MODERN MATHEMATICAL METHODS, MODELS AND INFORMATION TECHNOLOGY IN THE ECONOMY

Liashenko O. M., Professor
Lutsk National Technical University
Lutsk, Volyn region, Ukraine

Turski I. V., Associate Professor
Lutsk National Technical University
Lutsk, Volyn region, Ukraine

Kondratska L. P., Associate Professor
West Ukrainian National University
Ternopil, Ukraine

DOI: https://doi.org/10.30525/978-9934-26-107-7-22

THE PRINCIPLE OF UNCERTAINTY MAXIMUM IN RISKY DECISIONS MAKING IN PROJECT MANAGEMENT

In project management decision making process one has to choose from x_1, ..., x_m in the case when it is known that one of the two economic states θ_1, θ_2 is possible. For each of these states, are known following indicators: f_{kj}, $(k = \overline{1,m},\quad j = 1;2)$. However, the probabilities p_1 of state θ_1 and p_2 of state θ_2, $p_1+p_2=1$ are unknown. To determine the unknown probabilities in the case of insufficient statistical support, it is advisable to use the Gibbs-Jaynes principle of maximum uncertainty [1-3]. According to this principle, the unknown probabilities p_1 and $p_2=1-p_1$ must give the maximum value of the function:

$$H(p_1, p_2) = -p_1 \ln p_1 - p_2 \ln p_2. \qquad (1)$$

In the absence of other restrictions on p_1 and p_2, this maximum can be found by means of Fermat's theorem:

$$\ln\ (1-p_1)=\ln p_1;\ 1-p_1=p_1;\quad p_1=\frac{1}{2};\quad p_2=1-p_1=\frac{1}{2}.$$

Therefore, in the absence of restrictions on the probabilities of p_1 and p_2, we obtain the result proposed by Bernoulli. Let consider some cases of possible constraints on indefinite probabilities p_1 and p_2=1- p_1.

1) Suppose that in project management process for the solution x_k, $(k=\overline{1,m})$, it is known that its mathematical expectation of profitability does not exceed some value $\overline{B_k}$. We can assume that $k = 1$. So, let k=1 and $f_{11}p_1+f_{12}p_2\le\overline{B_1}$, or

$(f_{11}-f_{12})p_1\le\overline{B_1}-f_{12}$. If $f_{11}>f_{12}$, then $p_1\le\dfrac{\overline{B_1}-f_{12}}{f_{11}-f_{12}}$.

Here two cases are possible:

a) $\dfrac{\overline{B_1}-f_{12}}{f_{11}-f_{12}}\ge\dfrac{1}{2}$, $\overline{B_1}\ge\dfrac{1}{2}f_{11}+\dfrac{1}{2}f_{12}$. In this case the biggest value of the function H can be achieved, again, when $p_1=1/2$ and $p_2=1/2$ such a restriction does not give significantly new results.

b) $\dfrac{\overline{B_1}-f_{12}}{f_{11}-f_{12}}<\dfrac{1}{2}$, or $\overline{B_1}<\dfrac{1}{2}f_{11}+\dfrac{1}{2}f_{12}$. The function H as function of p_1 is defined on interval $\left(0;\dfrac{\overline{B_1}-f_{12}}{f_{11}-f_{12}}\right]$. We can prove that in this interval the function H grows monotonically and reaches its greatest value at $p_1=\dfrac{\overline{B_1}-f_{12}}{f_{11}-f_{12}}$; probability

$$p_2=1-p_1=1-\frac{\overline{B_1}-f_{12}}{f_{11}-f_{12}}=\frac{f_{12}-\overline{B_1}}{f_{11}-f_{12}}.$$

If $f_{11} < f_{12}$, then from $(f_{11} - f_{12})p_1 \le \overline{B_1} - f_{12}$ such inequality follows: $p_1 \le \frac{\overline{B_1} - f_{12}}{f_{11} - f_{12}}$.

Let $\frac{\overline{B_1} - f_{12}}{f_{11} - f_{12}} \ge \frac{1}{2}$, that is $\overline{B_1} - f_{12} \ge \frac{1}{2} f_{11} - \frac{1}{2} f_{12}$; $\overline{B_1} \ge \frac{1}{2} f_{11} + \frac{1}{2} f_{12}$. Than $p_1 = \frac{1}{2}$ i $p_2 = \frac{1}{2}$, when $\frac{\overline{B_1} - f_{12}}{f_{11} - f_{12}} > \frac{1}{2}$, that is $\overline{B_1} < \frac{1}{2} f_{11} + \frac{1}{2} f_{12}$, and $p_1 = \frac{\overline{B_1} - f_{12}}{f_{11} - f_{12}}$, since the function H decreases monotonically on the interval $\left[\frac{\overline{B_1} - f_{12}}{f_{11} - f_{12}}; -1\right)$.

$$p_2 = 1 - p_1 = 1 - \frac{\overline{B_1} - f_{11}}{f_{11} - f_{12}} = \frac{f_{11} - \overline{B_1}}{f_{11} - f_{12}}.$$

In the case when $f_{11}=f_{12}$ inequality $f_{11} p_1 + f_{12} p_2 \le \overline{B_1}$ does not provide additional information for finding probabilities p_1 and p_2, therefore, there is nothing left but to consider them equal $p_1 = p_2 = \frac{1}{2}$.

