THE CONCEPT OF NORMATIVE THEORY OF SYNTHESIS OF INFOR-MATION TECHNOLOGIES FOR DECISION SUPPORT UNDER DIFFER-ENT FORMS OF IGNORANCE: MAIN IDEAS

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Abstract. As a result of the analysis of the domestic and foreign experience in field of ignorance research, the basic concepts and theoretical background of modeling of different types of ignorance (vagueness, uncertainty, inhomogeneity, incompleteness, inaccuracy, etc.) are investigated. The analysis shows that there are no established criteria and procedures for the identification and formalization of different types of ignorance, under the influence of which a set of source data is formed or analyzed. There are a significant number of methods for ignorance modeling, but there are no effective procedures for their systematic application in order to obtain reliable solutions. Therefore, an important task in this context is the formalization of the processes of identification of different types of ignorance that can affect the processes associated with the acquisition and analysis of source data set and reasonable choice of mathematical apparatus capable of identification and modeling the analyzed types of ignorance. Which in turn provides a theoretical basis for the synthesis of information technologies for decision support under different forms of ignorance. The concept of the normative theory of the synthesis of information technologies of intellectual support for the decision-making process under various kinds of ignorance has been proposed. The development of such a theory contributes to the formation and development of theoretical

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and practical aspects of the synthesis of information technologies for decision support, as a set of unified mathematical models and methods of intellectual and information support of the processes of formation and decision making in various fields of human activity, presented in the form of models, methods, approaches, procedures, techniques and algorithms for solving various applied problems. Within the framework of the concept of synthesis of information technologies, a procedure for choosing ignorance modeling method is proposed. Which presupposes the formation of a system of criteria that make it possible to unambiguously characterize the considered methods of ignorance modeling. Based on the formed system of criteria, decision rules for choosing a modeling method are synthesized. If, as a result of the performed procedure, more than one method was chosen, then for each candidate method a system of criteria for evaluating the effectiveness and efficiency of it is formed. The value of each criterion is expressed in numerical form, after which the problem of multi-criteria optimization is solved to select the optimal solution (method). The example of synthesis of decisive rules for choosing the mathematical formalism of ignorance modeling to solve multicriteria choice problems is given.

1. Introduction

For making effective management decisions it is necessary to take into account the growing number of influencing factors that describe complex processes related, for example, to the state economy, the business of large enterprises, social trends in society, etc. The situation is aggravated by the presence of different types of ignorance, which have a negative impact on the processes associated with the acquisition and analysis of initial data (statistical, analytical, expert information, etc.). In such situation a person cannot guarantee the adoption of an effective solution at the heuristic level, taking into account all conflicting factors that affect the achievement of the goal of the problem (task) being solved.

In these conditions, there is a need to create and develop the foundations of normative (the word "norm" originates from the Latin "normatio" meaning the establishment of norms, standards, rules that streamline the sequence of actions) of the theory of synthesis of information technologies (IT) focused on the intellectual support of decision-making processes, the main purpose of which is to organize support of processes connected with decision-making under different types of ignorance. That is, the development of a complex of formalized mathematical models and a system of rules that will allow to organize processes associated with the receipt, processing, analysis and aggregation of source information (expert knowledge; statistical, analytical data, etc.) about the state of the investigated object, process or phenomenon, in order to prepare the received information to make informed and effective decisions.

The purpose of the approach is to consider the approach to creating the basic concepts of normative theory of the synthesis of ITs for decision-making support, based on the systematization of the most studied types of ignorance and a comparative analysis of methods for their modeling.

2. The basic concepts of the normative theory of the synthesis of information technologies for decision support

The foundation of a normative theory lies in systematic generalization of existing models and methods in the field of decision theory, which allow to identify problems associated with the solution of tasks under consideration; formulate goals and criteria for their achievement; generate valid alternative solutions to the problem, evaluate them (to establish order relations on them and highlight extreme ones), and justify the solutions obtained.

The development of a normative theory of the synthesis of information tech-nologies requires solving the following problems [9, p. 3]:

– systematization, formalization and identification of different types of igno-rance, the justified existence of which is determined by a number of methods for their modeling, based on traditional as well as new developing mathemati-cal theories;

– a comparative analysis of modern methods of modeling various types of ignorance in order to determine those that can be used to construct information technologies to support decision-making process under certain types of ignorance;

– formation of a system of conditions, criteria, rules, restrictions, etc., which allow to construct the unified algorithms for the synthesis of information technologies for decision-making support based on the solutions of the first two previous tasks.

Thus, the basis of the concept of the synthesis of IT for decision support is the intellectual integration of four basic aspects (Figure 1):

1. Initial information (expert knowledge, statistical, analytical, experimental data). Initial data can be presented in both quantitative and qualitative scales of measurement. The type of measurement scale determines the nature and structure (presentation form) of the source information.

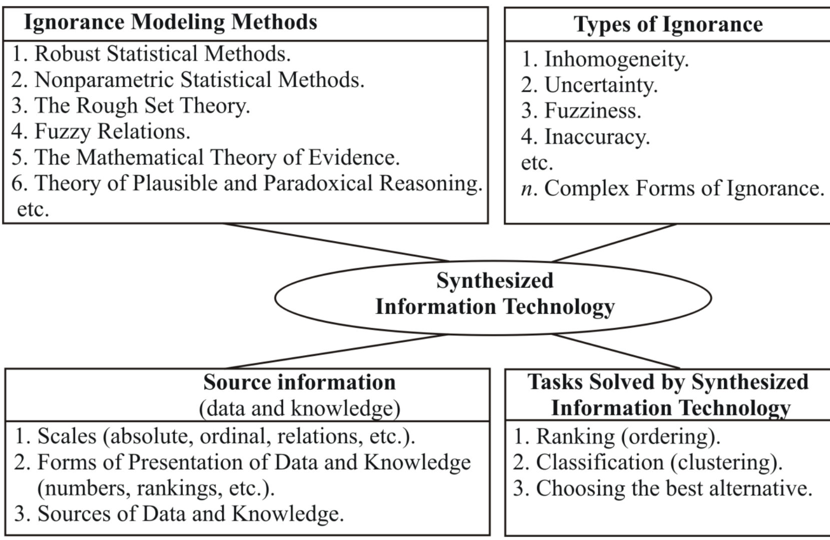

Figure 1. General structure characterizing the main components of synthesized in-formation technology

Data sources can be broadly classified into two categories: internal and external to the system. The source data must meet a set of requirements (criteria): timeliness, reliability, accuracy, richness, degree of compliance with the purpose of the study (relevance); and possess the following properties: objectivity, credibility, com-pleteness, topicality, worth, understandability.

