STRESSED-DEFORMED CONDITION AND DESTRUCTION OF TECHNOLOGICALLY DAMAGED REINFORCED CONCRETE STRUCTURES

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Abstract. The studies, conducted by the authors over the past 30 years have shown that reinforced concrete structures in the process of processing into products receive technological (hereditary) damage at the micro and macro levels. The resulting damage and structural imperfections affect the operation and behavior of structures during operation. They change the strength, deformability, stress-strain state, the nature of the appearance and the development of force cracks, as well as the durability of the structures and systems. Therefore, it is important to analyze the occurrence of technological cracks and the causes of their genesis in reinforced concrete structures at micro and macro levels, the effect of this damage on the stress-strain state, crack resistance, deformability and durability during the operation. The subject of the study is concrete mixtures with different ratios of initial components, reinforced concrete structures that have got initial (technological) damage and the stress-strain state of bent elements at the crack tip, taking into account the dispersed damaging of the material structure. For the description of non-homogeneity of the material, linear and structural mechanics, as well as micromechanics, were used. Micromechanics allowed us to describe the interaction processes that occur between the individual components during the operation of the material and structure. The photoelasticity method made it possible to determine the stress distribution in regular-structure composites before cracks appeared at the micro level with stress concentration on inclusions. This made it possible to use the relations of the linear theory of elas-

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ticity for a piecewise homogeneous body without discontinuities and cracks. We investigated the two-dimensional stress distribution: plane strain or plane stress state. To describe the two-dimensional stress state, we used the apparatus of the theory of analytical functions of a complex variable proposed by Kolosov and developed by Muskhelishvili. The graphoanalytical method was used when analysing the mechanism of structure formation of the materials at the macro level. When analyzing flat models of structural concrete, the shape of the placeholders is taken in the form of squares. An asymptotic study of stress fields in the vicinity of the tip of a force crack, taking into account the mixed stress, is an important task of nonlinear fracture mechanics, and in particular, taking into account the presence of a scattered damage field. Therefore, the study of the stress and strain field near the crack tip under tensile and shear loads is of particular relevance when calculating the bearing capacity and deformability of structures. The aim of the work is to study and generalize crack formation in the materials and reinforced concrete structures at micro and macro levels, the influence of technological damage on the development of cracks from external influences and loads, the determination of the stress-strain state at the crack tip from external influences and loads, and the influence of technological cracks on the bearing ability, deformability and operational characteristics of structures. The use of the apparatus of the theory of analytical functions of a complex variable made it possible to obtain numerical results, characterizing the initial stage of material operation (the formation of diffuse damage) for any characteristics of materials, forms of inclusions and external influences. The study of mechanisms of formation macrostructure concretes showed that the reasons of genesis and the development of discontinuity in hardening matrix material that degenerate in the internal surface of the process section or cracks are strain gradients. Following the algorithm, it is possible to obtain eigenvalues for constructing a multiscale, multilevel description of fracture processes in the vicinity of the crack tip that lead to loosening of the material, crack growth, and structural failure.

1. Introduction

It is known [1, p. 5-99] that reinforced concrete structures in the process of processing into products get technological damage at micro and macro levels. When analyzing the structures of materials, it is customary to distinguish certain types of them by size. Such an artificial identification of structural levels makes it difficult to assess the degree of danger of defects in the analysis of the physicomechanical properties of materials. It is considered dangerous to attribute defects, which size is larger than the size of the constituent structural elements. Therefore, a defect size that is safe at one structural level becomes dangerous for a lower scale level. At each selected level of structural heterogeneity, the material can be described using linear mechanics or micromechanics. Representation of the material as a complex medium involves the allocation of a volume in which the individual properties of the components are leveled (Figure 1).

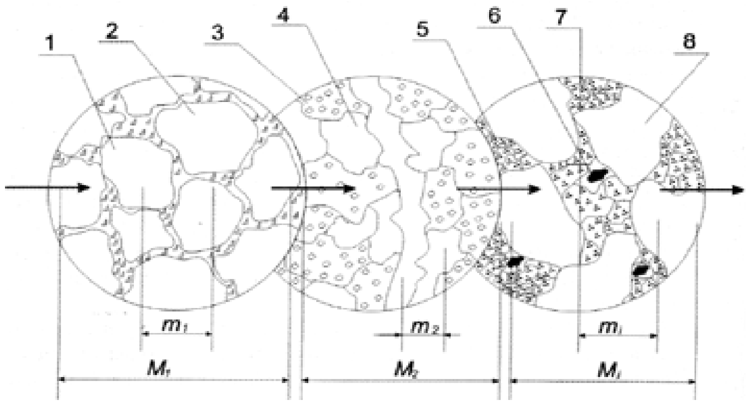


Figure 1. Theorganization of the structure of composite building materials:

1 – elementary structural element of a binder; 2– filler; 3 – cement stone; 4 – fine aggregate; 5 – macrostructural cracks; 6 – pores; 7 – soution part (matrix); 8 – coarse aggregate; m_1, m_2, M_1, $M_{2,,,,}$, M_i – structural scale levels, which are averaged individual structural inhomogeneity m_i

The identification of the features of the interaction of the individual components allows to reveal the mechanism of formation of average characteristics at the level M_i. In turn, the analysis of the interaction at levels structures M_1, M_2, ..., M_i (taking into account interaction of the components structures m_1, m_2, ..., m_i) reveals mechanism of the physicomechanical characteristics of composite building materials with layered structure organization.

