IMPROVING THE EFFICIENCY OF STUDENTS' INDEPENDENT WORK WHILE STUDYING DIFFERENTIAL EQUATIONS USING THE MAPLE SYSTEM

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INTRODUCTION

The nature of the work of specialists today brightly differs from the work of a specialist in the same field 10 years ago. This difference consists in the intensified implementation of modern information technologies in most spheres of human activity. This trend did not bypass the training of specialists in the educational field, and put forward new requirements for mathematical training of students in higher education. Intensive was the introduction of different systems of computer mathematics (SCM) in the training process. A prerequisite for such intensive introduction was also the improvement of technical computer base in educational institutions. Therefore, there are practically no obstacles to the application of SCM. Consequently, there is a question of creation, development and improve the quality of mathematical training in universities. The feasibility of applying SCM in the training process has been verified by many theoretical and experimental studies.

In addition, the trend of education is characterized by an increase in the proportion of independent work. Therefore, one of the urgent tasks of training is to form the abilities and skills of independent work of the student and its activation in different available ways.

1. The problem's prerequisites emergence and the problem's formulation

One of the conditions for improving mathematical training in higher education institutions is the active use of computer mathematical systems (CMS). Every year, the number of scientific and educational and methodological works devoted to the use of CMS in the process of higher mathematics training is increasing. The general questions of using information technologies in mathematics teaching at secondary and high schools are investigated in the works of V. Y. Bykov¹, M. I. Zhaldak², V. I. Klochko, S. G. Litvinov³, V. M. Mikhalevich, S. A. Rakov, Y. S. Ramski, O. V. Spivakovsky, S. A. Semerikov, Y. V. Trius⁴.

According to many experts, such as V.Y. Bykov¹, S.O. Semerikov⁴, Y.V. Tryus the presence of modern mathematical packages, such as Mathematica, MathCAD, Maple, create conditions for a radical review of the content, goals, forms, means and methods of mathematics learning. Such situation stimulates intensive searches in the specified field. The number of works with application of mathematical systems in teaching mathematics grows every year.

The system of computer mathematics gives an opportunity to enrich the subjects of natural-and-mathematical cycle, to extend their application, to influence significantly the mathematical activity. Therefore, the main content of mathematical training for students will be not mastering certain algorithms of problem solving (they, by the way, are quite effectively solved by computer), but mathematical competence, understanding, application of mathematical research methods⁵. All this should be taken into account in the development of methodological systems for teaching mathematically oriented subjects in higher education.

The importance of individualization of learning is noted in the works of O. V. Spivakovskyi, who believes that in the traditional system of education opportunities for individualization are rather limited, because today no society is able to provide each student with a personal teacher. In practice it turns out that the teacher's explanation, which is accessible to

¹ Биков В. Ю. Цифрова трансформація суспільства і розвиток комп'ютернотехнологічної платформи освіти і науки України. Матеріали методологічного семінару НАПН України "Інформаційно-цифровий освітній простір України: трансформаційні процеси і перспективи розвитку». 4 квітня 2019 р. / За ред. В. Г Кременя, О. І. Ляшенка. К, 2019. С. 20–26.

² Жалдак М. І. Педагогічний потенціал комп'ютерно-орієнтованих систем навчання математики. Комп'ютерно-орієнтовані системи навчання : НПУ імені М. П. Драгоманова. 2003. Вип. 7. С. 3–16.

³ Литвинова С. Г. Використання систем комп'ютерного моделювання для проєктування дослідницьких завдань з математики. Фізико-математична освіта. Суми, 2018. Вип. 1(15). С. 83–89.

⁴ К. І. Словак, С. О. Семеріков, Ю. В. Триус Мобільні математичні середовища: сучасний стан та перспективи розвитку. Науковий часопис НПУ імені М. П. Драгоманова. Серія 2. Комп'ютерно-орієнтовані системи навчання. 2012. С. 102–109.

