CURRENT ISSUES OF MECHANICS

DOI https://doi.org/10.30525/978-9934-26-115-2-4

MODELLING OF MIXING SCENARIO OF A VISCOUS FLUID INSIDE A RECTANGULAR CAVITY UNDER A COMPLEX VELOCITY DISTRIBUTION

Kurylko O. B.
PhD (physics and mathematics),
Assistant professor of the department of theoretical and applied mechanics of mechanics and mathematics faculty
Kyiv National University named by Taras Shevchenko
Kyiv, Ukraine

In the last few decades, a significant interest of the large number of researchers to the problem of mixing fluids by laminar flows is observed. The interest of specialists in this problem is caused not only by the widespread manifestation of mixing processes in nature but also by the use of various mixing technologies in industry and technology. The analysis of the scales of different flows allows us to conclude that at the initial stages of flows with different geometric scales, convective mixing mechanisms are predominant and mixing problems are reduced to the analysis of deformation processes of a particular fluid volume in the considered velocity field. In fact, this problem is related to the Lagrangian description of motion in hydrodynamics and is reduced to the analysis of the motion of a system of Lagrangian fluid particles in a known Euler velocity field. Such a problem is called *the fluid advection problem*, which in the two-dimensional case is described by a system of ordinary first-order differential equations [1, p. 275].

A fluid advection is associated with the study of the deformation over time of the selected volumes or areas (in a two-dimensional case) of the fluid that consist of an infinite set of physical fluid particles. Despite the fact that such systems have an infinite number of degrees of freedom, the study of the advection process of individual fluid particles in hydrodynamic flows is given special attention. Reducing the number of degrees of freedom allows not only to simplify the analysis of the phenomenon, but also to identify the basic properties and relationships of chaotic advection modes.

This work is devoted to modeling the process of advection of the selected volume of fluid in a rectangular cavity under the action of a complex distribution of tangential velocities applied to the upper and lower walls using the Poincare section technique. The movement of the walls is given by the law, which is considered in Ottino's experimental work [3, p. 469].

The study of chaotic mixing regimes that occur in some areas of the flow is reduced to the gradual solving of two problems. The first problem is to determine the velocity field inside a rectangular cavity for a given distribution of the boundary velocities. To solve the first problem one needs to solve the biharmonic equation with corresponding boundary conditions. This task aims to obtain a high-precision solution, because the second problem associated with obtaining trajectories of individual fluid particles of the selected region. The deformation of the boundaries of the selected fluid is associated with dynamically unstable fluid motion inside the cavity. To solve the first problem, the superposition method was used [2, p. 234]. The main idea of the superposition method is to construct the solution of the biharmonic equation in the form of the sum of two ordinary Fourier series with sufficient functional arbitrariness for fulfilling the boundary conditions on the sides of the cavity. This method also allows to find the asymptotics of unknown coefficients in a series of general solutions and thus allows to obtain almost any desired accuracy.

The advantages of the superposition method – relatively little calculations compared to other approaches and high accuracy of determination of the velocity field everywhere including the boundary with corner points and points of discontinuity of loading – become more evident when some examples are considered.

As an example, the periodic Stokes flow of a viscous incompressible fluid inside a rectangular cavity with a complex distribution of a given velocity at the upper and lower boundaries is considered. The size of the cavity and the velocity of the movement of the upper and lower boundaries of the cavity correspond to the data given in the Ottino's work [3, pp. 466-469]. The viscous fluid in the experiment is glycerin. This is a type of fluid whose Reynolds number is close to zero.

To model the process of mixing a viscous fluid inside a rectangular cavity under the action of tangential velocities according to a given sinusoidal law [3, p.469], the advection of the selected spot is considered, the size and position of which correspond to experimental data [2, p. 468].

The initial spot has a circular shape, which is filled with 1000 points inside the circle. The advection of each of the points was studied and their positions at the end of each period were recorded. As a result, a Poincare map was constructed, which simulated the mixing process. The resulting Poincare map allows us to draw certain conclusions about the nature of the mixing of the fluid inside the rectangular cavity. The obtained numerical results are in good agreement with the experimental results which are obtained in the Ottino's experimental work [3, p.483].

References:

1. Aref H. Chaotic advection of fluid particles / H.Aref // Philosophical Transactions of the Royal Society of London. – 1990. – Vol. A333. – P. 273–288.

2. Meleshko V. V. Biharmonic problem in a rectangle // Appl. Sce. Res. – 1998. – № 58. – P. 217–249.

3. Ottino J. M. Experiments on mixing due to chaotic advection in a cavity // Journal of Fluid Mechanics – 1989. – Vol. 209. – P. 463–499.