2. A complex of different types of ignorance, reflecting various aspects ("defects") of knowledge (incompleteness, inaccuracy, uncertainty, fuzziness, etc.) [1, p. 290; 3; 4; 17]. These types of ignorance are

inextricably linked with the processes of formalization and presentation of the existing knowledge in intelligent systems, and have a direct impact on the processes of extraction, analysis, presentation and processing of data and knowledge.

3. A range of analysis tasks. The nature of the problem(s) to be solved has a direct impact on the formulation of the research goal, the volume and structure of the used (initial) data, the way they are received, the form in which the results are presented, etc.

4. A range of modeling methods [3, p. 285; 20, p. 5; 20, p. 11; 22, p. 170; 23]. By modeling method we understand the mathematical formalism (theories, methods, approaches, technologies) used to obtain, represent, analyze and process knowledge and data. The choice of modeling method largely depends on the nature and characteristics of the problem (task) being solved. However, when choosing a research method / mathematical apparatus, it is necessary to conduct a thorough and in-depth analysis of all kinds of ignorance that characterize the corresponding problem area (the existing system of knowledge): identify them, determine how to formalize them, establish their significance and degree of influence on the processes associated with the acquisition, presentation, analysis and use of knowledge and data within the existing system of knowledge.

During the analysis of different types of ignorance, it is necessary to take into account which aspects of data or knowledge they are related to, since in relation to different categories of data and knowledge, the same types of ignorance can acquire different interpretations, for example, fuzzy data and fuzzy inference are different concepts, etc. The search of a mathematical formalism that allows to correctly operate with the available data and knowledge with different types of ignorance should be carried out at all stages of data and knowledge manipulation (production, analysis, presentation, processing, using). At the same time, one of the central problems of the normative theory of the synthesis of ITs for decision support is the formalization of the procedure of identification of various types of ignorance, including their complex forms generated by complex types of ignorance, characterized by the simultaneous presence of two or more types of ignorance, which can influence the processes associated with the acquisition and analysis of source data (statistical, analytical, expert information, etc.). An equally important task in this context is the justified choice of a method / mathematical apparatus for modeling the identified types of ignorance (including their possible combinations), capable of carrying out the correct mathematical processing of initial information in the presence of selected types of ignorance, to generate aggregated data and new information necessary for the decision-making process and the interpretation of new (received) data (knowledge).

The solution of these problems is based on a systematic (integrated, multi-aspect) approach to determining various types of ignorance, which creates the conditions for the correct selection and application of methods of analysis and structuring of initial information, and this, in turn, allows to obtain effective results for modeling of relevant subject and problem areas of knowledge.

The procedure of identification of various types of ignorance is based on the formation of a set of qualimetric characteristics (criteria, indicators, etc.) of recognition and identification the type of ignorance (and their combinations) that analyzed in the original system of knowledge, and the construction of a system of decisive rules for the identification (recognition) of types of ignorance on based on a dedicated set of such characteristics.

The choice of methods (mathematical apparatus) for ignorance modeling is determined by the specifics of the problem being solved, the nature (structure) of the accumulated data (knowledge), the purpose of the study, and is determined by a set of qualitative and quantitative characteristics that uniquely characterize the possibilities (the most important and essential aspects for solving this problem under given initial conditions) and the limitations of the method candidate.

The development and evolution of the normative theory of the synthesis of ITs of intellectual support for the decision-making process contributes to the establishment and development of the theoretical and practical aspects of the normative methodology of the synthesis of ITs for decision-making support, as a combination of systematic techniques and methods of intellectual and informational support of the processes of formation and making decisions in different areas of human activity, presented in the form of approaches / procedures / algorithms / methods / techniques for solving various applied problems.

3. Mathematical models of the process of synthesis of information technologies for decision support

The components of IT of ignorance modeling in general terms can be represented by the following tuple:

$$IT = < Z, ST, NF, MN, B >, \quad (1)$$

where $Z = \{z_i \mid i = \overline{1,k}\}$ is the set of tasks to be solved; ST is a data structure; $NF = \{nf_i \mid i = \overline{1,p}\}$ is the set of types of ignorance modeled in decision support sys-tem (DSS); $MN = \{M_i \mid i = \overline{1,n}\}$ is the set of ignorance modeling methods defined for solving each of considered problems, $M_i \subset MN$ is the set of methods $M_i = \{m_j^{(i)} \mid j = \overline{1,n^*}\}$, ($n^* \leq n$), allowing correct processing of data under defined type of ignorance nf_i; B is the procedure for choosing a mathematical apparatus (method) for ignorance modeling, which can be represented as follows

$$B = < T, D, ST, SP, G, F, KN, SRM >, \quad (2)$$

where T is the data structuring task type; $D = \{d_i \mid i = \overline{1,r}\}$ is the set of initial data presented in the form of *ST*; $SP = \{s_i \mid i = \overline{1,g}\}$ is the set of data measurement scales (nominal, ordinal, interval, and ratio); *G* is the procedure of ignorance type identification; *F* is the function of choosing the mathematical apparatus for ignorance modeling; $KN = \{K_j \mid j = \overline{1,o}\}$ is the set of criteria for choosing a modeling method $m_j^{(i)}$, $m_j^{(i)} \in M_i$ that uniquely describe the capabilities and limitations of the method, $|K_j| > 1$; SRM is the system of decision rules for choosing a method $m_j^{(i)}$, based on a formed set of criteria K_j.