The appearance and development of cracks in hardening materials depends on the number of interacting phases ν, their mechanical characteristics, the geometric parameters of the heterogeneous system, the geometric characteristics of the sample or structure, the magnitude and kinetics of the development of intrinsic volumetric deformations, the change in the rheological characteristics of the hardening system, the intensity and parameters of technological influences. Studies have shown that for a qualitative description of the physicomechanical processes of structure formation and destruction of materials and the establishment of qualitative relations, it is sufficient to distinguish two structures: micro and macro levels. At the micro level, the characteristic structure of the heterogeneity of the material is the "binder-filler". The macrostructure is represented by the solution-aggregate heterogeneity. Moreover, each selected structure consists of structures of a lower scale level, and sprouting from one to another scale levels, they interact with each other. The structure organization is most clearly traced in concrete: from submicrocrystals of the gel component with a size of 10^{-10}... 10^{-8} to heterogeneity "mortar-aggregate" with a size of 2×10^{-2} st.

2. The mechanism of cracks genesis in the microstructure of concrete

The analysis of the formation of structures at the micro level shows that the use of fillers will let increase the strength of composite building materials [2]. It is associated with an increase of the destruction energy of a heterogeneous medium. Dispersed particles are able to interact with the front of the cracks. Therefore, the fracture energy is associated with the fracture energy of the matrix material, the linear energy per unit length of the crack front and the distance between discrete particles. The size of a fracture crack depends on the specific surface energy, stress concentration in the region of the crack mouth, the module of elasticity of the material, and the maximum stress value, they harden the material by restraining the directional movement of dislocations. The line of dislocations bends between the particles in loops, that have different signs behind each particle, which causes them to close , annihilate, and form dislocation fields around the particles.

The effect of fillers on the strength of materials is valid for a group of materials consisting of an isotropic matrix in which filler particles of the same size are evenly distributed [2, p. 35–41]. In this case, the interface between the matrix material and inclusions can serve as a source of crack genesis and decrease in the strength of the material, or as a kind of stopper for the genesis and development of cracks, and increase the mechanical characteristics of the material [3, p. 23–27]. It can be concluded that the strength of composite building materials at the microlevel is determined by its heterogeneity. Therefore, the strength of such filled systems will depend on both the amount of fillers and their dispersion (Figure 2).

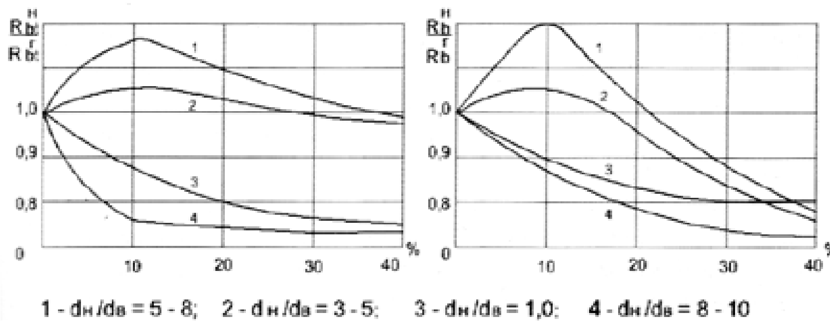


Figure 2. The effect of the amount and dispersion of the filler on the strength of the filled compositions a – n / a – constant; b – S_n – constant

$$R = f(V_{H,}S_{H,}\gamma_{M,}\gamma_{H}...) \quad (1)$$

It can be concluded that for composite building materials, the heterogeneity of which is determined by the internal interstructural interfaces, strength fulfills the function of internal hereditary cracks. In turn, the number of internal cracks and their extent depends on the ratio of surface activities of binder grains and filler particles that determines the dispersion and amount of filler. It can also be concluded that the strength of the microstructure of materials is a function of their heterogeneity that is determined by intercluster interfaces, and can be controlled by introducing a certain amount of fillers into the system that are characterized by surface activity and dispersion.

As a comparison of the results obtained experimentally (by the photoelastic method) for composites of regular structure shows [2, p. 25], the stress distribution in them before the appearance of cracks is fairly well described by a linear model. At the initial stage of the work of the material during the formation of cement stone, shrink stresses are manifested. Material properties do not depend on the load level yet, and stress concentration occurs at the inclusions. By inclusion we will mean an area with modified properties. This makes it possible to use the relations of the linear theory of elasticity for a piecewise homogeneous body without discontinuities and cracks. The latter is due to the fact that the appearance of microcracks indicates a significant nonlinear stage of the work of the material, and therefore the stage of the appearance of microcracks is excluded from the initial stage of the work of the material. In the future, we will study only the two-dimensional distribution of stresses: plane strain or plane stress state. On one hand, these rays are widespread in the practice of construction, and on the other, an exceptionally powerful apparatus is developed for describing a two-dimensional stress state in the mathematical theory of elasticity – an apparatus of the theory of analytic functions of a complex variable, first used by Kolosov and developed by Muskhelishvili [4, p. 270]. The use of this apparatus makes it possible to obtain numerical results, characterizing the initial stage of the composite for any characteristics of materials, forms of inclusions and external influences.