⁵ Михалевич В. М., Тютюнник О. І. Використання системи комп'ютерної алгебри для висвітлення ключових ідей симплекс-алгоритму. Теорія та методика навчання математики, фізики, інформатики. 2011. Вип. IX. С. 113–118.

some students, is not sufficient for others, and for others, on the contrary, seems very detailed. Consequently, low-achieving students when solving problems do not always get the help they need, and the more capable ones often have to be «stopped»⁶.

By means of a computer it is possible to provide individual training «in mass order», especially in the process of choosing a learning action (explanation, prompting, encouragement), take into account the learning of each particular student. Using a computer allows taking into account the peculiarities of the student's cognitive processes – perception, thinking, memory and helping the student with his/her individual abilities⁶.

In this article, the scientist shares his experience of using the Maple computer algebra system in teaching mathematics to students majoring in technical studies. Authors says that the course of mathematics for engineering students should be focused on an applied approach to mastering the mathematical apparatus, rather than computer technology. This approach teaches future engineers to use that skill to solve their job tasks. The authors have used Maple computing environment in teaching mathematics for several years. This graphic system can produce curve and surfaces graphics specified in different coordinate systems quickly and accurately. The animation of basic mathematical notions help learners succeed in course without sticking to complex computations. Visualized mathematical knowledge via Maple allows improving education⁷.

In this paper ⁸ presents the use of Maple as a multimedia tool in classrooms, in particular, in mathematics and related science classes for undergraduate students. Author says the mathematical computations and manipulations can be extensive and tedious for students in solving large and complex problems. He employ interactive Maple sessions in many classrooms such as in calculus, applications of abstract algebra, combinatorial, and linear algebra to solve for solutions in either numeric form or symbolic form. With explanation of the computational process and

⁶ Співаковський О. В. Теорія й практика використання інформаційних технологій у процесі підготовки студентів математичних спеціальностей : монографія. Херсон: Айлант, 2003. 229 с.

⁷ S. Suanmali, «Maple in Mathematics,» Fifth International Conference on Information Technology: New Generations (itng 2008), 2008, pp. 528-533, doi: 10.1109/ITNG.2008.146.

⁸ Salleh, Tuan & Zakaria, Effandi. (2012). Module for learning integral calculus with Maple: Lecturers' views. Turkish Online Journal of Educational Technology. 11. 234-245.

time saving involved, students find the integration of this technology into regular lectures make a significant improvement in their learning process.

A study to explore lecturers' views regarding a newly developed integral calculus with Maple software module was conducted. Engineering technology students can attain a meaningful mathematics learning if they are allowed to actively participate in hands-on activities. Nine lecturers with at least eight years of teaching experience were involved in the evaluation of the module. As a result, a module which emphasized on student-centered learning based on conceptual and procedural understanding and metacognitive awareness teaching approach will be produced. This module will be used to enhance students' procedural and conceptual understanding in learning integral calculus at the university.

The aim of the present paper ⁹ is to have an introduction trip focusing on the advantages of using powerful software like Maple in computational teaching. Some examples are focused on representing the objects and basic algebraic manipulation commands, very successfully used in teaching sciences. In this context, the resources, including the collection of online lessons plan, videos and other resources were created to provide a better understanding of CT. But the most important are software resources, because of their flexibility and wide application tools.

In the article¹⁰ authors shares the experience of using Maple computer algebra system in teaching mathematics to students majoring in technical studies. Universities need to change classical teaching methods to educate top graduates properly to get knowledge accumulated by mankind and modern technologies. The authors have used Maple computing environment in teaching mathematics for several years. This graphic system can produce curve and surfaces graphics specified in different coordinate systems quickly and accurately. The animation of basic mathematical notions help learners succeed in course without sticking to complex computations. Visualized mathematical knowledge via Maple allows improving education. It motivates to acquire knowledge and trains

⁹ Adela Ionescu. (2021). Why computational teaching? The advantage of using the maple software in teaching exact sciences. Atti Della Accademia Peloritana Dei Pericolanti, Classe di Scienze Fisiche, Matematiche E Naturali. Volume:99 Issue:S1 Supplement:S1 Article Number: A14 DOI: 10.1478/AAPP.99S1A14

¹⁰ N. V. Eyrikh, R. I. Bazhenov, N. V. Markova and L. V. Putkina, «Applying Maple Computing Environment in Teaching Mathematics to University Students Majoring in Technical,» 2018 IEEE International Conference «Quality Management, Transport and Information Security, Information Technologies» (IT&QM&IS), 2018, pp. 623-628, doi: 10.1109/ITMQIS.2018.8525118.

better professionals. The experiment increased qualitative academic performance by 16,9% in the experimental group.