The procedure of ignorance type identification can be presented in the form of the next model:

$$G = < D, NF, CN, SRN >, \quad (3)$$

where $CN = \{C_i \mid i = \overline{1,p}\}$ is the set of criteria for recognition and detection the defined type of ignorance; *SRN* is the system of decision rules for detection the type of ignorance nf_i, based on a formed set of criteria C_i.

Consider now the main stages of construction the IT of ignorance modeling:

Stage 1. Determination the type of decision-making problem.

The paper considers two main types of decision-making problems. The first task is to rank the elements of the set $\{D\}$, then the resulting binary relations can be characterized by a linear or weak order. The second task is to extract from the original set $\{D\}$ some subset $\{D^*\} \subseteq \{D\}$ so that its elements have common properties.

Stage 2. Determination the data structure. The data structure ST is determined by a given measurement scale $s_q \in SP$, $q = \overline{1,g}$. Initial data can be presented in one of four data measurement scales (nominal, ordinal, interval, and ratio) in form of qualitative and quantitative value respectively.

The data structure characterizes the model of the source data $D = \{d_i \mid i = \overline{1,r}\}$ and the form of it presentation in the form of labels, rankings, numbers, intervals, binary relations, etc. The form of presentation of the source data largely depends on the nature of the problem under consideration and the objectives of the research.

Stage 3. The formation of a set of source data $D = \{d_i \mid i = \overline{1,r}\}$ in accordance with a given measurement scale s_q.

Stage 4. Identification the types of ignorance present in the original data set $D = \{d_i \mid i = \overline{1,r}\}$, or arising in the process of data obtaining. The procedure for ignorance type identification consists a set of ignorance identification rules that used a given set of ignorance identification criteria.

Let $NF = \{nf_i \mid i = \overline{1,p}\}$ be a set of types of ignorance, that are modeled in a sys-tem (decision support system). Each nf_i corresponds to a set of crite-ria $C_i = \{c_j^{(i)} \mid j = \overline{1,t}\}$, $C_i \subset CN$, which allow to uniquely identify the presence of nf_i in the source dataset $D = \{d_i \mid i = \overline{1,r}\}$.

By forming a set of decision rules $SR_i = \{R_l^{(i)} \mid l = \overline{1,h}\}$, $SR_i \subset SRN$ it can be established the procedure for ignorance type nf_i identification in the original dataset:

$$R_l^{(i)}: \quad \left(c_j^{(i)}, D\right) \rightarrow nf_i, \tag{4}$$

If the initial dataset D is characterized by the presence of $c_j^{(i)} \in C_i$, then nf_i is identified.

In this case, the antecedent can be formed using operations $\vee$ and $\wedge$, and their combinations, for example:

$$R_1^{(i)}: \quad \forall c \in C_i : \left(\wedge c, D\right) \rightarrow nf_i; \tag{5}$$

$$R_2^{(i)}: \quad \exists c \in C_i : \left(c_j^{(i)} \vee c_{j+1}^{(i)}, D\right) \rightarrow nf_i; \tag{6}$$

and etc.

Stage 5. The choice of the mathematical formalism for nf_i modeling.

The procedure for choosing a mathematical apparatus can be represented by a method or a group of methods that allow to correctly process data (that are formed or analyzed) under identified type of ignorance nf_i, or their group. The structure of the source data affects the choice of ignorance modeling method.

4. Methodology of the process of synthesis of information technologies for decision support

The systematic methodology for the synthesis of ITs for ignorance modeling allows to generate ITs for decision support on the basis of the generated set of rules and parameters, such as the type of data structuring task, data structure, identified types of ignorance, or their combinations, etc. Figure 2 shows stages of the synthesis of information technology for decision support under different types of ignorance.

The IT synthesis methodology can be formally presented in the form of the following successive stages:

Stage 1. Definition of the purpose (goals) of the analysis (evaluation).

In system analysis two types of goals are distinguished. Qualitative is a goal whose achievement is expressed in a nominal or ordinal scale. Quantitative is the goal whose achievement is expressed in quantitative scales (interval and ratio). The form of presentation of the evaluation result depends on determination of the purpose of assessment.

Stage 2. Determination of the composition and structure of the analysis task.

There are considered five basic data mining tasks:

1. classification (recognition, ranking);
2. clustering (determination of the inherent decomposition into the analyzed data into homogeneous groups – clusters);
3. association;
4. time series analysis;
5. prediction.

There are three main tasks of decision making [12]: ordering a set of alternatives (ranking); grouping a set of alternatives to decision classes (clustering); choosing the best alternative.

The type of source data structuring task determines the type of data structuring procedure. So, for example, to solve the problem of choosing the best alternative, the ranking procedure of the initial set of alternatives can be used.

Stage 3. Determination of the method of obtaining initial data.

The methods of obtaining initial information can conditionally be divided into the following groups:

1. empirical – methods of obtaining empirical information (empirical data);
2. theoretical – methods of obtaining theoretical information;
3. mixed (semi-empirical) – methods for obtaining both empirical and theoretical information.