Summarizing the above, we conclude: if in inequality $f_{11} p_1 + f_{12} p_2 \le \overline{B_1}$ value of $\overline{B_1}$ satisfies the condition $\overline{B_1} \ge \frac{1}{2} f_{11} + \frac{1}{2} f_{12}$, than $p_1 = \frac{1}{2}$ i $p_2 = \frac{1}{2}$. If $\overline{B_1} < \frac{1}{2} f_{11} + \frac{1}{2} f_{12}$, than $p_1 = \frac{\overline{B_1} - f_{12}}{f_{11} - f_{12}}$, and $p_2 = \frac{f_{11} - \overline{B_1}}{f_{11} - f_{12}}$.

2) Let us now consider the case when the mathematical expectation of the project profit is not less than a certain value:

$f_{11}p_1 + f_{12}p_2 \geq \overline{B_1}$. If $p_1 = p_2 = 1/2$ and $\overline{B_1} \leq \frac{1}{2} f_{11} + \frac{1}{2} f_{12}$, then they must be taken to further selecton of the optimal decision. If $\overline{B_1} < \frac{1}{2} f_{11} + \frac{1}{2} f_{12}$, than p_1 and p_2 must be taken from the solution of the system of equations:

$$\begin{cases} f_{11}p_1 + f_{12}p_2 = \overline{B_1} \\ p_1 + p_2 = 1 \end{cases}.$$

Solving this system, we get:

$$f_{11}p_1 + f_{12}(1 - p_1) = \overline{B_1};$$

$$p_1 = \frac{\overline{B_1} - f_{12}}{ff_{11} - f_{12}}; \quad p_2 = \frac{f_{11} - \overline{B_1}}{ff_{11} - f_{12}}. \qquad (2)$$

In table 1 calculations according to formulas (2) are given in case when: $\overline{B_1}$ =15000 notional currency and $f_{11} = 30000$, $f_{12} = 10000$, probabipity of first economic state $p_1 = \frac{\overline{B_1} - f_{12}}{f_{11} - f_{12}} = 0{,}25$, second – $p_2 = 0{,}75$.

With such probabilities calculated according to the Gibbs-Jaynes principle, the mathematical expectation for the solution x_2 exceeds the corresponding expectation for the solution x_1. So, if the indicators f_{kj} mean possible profits, the decision x_2 is better. If we ignore the constraints $f_{11}p_1 + f_{12}p_2 \leq \overline{B_1}$, and use the Bernoulli principle, ie take the probabilities p_1 and p_2 equal to each other $p_1 = p_2 = 1/2$, we can make the wrong conclusion that the best decision is x_1 (Table 1).

Table 1

The results of a possible decision-making option in project management based on the Gibbs-Jaynes and Bernoulli principles provided that $f_{11} p_1 + f_{12} p_2 \le \overline{B_1}$

Jeynes	Probabilities		Mathematical expectation of project profit
	p_1	p_2	
	0.25	0.75	
Decision	Quantitative estimates (notional currency)		
x_1	30000	10000	15000
x_2	10000	20000	17500
Bernoulli			
	Probabilities		
	p_1	p_2	
	0.5	0.5	
Decision	Quantitative estimates (notional currency)		
x_1	30000	10000	20000
x_2	10000	20000	15000

3) If the constraint is known as an inequality with variance:

a) $D_1 \le \overline{D_1},\ (\overline{D_1} > 0);$ або б) $D_1 \ge \overline{D_1},$ where the variance D_1 is expressed by the formula:

$$D_1 = p_1 (f_{11} - (p_1 f_{11} + p_2 f_{12}))^2 + p_2 (f_{12} - (p_1 f_{11} + p_2 f_{12}))^2 .$$

After transformations we get: $p_1 (1 - p_1)(f_{11} - f_{12})^2 \le \overline{D_1}$. It can be seen that p_1=1/2 is the solution of this inequality if $\overline{D_1} \ge \frac{1}{4} (f_{11} - f_{12})^2 .$ The restriction $D_1 \le \overline{D_1},$ when $\overline{D_1} \ge \frac{1}{4} (f_{11} - f_{12})^2$ is not important, that is, with such a limitation, according to the principle of the Gibbs-Jaynes maximum, the probability $p_1 = p_2 = 1/2$.

References:

1. Jaynes E.T. (1963) Information Theory and Statistical Mechanics. Statistical Physics / Ford, K. (ed.). New York: Benjamin, p. 181.
2. Jaynes E.T. (1986) Monkeys, kangaroos and N In Maximum-Entropy and Bayesian Methods in Applied Statistics, J.H. Justice (ed.), Cambridge University Press, Cambridge, p. 26.
3. Swendsen R.H. (2008) Gibbs' paradox and the definition of entropy. *Entropy*, no. 10 (1), p. 15–18.