Practice indicates that the teacher, usually, in his own educational activity uses specifically those forms and ways of work, which personally was involved in the learning process. Consequently, to develop himself, to educate cognitive independence of his students can only be such a teacher of mathematics, which formed the ability and skills of independent work.

2. The analysis of existing methods for solving the problem and formulating a task for the optimal technique development

Independent work of students can be interpreted as a type of learning activities aimed at mastering new knowledge, improving the skills of independent cognition, the formation of practical skills necessary for the future professional activity. Independent work is organized and guided by a teacher, but takes place without his direct participation.

The main functions of independent work include educational, cognitive, prognostic, stimulating, implementing, diagnostic and self-educational. To fully implement these functions in the learning process, according to the conclusions of scientists, the work should be planned, systematic and meaningful.

In order to implement the task of forming the professional competence of the future teacher of mathematics it is important to create favorable pedagogical conditions, both internal and external. Internal are formed by the person himself, initiated by him, they can include: the nature of personality, health, knowledge, skills, experience, personal motives and tasks, etc. External are determined by the influence of the environment and express social needs, and this is the place of study, relationships with other subjects of activity, curriculum requirements, etc. The organization of students' independent work, as well as its management can be considered as external pedagogical conditions of learning.

The learning process is carried out in the «man-human» system. Organization of learning and cognitive activity of students is an essential component of this process. Independent work of students is one of the types of activities for mastering knowledge, skills and abilities. Therefore, based on the previous considerations, its organization should take into account:

- the appropriate order and relationship of the structural elements of the educational process;

 focus on the implementation of teaching and educational functions or the solution of cognitive tasks; - determination of the place and conditions of functioning of the system;

- the choice and use of means and methods to ensure the implementation of the tasks.

The organization of independent work should be built in accordance with the principles that define the order of achieving the goal, implementation of tasks, contribute to the effective management of this process, make it possible to provide the results and scientifically substantiate the technology of teaching students. Among the principles of the organization of independent learning activities are:

1. The principle of making the full cycle of cognitive actions.

2. The principle of interconnectedness of practical activity, cognition and communication.

3. The principle of conceptualizing and interpreting activity.

4. The principle of interconnection of cognitive activity and creative process.

5. The principle of transition of personal knowledge into social knowledge.

6. The principle of interaction of the subject and object of cognition.

7. The principle of functioning and development of the system of collective intersubjective activity of cognition ¹¹.

In general, the organization of students' independent activity is a constant learning process in higher education institution, which can be provided only by a whole system of principles, a variety of methodological forms, consistently leading students to the possibility of productive independent work. Consequently, during the organization of independent work of future teachers of mathematics, in our opinion, it is important to be guided by a set of principles, considering them as specific recommendations for the implementation of the system of basic laws and strategic goals.

Analyzing the sources, we can establish the fact that the capabilities of modern SCM in teaching mathematics students and the organization of independent work are not fully used. In particular, there are almost no works on creation of pedagogical software in the SCM environment, which components are programs for generating typical mathematical tasks and reproducing the step-by-step course of their solution, with properties that allow to some extent replace the elements of intellectual activity of the teacher, which usually refers to the routine work. Therefore, in this

¹¹ Солдатенко М. М. Теорія і практика самостійної пізнавальної діяльності : монографія. Київ : Вид-во НПУ ім. М. П. Драгоманова, 2006. 198 с.

part we will present Maple simulators, which allow to obtain a step-bystep solution of differential equations, as well as generators of typical tasks on the subject of differential equations.

The use of standard Maple SCM commands allows you to get the final answer of the mathematical problem. At the same time the use of the software environment of this system allows the teacher to create programs with which you can reproduce the solution of mathematical problems, not just get an answer.