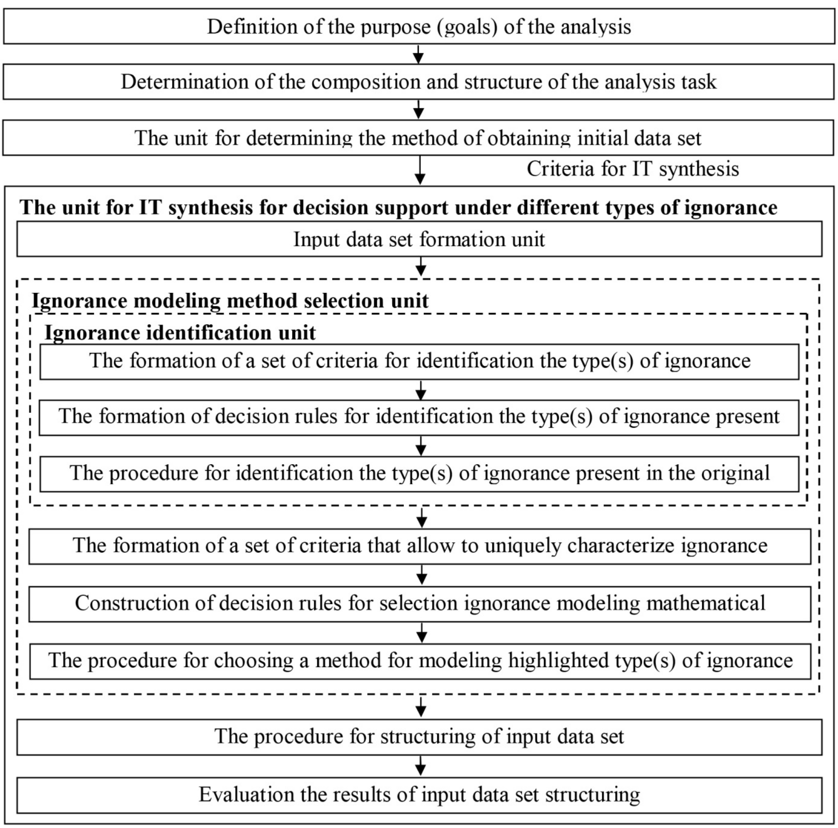

Figure 2. The procedure of IT synthesis for decision support under different types of ignorance

The most widespread are empirical methods of obtaining information, among which are the description, comparison, measurement, observation, experiment, analysis, etc. Examples of empirical data are research results, respondents' answers, expert estimates, observations, measurements, etc. One of the most common empirical methods is the expert judgment method. Distinguish between individual and collective expert assessments.

The information obtained can be both qualitative and quantitative in nature, and represents estimates in one of the four basic types of measurement scales (nominal, ordinal, interval, and ratio).

Numerical scales (ratio and interval) are used, for example, to measure the number of objects (alternatives, events, etc.), or show how many points the indicator of one object differs from the indicator of another object. The estimates obtained in such scales are ordinary natural numbers whose values lie in a certain interval.

A nominal scale consists of names, categories, names for classifying and sorting objects or observations by some criterion. In this scale numbers are used only as labels, i.e. just to distinguish between objects.

An ordinal scale (ordinal, rank, preference) is used to measure the ordering (ranking) of objects by one or a combination of signs. Rankings are formed using the ranks (numbers of the analyzed object in an ordered series). Numbers in the ordinal scale are used to establish order between objects using two types of relations: equivalence (~) and preference ($\succ$).

The choice of the method of obtaining the source information affects the structure of the source data.

Stage 4. Synthesis of IT for decision support under highlighted type(s) of ignorance.

The basis of the methodology for synthesis of IT for decision support is a model of the next form:

$$SIT = < V, PS, ITS, SGR, IP, R > , \tag{7}$$

where V is the vector of input parameters (primary information); PS is the set of parameters (criteria) for generating the IT; ITS is the procedure for the IT synthesis; SGR is the system of rules for the generation of IT for decision support; IP are the information processes; R is the vector of result parameters.

The vector of input parameters includes a set of source data, parameters, conditions and restrictions necessary to solve the problem.

The IT synthesis procedure is a set of interconnected, sequential processes associated with obtaining and processing (analyzing) a variety of source and synthesized (during the analysis) data and knowledge, interpreting the results obtained under various kinds of ignorance, and used to solve the problem (tasks analysis).

The parameters (criteria) for synthesis of IT generation rules $PS = \{Par_i \mid i = \overline{1, m}\}$ can be conditionally divided into two categories: a set of input / initial / source (PS^V) and a set of intermediate (PS^P) parameters, so that $PS = PS^V \cup PS^P$. A PS^V set can be formed based on the values of the vector of input parameters *V*. These parameters include: the nature of the problem under consideration (individual, collective choice; single, multi-criteria etc.), the type of data structuring procedure (choice, ordering, grouping, etc.), the form of the initial data presentation (crisp, interval, fuzzy data), the data structure (ordering, conditional gradations, words, numbers, binary relations, etc.), the method of obtaining the initial information, the form for presenting the result, etc.), and follow directly from the subject (goals) of the research, the limitations and conditions put forward, the provisions of the scenario (regulation) of the examination, etc., i.e. known at the initial stage (before the examination).

Since IT is a process that uses a combination of means and methods of collecting, processing, visualizing and transmitting data to generate new information, some parameters of IT synthesis (PS^P) can be formed directly in the process of obtaining and processing the accumulated information. These parameters include: distinguished types of ignorance; used mathematical apparatus for modeling highlighted type(s) of ignorance, etc. Taking into account that IT is a complex of interconnected processes aimed at processing information, some of which are sequential, these parameters cannot be obtained at the initial stage, since they are synthesized (formulated) as these processes proceed.