The solution of the posed problems is reduced to the determination of two functions φ(z) and ψ(z) to the complex variable z = x + iy. The components of the stress state are expressed in terms of φ(z) and ψ(z) by ratios help:

$$f(\theta = 0) = 1,\quad f'(\theta = 0) = -(\lambda + 1) / tg\left(M^{p}\pi / 2\right),$$
$$f''(\theta = 0) = A_2,\quad f'''(\theta = 0) = A_3. \tag{2}$$

Reφ′(z) – a material part of analytic functions φ(z).

And moving:

$$2\mu(u - iv) = x\phi(z) - z\overline{\phi'(z)} - \overline{\psi(z)} \tag{3}$$

The determination of the desired functions was carried out from the system of functional equations:

$$\frac{x}{\mu}\phi\left(t^{(m)}\right) - \frac{1}{\mu}\left[t^{(m)}\overline{\phi'\left(t^{(m)}\right)} + \overline{\psi\left(t^{(m)}\right)}\right] - \frac{x_0}{\mu_0}\phi_m\left(t^{(m)}\right) + \frac{1}{\mu_0}\left[t^{(m)}\overline{\phi'\left(t^{(m)}\right)} + \overline{\psi_m\left(t^{(m)}\right)}\right] = g\left(t^{(m)}\right);$$

$$g\left(t^{(m)}\right) = g_1(t) + ig_2(t); \tag{4}$$
$$t^{(m)} \in \Gamma^{(m)}; m = 0.1...$$

Note, that g (t) determines the amount of displacement that should be set at the point $t^{(m)}$ kernel $\Omega^{(m))}$ in order to combine it with the point $t^{(m)}$ element of the boundary $t^{(m)}$ domain Ω when the latter is in an undeformed state. For example, in the case of uniform shrinkage of circular elastic nuclei with an initial radius R into holes of unit radius $g\left(t^{(0)}\right)=(R-1)e^{i.}$, Θ is the polar angle of the point $t^{((m)}$ in the coordinate system associated with the center of the hole.

In addition to conditions (4), for each element of the boundary Γ, equilibrium conditions must be satisfied that lead to the second functional equation:

$$\phi\left(t^{(m)}\right)+t^{(m)}\overline{\phi'\left(t^{(m)}\right)}+\overline{\psi\left(t^{(m)}\right)}-\phi_m\left(t^{(m)}\right)-t^{(m)}\overline{\phi_m'\left(t^{(m)}\right)}-\overline{\psi_m\left(t^{(m)}\right)}=0 \quad (5)$$

The system of equations (4) and (5) together with (3) makes it possible to determine φ(z), ψ(z), ψm (z), (m =0,1,.., n =1) from the given g(t) and the conditions at infinity.

The maximum tangential stresses τ_{max} and principal stresses σ_1 and σ_2 at the point z with Ω are expressed in terms of the complex potentials φ(z) and ψ(z).

$$\tau_{\max}=\left[\bar{z}\phi'(z)+\psi'z\right], \quad (6)$$

$$\sigma_1=\phi'(z)+\overline{\phi(z)}+\left[\bar{z}\phi''(z)+\psi'(z)\right], \quad (7)$$

$$\sigma_2=\phi'(z)+\overline{\phi_1'(z)}+\left[\bar{z}\phi_1''(z)+\psi_1'z\right], \quad (8)$$

An infinite isotropic elastic body with a regular ring of circular inclusions is considered as a model describing the stress distribution in the composite. Therefore, they can be thought of as elastic cores with equal effective characteristics. The elastic properties of the core and matrix materials differ from each other and are described by the modulus of elasticity and Poisson coefficient υ or the Lyame constants λ and μ which are related to E and υ ratios.

$$\lambda=\frac{Ev}{(1+v)(1-2v)} \text{ and } \mu=\frac{E}{2(1-v)}$$

In addition, an elastic constant is introduced: $\chi=\dfrac{3-v}{1-v}$

The distribution of stresses in the composite caused by surges at the core boundaries is investigated. This simulates shrinkage phenomena occurring in both the matrix material and the core material. In any deformation, cracks do not occur between the cores and the matrix, and the centers of gravity of the cores coincide with the centers of gravity of the corresponding planes in the matrix (Figure 3).

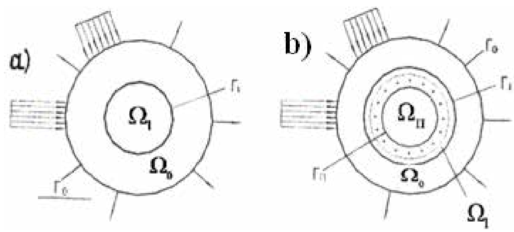


Figure 3. Design model of non-uniform material
a – even boundary; b – case of smooth change of properties

3. The mechanism of structure formation of the materials at the micro level

Previous studies have noted the effect of aggregate properties on concrete properties. At the same time, various models of concrete macrostructure were proposed, taking into account the volumetric concentration of aggregates, their shape, the ratio of the deformation and strength characteristics of the mortar part and aggregates. The analytical dependences of deformability and strength characteristics of concrete calculations, based on the rules or mixtures of conservation laws of additivity are proposed. As a rule, the calculated values and experimental results are far from satisfactory convergence.