Therefore, to improve the efficiency of independent work of students in the study of the topic of linear differential equations, a training Maple simulator (TMS) was developed which allows the student to get a step-bystep solution. A fragment of program code for a linear differential equation:

```
restart;
qq := [(-2*x+1)/(x*x-x), 2*x];
y^{*}, +y^{*}qq[1] = qq[2];
print(`Зробиму заміну `);
v = U*V:
v*`'` = U*`'`*V+UV*`'`:
U^*, V^U V^*, U^V Q^{[1]} = qq[2];
U^{*}(V^{*}, +V^{*}qq[1])+U^{*}, +V = qq[2];
print(`a) Прирівняємо вираз у дужках до 0, маємо: `);
V^*, +V^*qq[1] = 0;
dV/dx + V^*qq[1] = 0;
dV/dx = -V*qq[1];
dV/V = -qq[1]*dx;
print(`Проінтегруємо ліву та праву частину `);
Int(1/V, V) = Int(-qq[1], x);
int(1/V, V) = int(-qq[1], x);
VV := solve(int(1/V, V) = int(-qq[1], x), V);
V = VV;
print(`б) маємо:);
VV*U*`` = qq[2];
U^{*,*} = qq[2]/VV; U = Int(qq[2]/VV, x);
UU := int(qq[2]/VV, x)+c;
U = UU: v = UU*VV:
```

Consider a brief description and operation of the author's TMS on the topic: «Linear differential equation» (Fig. 1.). In this simulator and in all other simulators it is enough for the student to connect the corresponding

author's library and enter his example, as a result of the work of the simulator the student will receive a step-by-step solution of his example. If the student has a task with initial conditions, then he should use the other author's procedure, in which he additionally specifies the initial conditions, a fragment of which is shown in Fig. 2.

The purpose of creating the simulator was to provide a high level of training in mathematical analysis, as well as reducing the routine load on the teacher. With the simulator in place, the student's independent work becomes more effective. And the instructor's role is to provide consultative assistance.



Fig. 1. TMS's work on «Linear differential equations»



Fig. 2. TMS's work on «Linear differential equation with initial condition»

In the presence of TMS practical classes actually turn into independent work of students under the guidance of an instructor. His role is to select tasks and provide advisory assistance. Importantly, the instructor is largely relieved of most of the routine checks, partially transferring this work to the students themselves. Students, in turn, are given a greater degree of independence. In doing so, they are able to:

- determine at what step they made a mistake if the final answer was wrong;

 look at the next step or the whole course of solving a typical problem in case the degree of their knowledge, skills and abilities have not reached the necessary level of assimilation;

- independently determine the pace of learning, depending on the level of proficiency;

- to change the condition of the example in accordance with the questions they have, and to get acquainted with the changes in the method of solution, which bring elements of the game into the training.



Fig. 3. Work of TMS on the topic «Bernoulli's equation»

Continuing the topic of simulators, we will demonstrate a fragment of the operation of the simulator on «Bernoulli's equation,» which is given in Figure 3. Similar to the previous ternary simulators, the student only needs to execute the author's procedure *«mysistemdifurbernuli(y`'* $+y/x=x^3y^2$);», where he/she needs to write his/her function.

The use of the training simulator is also appropriate when studying the topic «Systems of linear differential equations».

Simulator procedure *«difursystem(x'=4x-6y; y'=2x+11y);»*, fragments of which are shown on Figure 4, step by step solves the system of differential equations by method of elimination. The student only needs to execute the author's *«difursystem»* procedure and write the two equations in the function parameters.

If the student solved the example on his own, but doubts whether the answer is correct or not, he can use the simulator as a check on his steps and in case of errors, without the teacher's assistance to locate them. And also change the condition of the example and observe how the solution changes.