For construction the IT generation rules, it can be used either one criterion Par_i or their combination $\wedge Par_i$, $i \le m$. The IT generation rules can be represented as follows:

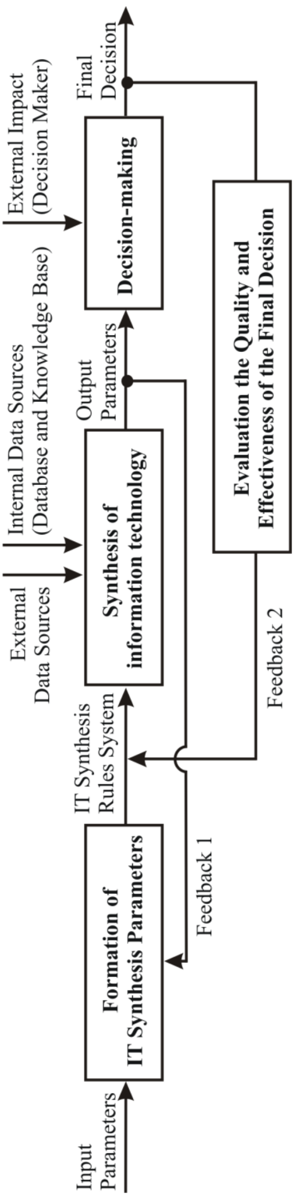

Figure 3. A generalized structural diagram of the synthesis of information technologies for decision support

– one criterion for IT synthesis is highlighted:

$$PIT_j : \quad Par_i \to IT_j , \qquad (8)$$

– a number of criteria are highlighted:

$$PIT_{j+1} : \quad \wedge Par_i \to IT_{j+1} , \ i \le m , \qquad (9)$$

It is used one or a combination of criteria (parameters) for IT synthesis as antecedent; as in the role of a consequent, the generated IT, taking into account the formed criteria, is used.

The vector of parameter-results is a set of output parameters of the system, determined by the content and specificity of a particular (solvable) task or problem, on the basis of which a management decision is synthesized. Figure 3 shows a generalized structural diagram of the synthesis of ITs for decision support.

In the IT synthesis parameters formation block a list of IT synthesis parameters PS^V is developed based on the received input data V. These parameters form the basis of the rules for generating IT at a preliminary stage. Then, already in the block for synthesis of IT, a set of parameters PS^P is produced, and the resulting IT synthesis rules are formed.

The following sequential tasks are solved in the IT synthesis block:

the for-mation of a set of initial data, the identification of the type of ignorance, the choice of the mathematical apparatus for ignorance modeling, the analysis of the obtained expert information, its visualization and preparation for decision-making. When the expert procedure is conducted from several rounds, the first three tasks are repeatedly performed.

The procedure of forming a set of source data largely depends on the selected method of obtaining source information. At this stage, the source data takes the form of numbers, rankings, paired comparisons, intervals, etc., depending on the selected measurement scale. Thus, the structure of the source data is formed. Input data can be obtained from both internal and external sources.

The choice of ignorance modeling methods depends on the structure of the source data and the types of ignorance that influenced the process of extracting information and forming a set of source data, or contained in the received information (data set). The result of the processes occurring in this block is structured information for decision-making, which meets the stated goals of the analysis. If for some reason the output parameters do not meet the requirements (feedback 1), then changes are made to the criteria for the IT synthesis. In the worst case, the procedure for generating a set of input data is repeated (in this case, the set PS^V can be changed, for example, by changing the composition of the expert group, the form of presentation of the input data can be changed, etc.).

A final analysis of the structured expert information, an interpretation of the obtained results, and a final managerial decision are made in the next block. Next, quality and effectiveness assessment, control and adjustment of the decision are performed. In general, an assessment of the quality of a decision can be made at the stages of development, adoption and implementation of a decision. As an indicator of the quality of the decision made, for example, an indicator of the economic efficiency of its implementation can be used. The results can be stored in the knowledge base and displayed in the system of decision rules for the synthesis of IT (feedback 2).

The basic principles of the synthesis of IT are invariant to the type of problem being solved and the method for identifying and presenting initial information.

5. Construction of decision rules for selection ignorance modeling mathematical formalism

The procedure for choosing the ignorance modeling method in general terms can be represented by the tuple (2).

Let $MN = \{M_i \mid i = \overline{1, p}\}$ be a set of ignorance modeling methods, where $M_i \subset MN$ is a set of methods $M_i = \{m_j^{(i)} \mid j = \overline{1, p^*}\}$, ($p^* \le p$), that allow to correctly process data under highlighted type(s) of ignorance nf_i. Each method $m_j^{(i)}$ is assigned a set of criteria $K_j = \{k_l^{(j)} \mid l = \overline{1, o^*}\}$ to uniquely characterize the capabilities and limitations of the method $m_j^{(i)}$ used for solving the analyzed problem T under highlighted type(s) of ignorance nf_i in initial data set $D = \{d_i \mid i = \overline{1, r}\}$. Thus, a matrix of characteristics of decision-making methods $KN = \{K_j \mid j = \overline{1, o}\}$, ($o^* \le o$) is formed. Each row of such matrix represents the values of the characteristic properties (criteria) K_j for method $m_j^{(i)}$, each column contains labels of methods $m_j^{(i)}$, data at the intersection of corresponding row and column contains the values of criterion $k_l^{(j)} \in K_j$ for $\forall m_j^{(i)} \in M_i$. Based on the *KN* matrix values, a system of decision rules for selection ignorance modeling methods is generated.

The modeling method $m_j^{(i)} \in M_i$ is determined based on a given selection function such as

$$F = f(T, ST, K_j, s_q, nf_i), \tag{10}$$

where T is the data structuring task type; *ST* is data structure; K_j is a set of criteria for ignorance modeling method $m_j^{(i)} \in M_i$ selection under highlighted type(s) of ignorance nf_i, $K_j \subset KN$; s_q is a given data measurement scale $s_q \in SP$; nf_i is an identified (highlighted) type of ignorance.

Let $M_i = \{m_j^{(i)} \mid j = \overline{1, p^*}\}$, $M_i \subset MN$, be a set of modeling methods allowing one to correctly process a set of initial data under highlighted type of ignorance nf_i, and a set of criteria $K_j = \{k_l^{(j)} \mid l = \overline{1, o^*}\}$ allowing to characterize the modeling method $m_j^{(i)}$, $m_j^{(i)} \in M_i$. Then, for each $M_i \subset MN$, it can be constructed a set of decision rules $RP_i = \{Rm_j^{(i)} \mid j = \overline{1, p^*}\}$, $RP_i \subset SRM$, where $Rm_j^{(i)}$ is a decision rule (or set of rules) for selection the method $m_j^{(i)} \in M_i$ for processing data under highlighted type of ignorance nf_i.