In our studies, the analysis was carried out by the graphoanalytical method. When analyzing flat models of structural cells of concrete, the shape of the aggregates is taken in the form of a square. The reduced distance between the aggregates *h* for different orientations is accepted h=0,2R, where R – is the reduced radius (Figure 4a). Analysis was performed for a case where in the adhesion quantity, R_a the matrix material to the surface of the aggregates is less than its cohesive strength R_k, $R_a < R_k$. Changing the shape of the aggregates changes the distribution of strains in the matrix material (Figure 4.b, c).

A comparison of the model with square-shaped aggregates was compared with the strain distribution in the model with disk-shaped aggregates. The analysis showed that when the shape of the aggregate changes, the character of the distribution of deformations of the hardened matrix material changes. The influence of the shape of the fillers on the strain gradients is especially noticeable when the orientation of the fillers changes are relative to each other (Figure 5). The violation of symmetry in the macrostruc-

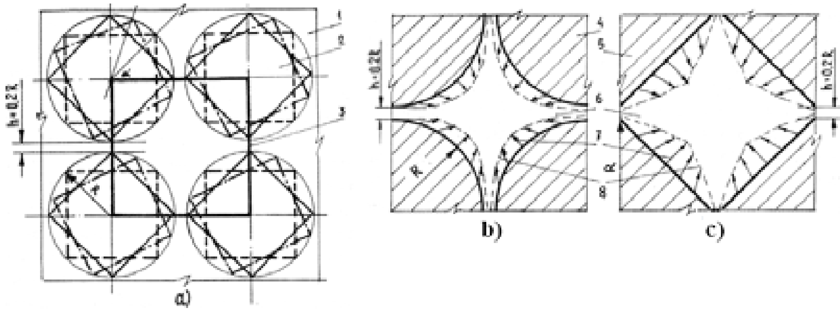


Figure 4. The effect of an aggregate shape on the distribution of shrinkage strain in the matrix

a – geometric characteristics of the model of cellular concrete structure;
b – a model with placeholders in the form of a circle;
c – a model with square placeholders.
1 – matrix; 2 – placeholders; 3 – elementary cell structure;
4 – placeholders in the form of a circle; 5 – placeholders in the form of a square;
6 – matrix; 7 – direction of deformations; 8 – strain diagram

ture due to a change in the orientation of the fillers in the form of a square causes an asymmetric distribution of intrinsic deformations at the level of structural inhomogeneity.

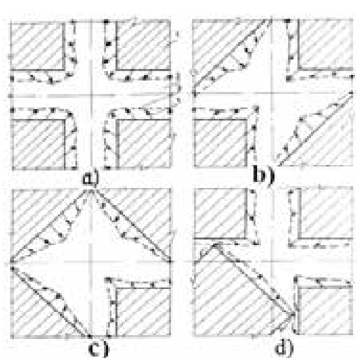


Figure 5. The effect of filler orientation on shrinkage distribution

a, b, c, d – ways of orienting placeholders
1 – placeholders; 2 – matrix; 3 – diagrams of deformations;
4 – direction of deformations

The distribution of intrinsic strains and their gradients in the hardening matrix material depends on the number of fillers that determines the distance between them, the shape of the fillers and their orientation relative to each other. The influences of the shape and relative orientation of the barriers are so significant that they can lead to a change in the nature of the distribution of technological strains in each structurally repeating cell. The structural cell includes a group of aggregates, in a certain way distributed in the matrix material. It is assumed that the characteristics of the cells are invariant that ensures the invariance of the properties, including the distribution of intrinsic deformations of the matrix material. Therefore, concrete, as a coarse heterogeneous composite, is represented as a set of structural cells that are identical in properties and parameters. In this regard, the adopted models of the macrostructure include the aggregates of the same shape, therefore, they were automatically transferred to the entire system. Our analysis showed that in the case of a change in the orientation of the aggregates, the shape of which differs from the circle for flat models, the nature of the distribution of intrinsic deformations in the hardening matrix material changes. This leads to individual characteristics of the formation of the macrostructure of concrete as a set of structural cells. The individuality of structural cells is also manifested in cases where the level of adhesion of the matrix material to the aggregates changes and in the case of a change in the surface topography of the aggregates. Figure 6 shows the integral and local deformations arising during the hardening of the matrix material for the case $R_a < R_k$.