Fig. 4. Work NMT on «Systems of linear differential equations»

Improvement of curricula, new approaches to teaching mathematics, new manuals, as practice shows, do not automatically eradicate typical mathematical errors of students. That is why this problem and at the present stage of development of education requires proper attention of methodologists and practicing teachers. It is especially important to prevent students' errors, which should be based on the teacher's knowledge of the peculiarities of students' training material, the content of typical errors made by students, understanding the reasons for their occurrence. After all, any error indicates an insufficient level of mastering the corresponding course material. Made once, it can be repeated again, and the student's misunderstanding of the training material to consolidate.

As we know, when studying the topic «Linear inhomogeneous differential equations of the second order with fixed coefficients with a special right part» students often make typical mistakes, especially those related to the disclosure of brackets. Prevention of possible mathematical errors of students, analysis and correction of mistakes made should be considered as an important component of the educational process, aimed at the assimilation of a strong and deep knowledge of mathematics. Therefore, TMSs on this topic provide an opportunity for the student to check their solution and identify their mistakes, if any, during the solution (Figure 5.). There is a limitation in this topic's simulator: it automatically recognizes the right part if it is written as a polynomial of *n*-th power or if it is written as (ax+b)exp(2x). In the future improvement of this TMS it is planned to extend the functions of the right part.

To consolidate the section «Differential equations» was developed a generator of tasks for student's independent work, allowing the teacher in a short time to generate the required number of individual tasks (Figure 6.). Read the author's generator and run it, specifying only the number of options you need.



Fig. 5. Linear inhomogeneous second-order differential equations with steady-state coefficients with special right part



Fig. 6. Generation of individual tasks

CONCLUSIONS

As a result of the introduction to the training process in mathematics using TMS increases cognitive activity of students due to greater independence. Besides, in many cases the student gets rid of the need to spend time on getting advice from the teacher or finding his error in identical transformations of expressions.

When students work independently and work on errors using TMS through the formation of self-control skills provides a more meaningful analysis of them and the analysis of personal actions to solve specific problems. This affects the quality of knowledge acquired and stimulates the development of logical thinking, its characteristic indicators: criticality, evidence, activity, depth and flexibility.

By application of TMS students' independent work becomes more active. Very often in practical classes, when answering at the board, students have to be afraid to make a mistake. This fear often abolishes the student's initiative in solving examples. Afraid to make a mistake, he/she will not solve the problem on his/her own, but will wait for assistance from the instructor. But without solving problems of increasing complexity on your own, you cannot develop intellectually. The computer mathematics system Maple is particularly suitable for: organizing the student's independent work; for student self-testing; and for reinforcing the training material learned in higher mathematics classes. In this case, the role of the teacher is based on the guidance of the correct application of the chosen mathematical package. Our research shows that the use of SCM in modern conditions significantly changes the role and functions of the teacher and students, significantly affects all components of the learning process: the very nature, place and methods change.

The use of computer mathematics systems in the process of fundamental mathematical training of students is optimal if the following pedagogical approaches are taken into account and observed: competencebased, system-based, integrative, synergetic, knowledge-based, personcentered, learning and research-based.

Thus, we can see that the use of ICT tools, namely Maple SCM is one of the ways to optimize the training process by creating conditions for the organization of active independent learning activities, to implement a differentiated and individualized approach in teaching students. The research shows that the use of SCM in modern conditions significantly changes the role and functions of the teacher and students, significantly affects all components of the learning process: the very nature, place and methods change.

SUMMARY

In the modern world, the rapid development of information technology and computerization of all spheres of activity - scientific, educational significantly increases the requirements for the introduction and systematic use of new information technologies in the learning process. This work is devoted to the problem of adaptation of computer mathematics system Maple in the training process. The main pedagogical approaches to the use of computer mathematics system Maple in the training process are disclosed: systemic, integrative, personality-oriented, activity-based approaches. The concept of adapting Maple to study the section «Differential equations» in order to activate their independent work and a more effective mastering of the material by students is suggested. The use of learning simulator activates the cognitive activity of students. Step by step solutions of problems from the sections «Differential equations» by means of simulators created in the Maple environment are shown. The work of these simulators on separate examples is investigated and the conclusion about the positive effect of their use is made. The system of computer mathematics Maple is an effective tool for teaching mathematics to students of higher education institutions.

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