The procedure for choosing the nf_i modeling method is carried out in three stages as follows. At the first stage, the set of analyzed (available) meth-ods $MN \Rightarrow \{\{Gr_1\}, \{Gr_2\}, \ldots, \{Gr_z\}\}$, ($Gr_z \subseteq MN$, $\{Gr_z\} = \{m_1, \ldots, m_r\}$, $t \ge z \ge 1$), $t = |MN|$ are grouped for a number of features (criteria) $K^* \subset KN$

that characterize the initial parameters (data) of the problem under consideration.

Such criteria may include the type of solution required (choosing the best alternative, highlighting a group of (equivalent) alternatives, ordering alternatives, etc.); time limit for solving the problem under consideration; the structure of the source data (type of scale, nature of estimates); the structure of the set of alternatives (finite set, the possibility of the appearance of new alternatives during the problem solving process); type of selection procedure (single, multiple); the presence of selection criteria (non-criteria (selection based on binary relations, the use of selection functions) and criteria (single / multi-criteria) problems of structuring; the number of decision-makers / experts (individual, group choice).

Among the indicated set of criteria a set of basic (K_1^*) and a set of auxiliary (K_2^*) criteria can be distinguished, such as $K^* = K_1^* \cup K_2^*$.

On the basis of the mathematical apparatus of the theory of rough sets [22, p. 169], the initial analyzed set of methods can be divided, in accordance with a given attribute / criteria (for example, by the type of problem under consideration), into groups of methods used to solve the problem under consideration ($\forall k \in K_1^* : (\wedge k) \to Gr_z$), based on a set of basic characteristics K_1^*. Also a set of methods that can potentially be used to solve the problem under consideration, based on a set of auxiliary characteristics K_2^*, can be highlighted.

At this stage, it is possible to filter out a part of the methods unsuitable for solving the formulated problem under the imposed restrictions, based on the state-ment of the problem, the formulation of the analysis goal and the structure of the initial data. Further analysis is carried out among a group of methods that satisfy the given conditions and constraints.

At the second stage, a mask of values of characteristic properties (qualitative, quantitative criteria) of potential modeling methods is formed. The values of the obtained mask are used as threshold values in the resulting system of decision rules, forming a set of candidate methods that correctly operate with a set of source data $D = \{d_i \mid i = \overline{1,r}\}$. To avoid conflicts and reduce the dimension of the set of candidate methods, the priorities of the mask elements can be determined.

Based on the results of this stage, an *SRM* system of decision rules for choosing a modeling method is formed, which are entered into the system knowledge base in next form

$$Rm_j^{(i)}: \quad \forall k_l^{(j)} \in K_j : \left(\wedge k_l^{(j)}, nf_i \right) \to m_j^{(i)}, \tag{11}$$

or

$$Rm_j^{(i)}: \quad \exists k_l^{(j)} \in K_j : \left(k_l^{(j)}, nf_i \right) \to m_j^{(i)}, \tag{12}$$

or

$$Rm_j^{(i)}: \quad \exists (k_l^{(j)}, k_c^{(j)}) \in K_j : \left(k_l^{(j)} \vee k_c^{(j)}, nf_i \right) \to m_j^{(i)}, \ \forall (l,c) < o^*, \tag{13}$$

where $m_j^{(i)} \in M_i$, $M_i \subseteq Gr_z$, $K_j \subseteq KN \setminus K^*$, $Rm_j^{(i)} \in SRM$.

At the third stage, the choice of the best method (group of alternative methods) is performed. If, according to the results of the second stage, more than one method was chosen, then at the third stage, a set of criteria of effectiveness and efficiency of the methods candidate are formed. These criteria include: reliability (the possibility of obtaining reliable and reproducible results); validity / adequacy (degree of conformity of the method to its intended purpose); sensitivity of the method, laboriousness (time and other resources); objectivity of the results; accessibility and ease of use; effectiveness of goal achievement; robustness, etc. The value of each criterion is expressed in numerical form, after which the problem of multi-criteria optimization is solved to select the optimal solution (method).

The obtained data are entered into the system knowledge base. Thus, a library of modeling methods for solving the most common decision-making tasks (problems) under different types of ignorance can be formed.

6. Example of choosing a method for solving the multi-criteria selection problem

In general terms, the statement of the problem of multi-criteria choice can be formulated as follows. Let a global goal, a set of acceptable objects (alternatives) $\mathrm{A} = \{a_i \mid i = \overline{1,n}\}$ and a vector $f = (f_1, f_2, ..., f_m)$, whose elements are numerical functions (criteria) that characterize the main parameters of a given set of objects *A*, are given. The value $\overline{F}(a) = \{f_j(a) \mid j = 1, m\}$ of the vector criterion f for a certain $a \in A$ is a vector objective function (vector estimate) of a possible solution *a*. The problem boils down to finding such solution $a^* \in A$ when $\overline{F}(a^*) \to \underset{a^* \in \mathrm{A}}{extr}$.

In some cases, the best (optimal) alternative is such a^*, which provides the maximum value of the function $\overline{F}(a^*)$, i.e. $a^* = \arg\{\max_i \overline{F}(a_i)\}$, $a^* \in A$.

Suppose that the problem of multi-criteria choice to be solved is characterized by the following initial conditions, Table 1.

Table 1

A set of initial conditions

Name	Value
type of solution required	choosing the best alternative, highlighting a group of (equivalent) alternatives
type of selection procedure	single
the presence of selection criteria	multi-criteria decision-making problem
the number of decision-makers / experts	1
structure of the source data	hierarchy
the way in which expert judgments might be obtained	binary relations

To solve this problem, the following methods can be applied: the analytic hierarchy process (AHP) [18, p. 73], the analytical procedure for structuring a set of alternatives [10, p. 82–84], the criterial method of the analytic stochastic procedure [11, p. 81–82].