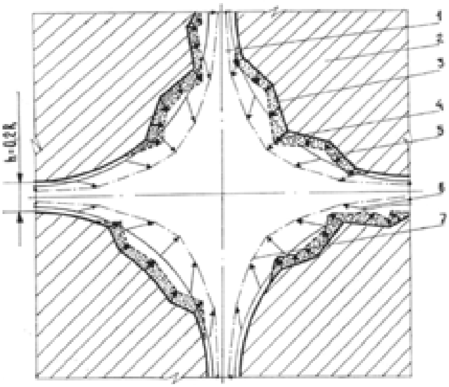


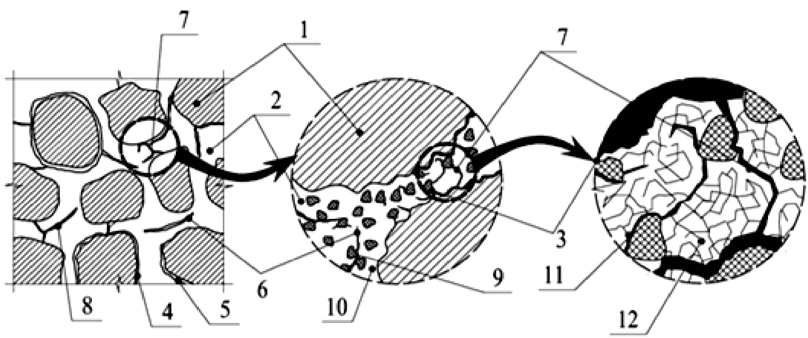
Figure 6. The effect of the surface topography of aggregates on the nature of the distribution of shrinkage deformations

1 – matrix; 2 – placeholders;
3 – relief surfaces of the fillers;
4 – direction of local deformations;
5 – diagrams of local deformations;
6 – direction of integral deformations;
7 –a plot of integral deformations

At the level of the structural cell, as well as at the level of the inner interface, the deformation gradients arise in magnitude and direction. Integral deformations are distributed fairly evenly compared to local deformations at the interface. A change in the surface topography of each aggregate causes a change in local deformations. Multidirectional deformations are also observed on the surface microroughnesses.

The analysis showed that at the macrostructure level there are adhesion cracks at the interface of the matrix material with large and small aggregates; starting from the crack of adhesion and developing into the mortar; cracks in the matrix that develop between the fillers.

In the general case, the concrete structure model includes internal interfaces formed as a result of the interaction of the mortar with a large aggregate (Figure 7), cement stone with fine aggregates (Figure 7b) and internal interfaces in the form of intercluster interfaces at the level of cement Figure 7c).



a b c

Figure 7. The model of concrete structure:
a – macrostructure; b –a level mortar part – a coarse aggregate;
c – level cement stone – fine aggregate.

1 – coarse aggregate; 2 – solution part; 3 –a fine aggregate; 4 – adhesion cracks over the entire surface of the aggregate; 5 – adhesion cracks located on separate surface areas of the aggregates; 6 – cracks located between the fillers; 7 – cracks that close on the banks of other cracks; 8 – cracks that are not completed in their development; 9 – a modified surface layer on the inner interfaces and crack faces; 10 – a cement stone; 11 – cracks resulting from the interaction of small aggregates and a cement stone; 12 – structural blocks of a cement stone.

As studies have shown [5], the mechanism of formation of the macrostructure of concrete is determined by its geometric characteristics – the quantitative and qualitative composition of aggregates, and the level of interaction of the hardening matrix material with aggregates. The analysis of the structure of concrete as a coarse heterogeneous material and the mechanisms of structure formation at micro and macro levels allowed us to propose a model of the structure of concrete that includes aggregates, matrix material, a modified layer at the interface of the matrix and aggregates, pores and capillaries, as well as technological cracks and internal interfaces.

4. Determination of stress state by methods of destruction mechanics

Analyzing projects [6, p. 77; 7, p. 123], it is possible to state questions concerning mixed deformation samples with breaches and defects are the subject of acute attention and intensive study in a modern on linear destruction mechanics. Specialists' interest in the mechanics field destruction mechanics is increasing towards tasks about inclined breaches and the breaches within the conditions of normal tensile tensioning and cross cut move attached simultaneously and relating to mixed forms stress in the destruction mechanics [8, p. 412].

There are three types of breaches in the destruction mechanics responding to the three types of stress: normal uplift breaches, cross cut and non linear move ones.

Marking the three types of breaches refers to line destruction mechanics where the composition of results had been got for the three stress types gives the true solution near the breach top for voluntary stress on the defected body:

$$\sigma_{ij}(r.\theta) = \sum_{n=1}^{3} \sum_{k=-\infty}^{\infty} \alpha_k^m f^{m,i,j}(\theta) r^{k/2-1} \tag{9}$$

r,θ – polar coordinates with the pole at the breach top.

Indicated stress types on the element of the construction with the breaches in line-elastic materials in ideal plastic bodies are studied quite well. However, tensioning deformation and moving divide near a defect top within the conditions of mixed breach stress in the non linear materials (for hardens according to exponential material law for materials following exponential Norton's law of the theory of steady creeping) is not studied well. Probably, in a modern science literature there is no any investigation's results of the effect of bloomed defect conservation on the direction of the breach extension within the conditions of mixed stress. There are various criteria of the direction of the breach spreading, for example, local criteria based on asymptotic tensioning divide at the breach top or a sharp gain. That's why asymptotic poles investigation of poles at the apex within the conditions of the mixed breach stress is an important in the modern non linear destruction mechanics and the mixed stress of the elements of defective construction is an object of constant attention of a scientific community and studied by many foreign mechanic recent years. Probably, first appeal to the investigation of mixed deformation forms was done in projects Si where tensioning and deformation poles was examined near the breach top that is under the influence of tensile and shift external forces (so it was identified the mixed stress referring to the breaches of I and II types). Si was the first who discovered the index of the stress (10):

$$M^{p} = \frac{2}{\pi} arctg \left| \lim_{r \to 0} \frac{\sigma_{\theta\theta}(r, \theta = 0)}{\sigma_{r\theta(r, \theta = 0)}} \right| \qquad (10)$$

The index of stress combination M_p takes zero amount $M_p=0$ for a fine cross cut sliding and the index is equal to one $M_p=1$, for usual uplift; $0 < M_p < 1$ for all the intermediary types enclosed external forces.

