifies three fundamental methodological approaches to measuring social cohesion: the factor method, outcome-based assessment and social network analysis. On the basis of the brief review of methodological approaches, it is concluded that it is appropriate to consider the algebra of social phenomena as a system of clear dependencies between estimates (measures) of phenomena and an estimate (measure) of a complex phenomenon.

The presented algebra of social phenomena is based on the nature of social processes and a partial analogy with the probability space and event algebra. Building a complete analogy is difficult due to the lack of the concept of an elementary phenomenon. In this article, the measure of a social phenomenon is understood as the share of support for the phenomenon among the subjects of a certain community. The concepts of social phenomenon, measure of social phenomenon, space of measures of phenomena, universal phenomenon, additional phenomenon, complex phenomenon are introduced within the framework of algebra. The operations of logical negation, union and intersection of phenomena are described. The analogy with the theory of probability allows to derive formulas for calculating the measure of a complex phenomenon from the measures of simple phenomena. The basics of the algebra of social phenomena are presented on the example of electoral support for abstract candidates.

To summarise, algebraisation involves certain discussion points that require their definition. As a result, in this paper, the objective is to identify the links between social phenomena, dealing with the issue of correlating theoretical calculations with reality, as well as adjusting the sum of the components of a social phenomenon using weighting coefficients (19). These promising tasks are yet to be addressed in further work. At the same time, the algebraic approach opens up new opportunities for establishing relationships between social phenomena, analysing and modelling them.

References

[1] Berner, Heiko & Bertrand, Romain (2023). Social Cohesion. A Basis for the Primary Prevention of Radicalisation in Cities. *Journal for Deradicalization,* 37, 72–104.

[2] Cooley, Charles Horton (1909). Primary Groups. *Social Organization: A Study of the Larger Mind*, 23-31. New York, NY: Charles Scribner's Sons.

[3] Durkheim, Emile (1897). *Suicide: a study in sociology*. New York, NY: F. Alcan.

[4] Dvoretska, H. V. (2002). *Sociology.* Kyiv: KNEU.

[5] Xavier Fonseca, Stephan Lukosch & Frances Brazier (2019). Social cohesion revisited: a new definition and how to characterize it. *Innovation: The European Journal of Social Science Research,* 32:2, 231–253. DOI: https://doi.org/10.1080/13511610.2018.1497480

[6] Freud, Sigmund (1921). *Mass psychology and ego analysis.* Wien: Internationaler Psychoanalytischer Verlag.

[7] Granovetter, Mark S. (1973). The Strength of Weak Ties. *American Journal of Sociology,* 78, 1360–1380.

[8] Green, A. and Janmaat, J. G. (2011). *Regimes of Social Cohesion: Societies and the Crisis of Globalisation*. NewYork: Palgrave Macmillan.

[9] Homans, George C. (1958). Social Behavior as Exchange. *American Journal of Sociology,* 63, 597–606.

[10] Horodianenko, V. H. (2008). *Sociology.* Kyiv: Akademiia.

[11] Horsham, Zoe, Abrams, Dominic, Davies, Ben, & Lalot, Fanny (2024). The Causal Relationship Between Volunteering and Social Cohesion: A Large Scale Analysis of Secondary Longitudinal Data. *Social Indicators Research.* DOI: https://doi.org/10.1007/s11205-023-03268-6

[12] Kartashov, M. V. (2007). *Probability, processes, statistics.* Kyiv: Kyivskyi universytet.

[13] Sahharon, Hamizah, Bolong, Jusang & Omar, Siti (2023). Exploring the Evolution of Social Cohesion: Interdisciplinary Theories and their Impact. *Forum Komunikasi,* vols. 18, no. 2, pp. 58–73.

[14] Larsen, Christian Albrekt. (2013). *The Rise and Fall of Social Cohesion: The Construction and Deconstruction of Social Trust in the US, UK, Sweden and Denmark.* Oxford: OUP.

[15] Lewin, Kurt. (1946). Behavior and Development as a Function of the Total Situation. *Manual of Child Psychology,* pp. 791–844. Hoboken, NJ: John Wiley & Sons Inc.

[16] Lockwood, David (1999). Civic Integration and Social Cohesion. *Capitalism and Social Cohesion,* pp. 63–84. London: Palgrave Macmillan.
[17] Lott, Albert J., and Bernice E. Lott (1966). Group Cohesiveness and Individual Learning. *Journal of Educational Psychology,* 57, 61–73.
[18] Maxwell, Judith (1996). *Social Dimensions of Economic Growth*. Edmonton: Department of Economics, University of Alberta.
[19] Putnam, R. (1993). *Making Democracy Work: Civic Traditions in Modern Italy*. Princeton, NJ: Princeton University Press.
[20] Stanley, D. (2003). What Do We Know about Social Cohesion: The Research Perspective of the Federal Government's Social Cohesion Research Network. *Canadian Journal of Sociology,* 28(1), 5–17.

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