Then the next additional restrictions are imposed, Table 2.

Table 2

An additional restrictions

Name	Value
the structure of a set of alternatives	finite set
the form of presentation of expert judgments	no restrictions

To solve this problem, various modifications of the AHP technique can be used [2, p. 151; 6–8; 21, p. 746].

Let $M_{AHP} = \{m_j^{AHP} \mid j = \overline{1,r}\}$ be a set of modifications of the AHP technique, and $K_{AHP} = k_i^{AHP} \mid i = \overline{1,z}\}$ be a basic set of criteria that allow to unambiguously characterize each of the considered method m_j. To select a modification of the AHP technique, the following criteria can be used [13, p. 48–52]: a type of hierarchy (k_1^{AHP}), which determines the nature of the relationship between the criteria and alternatives; a method of forming pairwise comparison matrices (PCM), k_2^{AHP}; a method of obtaining an eigenvector (k_3^{AHP}); a method of synthesis of the final solution (k_4^{AHP}).

Based on the data obtained, it is possible to create a matrix of criteria KN_{AHP} for modifications of the AHP technique selection modifications.

Thus, the problem of choosing the AHP technique modifications can be divided into three subproblems: the choice of the method of forming pairwise comparison matrices, the choice of the method of obtaining an eigenvector, the choice of the method of synthesis of the final solution. Let us consider each of them sequentially:

1. Choice of the PCM formation method.

Let a set of methods for the PCM formation $M_{mps} = \{m_j^{mps} \mid j = \overline{1,q}\}$, and a basic set of criteria $K_{mps} = k_i^{mps} \mid i = \overline{1,t}\}$ that allow to uniquely characterize each of the consid-ered method m_j are given. Let us formulate a set of criteria that allow to characterize various ways of PCM forming: a restriction on the number of elements at each level of the hierarchy (k_1^{mps}); a mutual exclusion of objects (k_2^{mps}); a mutual exhaustibility of objects (k_3^{mps}); the presence of identical objects in one or more of the analyzed properties (k_4^{mps}); the ability to present objects at time intervals (k_5^{mps}); a form of expert judgments presentation (k_6^{mps}); the ability to highlight groups of alternatives (k_7^{mps}); a prerequisite for evaluating all analyzed objects (k_8^{mps}). A number of PCM formation methods has been discussed in [2, p. 155; 6–8; 18, pp. 18–19; 21].

Thus, it is possible to create a matrix of criteria KN_{mps} for choosing the PCM formation methods. Table 3 shows the incomplete KN_{mps} matrix for crisp expert judgments.

2. Choice of methods for obtaining an eigenvector.

Let a set of methods for obtaining an eigenvector $M_{vp} = \{m_j^{vp} \mid j = \overline{1,l}\}$, and a basic set of criteria $K_{vp} = k_i^{vp} \mid i = \overline{1,p}\}$ that allow to uniquely characterize each of the considered method m_j are given.

Let us single out a number of criteria that allow to characterize various ways of obtaining the eigenvector: a restriction on the number of elements at each level of the hierarchy (k_1^{vp}); the PCM formation method (k_2^{vp}); a degree of consistency of the PCM (k_3^{vp}); the complexity of the method (resources expended), k_4^{vp}; method sensitivity (k_5^{vp}); a prerequisite for the

Table 3

The value of the selection criteria for the method of PCM construction based on crisp expert judgments

A set of criteria for the method of PCM construction								The method of PCM construction
k_1^{mps}	k_2^{mps}	k_3^{mps}	k_4^{mps}	k_5^{mps}	k_6^{mps}	k_7^{mps}	k_8^{mps}	
7–9	+	+	–	–	crisp	–	+	Saaty method
7–9	+	+	+	–	crisp	–	+	Method of copying
No limit	+	+	–	+*	crisp	–	+	Method for comparing objects with respect to standards
No limit	+	+	+	–	crisp	+	–	– DS/AHP knowledge matrix
No limit	–	+	+	–	crisp	+	–	– DSmT/AHP knowledge matrix

*it is not possible to compare objects in pairs

absence of zero values of weights (k_6^{vp}) [2, p. 155; 6–8; 13, 21] discussed a number of methods for obtaining an eigenvector.

3. Choice of method of synthesis of the final solution.

Consider two main methods of aggregation (synthesis of the final solution): a convolution method [15, p. 75; 16] and an evidence combination rules [5; 19, pp. 15–26; 20, pp. 5–10; 20, p. 16–18].

The choice of a method of convolution of criteria is carried out on the basis of the available information about the problem under consideration and/or based on simplicity of solving the resulting scalar problem [14, p. 3–8]. In [14, p. 3–8], the conditions of applicability of different scalarization methods for solving multi-criteria selection problems are considered. The choice of rules for combining evidence depends on: the analysis model (Dempster model, Dezert-Smarandache model), the nature of the analyzed data (information about conflicts and consensus; the degree of interaction and structure of expert evidence, etc.); structure of a set of alternatives.

Let a set of methods of synthesis of the final solution $M_{sr} = \{m_j^{sr} \mid j = \overline{1,s}\}$, and a basic set of criteria $K_{sr} = k_i^{sr} \mid i = \overline{1,g}\}$ that allow to uniquely characterize each of the considered method m_j are given.

Let us single out a number of criteria that allow to characterize various methods of synthesis of the final solution: a form of expert judgments presentation (k_1^{sr}); a PCM formation method (k_2^{sr}); a degree of con-sistency of the PCM (k_3^{sr}); a result presenting form (k_4^{sr}); a structure of initial data (agreed, arbitrary compatible and other expert evidence), k_5^{sr} ; local conflict information (k_6^{sr}); the use of additional coefficients (k_7^{sr}), for example, the optimism coefficient in the transition from the interval scale to crisp estimates.