There is a divide of tensioning, defomation and moving at the breach top in the material that follows an exponential law of the deformation theory of plastic properties for particular data of mixed stress index within the conditions of a flat deformation. Systematic calculation treatment of angle tensioning and deformation divide at the breach top in the material with the exponential determinative law for various index amounts of tensioning combination is indicated in [6, p. 77], where a system is designed and there are calculation results of elastoplastic rates of tensioning intensity in a complete diapason of mixed deformation forms from a right uplift to a full move and this allowed to study the form of the forthright oriented breach in a mathematical cut in a case of two-axial stress of the various intensity. On the ground of carried calculations it was arranged the character of the influence of mixed stress forms and plastic material features described by the index of deformation hardening.

In the project [9, p. 70] there is a numerical way of the determination of the full variety from proper amounts in non linear task to proper amounts following from the problem of determination of tense deformed condition near the breach top within the conditions of mixed deformation for the material with exponential determinative equation (the exponential law of a deformation theory of plasticity, the exponential law of a steady crawling). An author suggests the method that can be used for finding an intermediary asymptotic of tensioning poles in related (the crawling is a damage) task about the breach within the conditions of mixed stress in the material with exponential determinative equation of the set crawling theory.

It mentioned that the class of non linear tasks to proper accounts arising in non linear destruction mechanics is an important item due to the necessity of use multiscale, multilevel patterns [10, p. 1332; 11, p. 70] according to which at the breach top it is necessary to consider the complex of fields with a predominant activity of various asymptotic of tensioning poles and to undertake the splice procedure requires to know the whole variety of the o9wn amounts and probably these problems are still not solved. In literature there are only two completed mathematically splice procedures: classical solving by Rice for the breach of an out-of-plane shearing [12, p. 191] and analitical solving for the breach of a finite length in an endless battern.

To illustrate the local damage rate on various levels and the treatment of multiscale destruction character it is possible to use the scheme on figure 8 and first described in [13, p. 5]. It is mentioned in [14, p. 7] the conception development of mesomechanics requires more consistent way to solve the problems in cross-subject fields such as materiology and the mechanics of solid fields. Figure 8 shows the diagram of various big-and small-sized defects tensioning singularities near dislocations, micro- and macrodefects.

This diagram proves the necessity of applying the multilevel approach requiring the forming-up of hierarchic chain of fields with different characteristics of the tensioning pole near the breach top and from mathematical point of view it leads the determination of the whole variety of proper amounts and as a rule in non linear tasks to proper amounts had been got as a result of use the method of an expansion in analyzing tensioning, deformation and displacement poles near the breach top.

On the cause of [14, p. 7] let's show mathematical task organization following from the problem of deformed-tensely condition determination at the breach top within the conditions of mixed construction element deformation in the material with power equation:

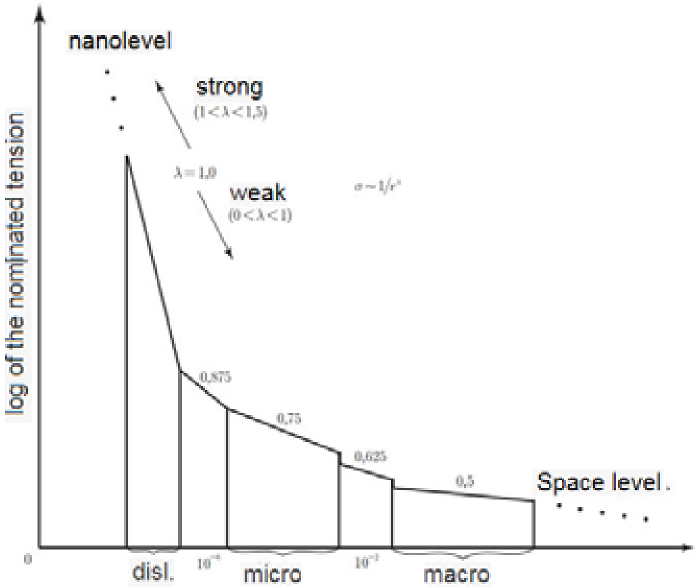


Figure 8. The orders of tensioning poles singularity near the breach top

$$\varepsilon_{ij} = 3B\sigma_e^{n-1}s_{ij} / 2, \quad (11)$$

where εij – tensor component deformation, s_{ij} – components of tensor's deviator stress, $\sigma_e = \left(3s_{ij}s_{ij} / 2\right)^{1/2}$ – the intensity of tangential stress, B, n – mass constants. This task leads to the necessity of the investigation of balance equation (in the polar system coordinates with a pole at the breach top):

$$r\sigma_{rr,r} + \sigma_{r\theta,\theta} + \sigma_{rr} - \sigma_{\theta\theta} = 0, \;\; r\sigma_{r\theta,r} + \sigma_{\theta\theta,\theta} + 2\sigma_{r\theta} = 0 \quad (12)$$

And conditions of deformation :

$$2\left(r\varepsilon_{r\theta,\theta}\right), r = \varepsilon_{rr,\theta\theta} - r\varepsilon_{rr,r} + r\left(r\varepsilon_{\theta\theta}\right), rr \quad (13)$$

with determinative equations (9).

An author indicates that one of the most common methods of determination of the deformed-tensely condition at the breach top is the method of dissolution (contamination) to own functions according to asymptotic stage performance of stress function Eri χ (r,θ) near the breach top (r→0) is determined as:

$$\chi(r,\theta) = \mathrm{K} r^{\lambda+1} f(\theta). \quad (14)$$

The components of tensor's stress near the breach top the form $\sigma_{ij}(r,\theta) = r^{\lambda-1}\tilde{\sigma}_{ij}(r,\theta)$ or take:

$$\sigma_{rr} = r^{\lambda-1}\left[(\lambda+1)f + f''\right],\ \sigma_{\theta\theta} = r^{\lambda-1}\lambda(\lambda+1)f,\ \sigma_{r\theta} = -r^{\lambda-1}\lambda f'. \quad (15)$$

Asymptotic performance the intensity of the tangential stress within the conditions of flat deformed state at the breach top has the following form $\sigma_e(r,\theta) = r^{\lambda-1} f_e(\theta)$, where

$$f_e(\theta) = \sqrt{\left[f''(\theta) + \left(1-\lambda^2\right)f(\theta)\right]^2 + 4\lambda^2\left[f'(\theta)\right]^2}.$$

Thus, because of the strength (11), (14), and (15) deformation tensor components with the flat deformed state condition near the breach top have the following structure:

$$\varepsilon_{rr}(r,\theta) = -\varepsilon_{\theta\theta}(r,\theta) = Br^{(\lambda-1)n}\tilde{\varepsilon}_{rr}(\theta),\ \ \varepsilon_{r\theta}(r,\theta) = Br^{(\lambda-1)n}\tilde{\varepsilon}_{r\theta}(\theta), \quad (16)$$

where

$$\tilde{\varepsilon}_{rr}(\theta) = 3f_e^{n-1}\left[f''(\theta) + \left(1-\lambda^2\right)f(\theta)\right]/4,\ \ \tilde{\varepsilon}_{r\theta}(\theta) = -3f_e^{n-1}\lambda f'(\theta)/2.$$

Setting in the condition (16) of consistency (13) allows to get common non linear differential equations of the forth order as relating to f(θ) , (17):

$$2\left[(\lambda-1)n+1\right]\tilde{\varepsilon}_{r\theta,\theta} = \tilde{\varepsilon}_{rr,\theta\theta} - (\lambda-1)n\left[(\lambda-1)n+2\right]\tilde{\varepsilon}_{rr} \quad (17)$$

Or in the function terms f(θ), (18):

$$\begin{aligned}
& f_e^2 f^{(4)}\left\{(n-1)\left[\left(1-\lambda^2\right)f + f''\right]^2 + f_e^2\right\} + (n-1)(n-3)\times \\
& \times\left\{\left[\left(1-\lambda^2\right)f + f''\right]\left[\left(1-\lambda^2\right)f' + f'''\right] + 4\lambda^2 ff''\right\}^2\left[\left(1-\lambda^2\right)f + f''\right] + \\
& +(n-1)f_e^2\left\{\left[\left(1-\lambda^2\right)f' + f'''\right]^2 + \left[\left(1-\lambda^2\right)f + f''\right]\left(1-\lambda^2\right)f'' + 4\lambda^2\left(f''^2 + ff'''\right)\right\} \quad (18) \\
& \left[\left(1-\lambda^2\right)f + f''\right] + 2(n-1)f_e^2\times\left\{\left[\left(1-\lambda^2\right)f + f''\right]\left[\left(1-\lambda^2\right)f' + f'''\right] + 4\lambda^2 ff''\right\} \\
& \left[\left(1-\lambda^2\right)f' + f'''\right] + \square_1(n-1)f_e^2\left\{\left[\left(1-\lambda^2\right)f + f''\right]\left[\left(1-\lambda^2\right)f' + f'''\right] + 4\lambda^2 ff''\right\}f' + \\
& +\square_1 f_e^4 f'' - \square_2 f_e^4\left[\left(1-\lambda^2\right)f + f''\right] + f_e^4\left(1-\lambda^2\right)f'' \ \square\ \square\square
\end{aligned}$$

where the statements are established (19):

$$C_1 = 4\lambda\left[(\lambda-1)n+1\right],\ \ C_2 = (\lambda-1)n\left[(\lambda-1)n+2\right]. \quad (19)$$

The solution to non linear equation (18) has to suit border conditions (21) emerging from the require of absence of pertaining surface efforts at6 the breach shores (20).