Formation of decision rules for choosing a modification of the AHP method. According to Table 1, as an example the follow system of decision rules for choosing the method of PCM construction could be generated:

1. (k_1^{mps} = „7–9") $\wedge$ (k_2^{mps} = „+") $\wedge$ (k_3^{mps} = „+") $\wedge$ (k_4^{mps} = „–") $\wedge$ (k_5^{mps} = „–") $\wedge$ (k_6^{mps} = „crisp") $\wedge$ (k_7^{mps} = „–") $\wedge$ (k_8^{mps} = „+") $\rightarrow$ „Saaty's method".

2. (k_1^{mps} = „–") $\wedge$ (k_2^{mps} = „+") $\wedge$ (k_3^{mps} = „+") $\wedge$ (k_4^{mps} = „+") $\wedge$ (k_5^{mps} = „–") $\wedge$ (k_6^{mps} = „crisp") $\wedge$ (k_7^{mps} = „+") $\wedge$ (k_8^{mps} = „–") $\rightarrow$ „DS/AHP knowledge matrix".

3. (k_1^{mps} = „–") $\wedge$ (k_2^{mps} = „–") $\wedge$ (k_3^{mps} = „+") $\wedge$ (k_4^{mps} = „+") $\wedge$ (k_5^{mps} = „–") $\wedge$ (k_6^{mps} = „crisp") $\wedge$ (k_7^{mps} = „+") $\wedge$ (k_8^{mps} = „–") $\rightarrow$ „DSmT/AHP knowledge matrix".

Let us consider an example of construction the decision rules for choosing a modification of the AHP method.

None type of ignorance have been identified in initial data set:

1. (k_1^{AHP} = „type A") $\wedge$ (k_2^{AHP} = „Saaty's method") $\wedge$ (k_3^{AHP} = „linear convolution of criteria") $\wedge$ (k_4^{AHP} = „geometric mean method") $\rightarrow$ „AHP by Saaty".

2. (k_1^{AHP} = „type B") $\wedge$ (k_2^{AHP} = „DS/AHP knowledge matrix") $\wedge$ (k_3^{AHP} = „Beynon's method") $\wedge$ (k_4^{AHP} = „Dempster's rule of combination") $\rightarrow$ „DS/AHP".

A fuzziness has been identified in initial data set [6; 8, p. 50]:

3. (k_1^{AHP} = „type A") $\wedge$ (k_2^{AHP} = „modified AHP (triangular fuzzy numbers)") $\wedge$ (k_3^{AHP} = „Chang's method") $\wedge$ (k_4^{AHP} = „linear convolution of criteria") $\rightarrow$ „Chang's fuzzy AHP)".

The additional coefficients have been used [6, pp. 63–65]:

4. (k_1^{AHP} = „type A") $\wedge$ (k_2^{AHP} = „modified AHP (triangular fuzzy numbers)") $\wedge$ k_3^{AHP} = „–") $\wedge$ (k_4^{AHP} = „calculation of entropy coefficients of a weighted fuzzy matrix") $\rightarrow$ „Chang's entropy method".

High level of conflict of expert evidence (lack of consistency) [2, p. 155; 7, p. 3]:

5. (k_1^{AHP} = „type B”) ∧ (k_2^{AHP} = „AHP knowledge matrix”) ∧ (k_3^{AHP} = „Beynon's method”) ∧ (k_4^{AHP} = „proportional conflict redistribution rule”) → „DS/AHP”.

6. (k_1^{AHP} = „type B”) ∧ (k_2^{AHP} = „AHP knowledge matrix”) ∧ (k_3^{AHP} = „Dezert-Smarandache method”) ∧ (k_4^{AHP} = „proportional conflict redistribution rule”) → „DSmT/AHP”.

An acceptable level of consistency, but an additional condition is imposed [19, p. 26; 20, p. 10], Table 4.

Table 4

An added restrictions

Name	Value
the structure of a set of alternatives	the power of a set of alternatives can be changed
during the analysis	

7. (k_1^{AHP} = „type B”) ∧ (k_2^{AHP} = „AHP knowledge matrix”) ∧ (k_3^{AHP} = „Beynon's method”) ∧ (k_4^{AHP} = „Smets's rule of combination”) → „DS/AHP”.

The following notation has been used: DS/AHP is the Dempster Shafer-Analytical Hierarchy Process [2, p. 155]; DSmT/AHP is the Dezert-Smarandache-Analytical Hierarchy Process [7, p. 3]. Type A includes hierarchies in which each criterion is associated with all available alternatives. Type B includes hierarchies in which each criterion may not be associated with all available alternatives.

7. Conclusions

A procedure for the selection of ignorance modeling methods has been pro-posed. This approach allows to generate a system of decision rules for an informed choice of the mathematical formalism of modeling various types of ignorance that affect the processes of obtaining information and analyzing a set of source data. The IT generation rule is an algorithm for solving the stated analysis problem based on the mathematical apparatus used, taking into account the specifics of the source data. It is proposed to use data structuring task type, data structure, highlighted type(s) of ignorance, mathematical formalism of modeling highlighted type(s) of ignorance, form for presenting the result, etc., as parameters (criteria) for construction of IT generation rules.

The concept of the normative theory of the synthesis of information technologies focused on the intellectual support of decision making processes has been proposed. The basic principles of the synthesis of information technologies are invariant to the type of problem being solved and the method for identifying and presenting initial information. A set of rules and formalized mathematical models of the process of synthesis of information technologies for decision support under various types of ignorance has been developed.

The basis of the proposed concept is the integration of four basic components (a set of initial data and knowledge (system of knowledge); a complex of kinds of ignorance affecting the processes of obtaining, processing, analyzing initial data and knowledge, synthesis and interpretation of the final solution; a range of tasks analysis; a range of methods of ignorance modeling) and the accounting of system-forming links between them, with the aim of organizing support of decision making processes and generation of effective optimal decisions synthesized under ignorance of the various nature.

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