The condition of the absence of pertaining surface efforts at the breach shores:

$$\sigma_{\theta\theta}\left(r,\theta = \pm\pi\right) = 0,\ \ \sigma_{r\theta}\left(r,\theta = \pm\pi\right) = 0. \tag{20}$$

Border conditions (21):

$$f\left(\theta = 0 \pm \pi\right) = 0,\ \ f'\left(\theta = \pm\pi\right) = 0. \tag{21}$$

Thus, for the equation (18) the solution of which depends on border conditions (21) was formed non linear task for proper value: it is necessary to find proper value of λ responding non-trivial solutions (18), suiting border conditions (21). During the breach investigation of normal uplift and cross cut move for numerical equation solution (18) is used the conditions of symmetry or antisymmetry and the equation (18) that is integrated on the length [0,π] with initial conditions: for the breaches with normal uplift (22):

$$f\left(\theta = 0\right) = 1,\ \ f'\left(\theta = 0\right) = 0,\ \ f''\left(\theta = 0\right) = A_2,\ \ f'''\left(\theta = 0\right) = 0. \tag{22}$$

And for the breaches of cross cut move (23):

$$f\left(\theta = 0\right) = 0,\ \ f'\left(\theta = 0\right) = 1,\ \ f''\left(\theta = 0\right) = 0,\ \ f'''\left(\theta = 0\right) = A_3, \tag{23}$$

According to the work [9, p. 70] we describe the scheme of numeric solution of the whole variety of proper value in the non linear task for proper value arising from the problem of determination of the tense deformed condition at the breach top in the material with exponential equations in the conditions of mixed deformation within mixed deformation forms: from clear move to normal uplift.

The author states that in case of mixed deformation view of symmetry and antisymmetry can't be used and it is necessary to search the equation solution (18) on the length [~π,π]. In the conditions of mixed stress on numeric equation solution (18) the integrating length [~π,π] can be divided into two length [~π,0] and [0,π] Koshi with initial conditions. First the equation (18) is integrated on the length [0, π] and border task goes to task Koshi with initial conditions (24).

$$f\left(\theta = 0\right) = 1,\quad f'\left(\theta = 0\right) = -\left(\lambda + 1\right) / tg\left(M^{p}\pi / 2\right),$$
$$f''\left(\theta = 0\right) = A_2,\ \ f'''\left(\theta = 0\right) = A_3. \tag{24}$$

The value of first-order derivative originates from given form of mixed stress (the value of mixed stress term (10) giving the type of the stress is unknown). The unknown variables A_2 and A_3 are found in such a way in order to make border conditions on the top breach shore (25).

$$f(\theta = \pi) = 0, \quad f'(\theta = \pi) = 0. \tag{25}$$

After constants A2 and A3 determination the equation (18) is intergrated on distance [~π, θ] for that two-point border task for the equation (18) with border conditions (26) .

$$f(\theta = -\pi) = 0, \quad f'(\theta = -\pi) = 0,$$
$$f(\theta = 0) = 1, \quad f'(\theta = 0) = -(\lambda + 1) / tg\left(M^p \pi / 2\right). \tag{26}$$

Is replaced Koshi task with initial conditions (27):

$$f(\theta = -\pi) = 0, \quad f'(\theta = -\pi) = 0, \quad f''(-\pi) = B_2, \quad f'''(\theta = -\pi) = -B_3. \tag{27}$$

The unknown constants B_2 and B_3 are chosen in such a way to follow element balance conditions that is placed on the half-line θ=0. Balance equation of this element requires persistence tensor component of the 4 efforts on the half-line θ=0, that leads to the function persistence (and as a result border conditions (26)). That's why two unknown constants B_2 and B_3: are determined in such a way the resolution solving on the distance [~π,0] was in accord with border conditions with θ=0.

So the determination of the full variety of proper values of non linear task to proper values gives the opportunity to describe the structure of the top breach area within the conditions of mixed deformation and also to build up the configurations of the area of deformed material surrounding the breach top.

The author illustrates angle breakdown tensor component of efforts had been found as a result of the solution of a numerical problem according to the scheme described above for various terms of hardening index.

Following the scheme described in this work the opportunity to get proper values is opened, those ones can be used to build up multiscale, multilevel description of the destruction processes near the breach top. It is possible to conduct numerical analysis that enables to assess asymptotic of machinal poles at the breach top reinforced concrete at the distance balancing the shredding zone (with the area of dispersed material).

The stages of the numerical finding of the whole variety of proper values in the non linear tasks for values originating from the problem of determination of the tense-deformed condition at the breach top in the material with the exponential determinative equation within the conditions of mixed deformation forms from clear move to normal separation are offered.

5. Conclusion

The use of the apparatus of the theory of analytical functions of a complex variable made it possible to obtain numerical results, characterizing the initial stage of material operation (the formation of diffuse damage) for any characteristics of materials, forms of inclusions and external influences.

The study of mechanisms of formation macrostructure concretes showed that the reasons of genesis and the development of discontinuity in hardening matrix material that degenerate in the internal surface of the process section or cracks are strain gradients.

Following the algorithm, it is possible to obtain eigenvalues for constructing a multiscale, multilevel description of fracture processes in the vicinity of the crack tip that lead to loosening of the material, crack growth, and structural failure.

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