

IMPROVING THE LEARNING EFFECTIVENESS OF TECHNICAL HEA STUDENTS BY IMPLEMENTING ICTS

Tiutyunnik O. I., Klicopa I. A.

INTRODUCTION

Information and communication technologies (ICTs), electronic services, social networks, the Internet are firmly included in all spheres of society. By means of informatization tools everyone can have access to various information resources. Creative capabilities are always in demand in training and activities. Methods of using ICTs must include goals and principles of use; methods, techniques, forms and ways of teaching; training and methodical materials with a description of the appropriate methodology¹.

The main goal of the suggested methodology of using computer mathematics systems (CMS) is to provide an opportunity to improve the process of learning linear programming (LP) by applying these systems to perform computational and graphical operations.

The methodology of using CMS in the process of teaching LP is based on two basic principles.

1. The principle of gradual and non-antagonistic integration of ICT into the existing didactic systems. According to this principle, the use of ICT must not necessarily result in «principally new (purely «computer-based») methods of teaching, but involves an organic combination of familiar forms and techniques with innovative approaches and methods, creating an environment for expanding the methodological tools of the teacher, but not its destruction».

2. The principle of using a learning linear programming problem (LLP) of a new type. Its essence is not to transfer traditionally formed training tasks to the computer, but to reconstruct them in accordance with new potential uses of CMS through the provision of a wide range of services in accordance with their inherent computational, reference, communicative, programming and constructive-combinatorial functions.

¹ Герасименко І. В. Використання технологій дистанційного навчання в підготовці майбутніх бакалаврів комп'ютерних наук . *Інформаційні технології і засоби навчання*. 2014. Т. 41. № 3. URL: <http://journal.iitta.gov.ua>

The computational functions can include those that provide numerical calculations, symbolic transformations and graphical constructions of all kinds of graphs. Own environment for the development of programs of any complexity creates prerequisites for the use of CMS to create software products for educational purposes. Reference and information functions provide quick access to large amounts of information in a structured form. The group of communicative functions can provide organizational forms of learning, the choice of modes of communication and interaction between students and teacher, translation of subject content and communication between all participants in the learning process. «The group of constructively combinatorial functions is connected with the realization of the possibilities of using CMS as means of presenting subject content. Due to this group of functions the functioning of the subject environment as a world of objects with which the user works is provided»².

The principle of using new type of learning tasks means the necessity of analyzing the learning process of mathematical subjects to identify losses caused by deficiencies in its organization and restructuring traditional learning tasks, taking into account the use of CMS in order to eliminate the identified shortcomings.

1. The current state of application of information and communication technologies and principles of designing training tasks of linear programming of a new type in the conditions of using CMS

Professional training of students of HEI assumes formation of high level of mathematical and information culture, scientific outlook, understanding of essence of applied and practical orientation of mathematics, mastering of methods of mathematical modeling and ICT solutions of practical tasks in industrial, commercial and administrative spheres of activity. Mathematical programming, the content module of which is linear programming, plays an extremely important role in the professional training of HEI students as one of the main tools of management of economic systems, consisting in the development of methods for solving optimization problems and research of the obtained solutions. This is due to the fact that the formation of HEI students' ability

² Гончарова О. М. Шляхи і принципи системного введення комп'ютерних математичних систем у навчальний процес вищого навчального закладу. *Зб. наук. статей Національного педагогічного університету імені М. П. Драгоманова «Комп'ютерно-орієнтовані системи навчання»*. Київ. 2011. Вип. 11. С. 3–6.

to make effective management decisions, choose the best options is impossible without the ability to formulate and investigate mathematical models using ICT.

In recent years, most school leavers have continued their education in higher education institutions. This raises the issue of the popularization of higher education, which reduces the average level of ability of school leavers to continue their education, the lack and heterogeneity of training.

It should be noted that the problem of mass education is actually understood as a problem of mass higher education³.

The increase in the average size of the group of students and the marked heterogeneity of their training complicates the organization of the educational process and increases the workload on teachers. On the other hand, from a number of objective and subjective factors, teachers are unable to prepare timely and high-quality teaching and learning support.

The most promising approach to solving the problem of students' information load is modernization of education on the basis of informatization and computerization.

The problem of substitution of a task in one subject by a task in another subject in conditions of using ICTs is presented from the perspective of the theory of training tasks. On an example of linear programming tasks it is shown that the way of action of the student at the decision of a training task is defining on identification of a training task on a particular subject: linear programming; computer science; mathematical modeling; methods of optimization; theory of automatic control; numerous methods, etc. The necessity of updating the training tasks of linear programming in order to free students from cumbersome arithmetic calculations and notes, which more often than not become a barrier to a deeper understanding of the key ideas that underlie the algorithms they use.

It should be noted that some components of the problem of information overload of students due to excessive humanization of education, as a rule, in the first decade of the XXI century, due to optimization of credit distribution between the cycles, now lost relevance. Teaching (humanities, science, and general emphasis) in the Bachelor of Engineering and Economics renewal program. Due to the informatization of education, the interaction between teachers and students is increasingly mediated through the modern educational environment.

³ Зимина О. В. Предметный сегмент образовательной информационной среды и методика его использования в математическом образовании инженеров: дис. ... доктора пед. наук : 13.00.02 . Москва, 2004. 372 с.

By analyzing the processes that have occurred in training of mathematics over decades, the authors⁴ have identified the following trends:

- a reduction in the number of hours devoted to mathematics;
- deepening of the gap between the mathematical training of school and university graduates;
- deepening of the gap between the mathematical training of HEI graduates and the objective needs of science, engineering, and technology.

The era of computer symbolic mathematics falls in the early 1960s. This period was associated with the formation of a new field of computer mathematics, which, according to, received a less precise but sound name: computer algebra. This refers to the possibility of creating computer systems capable of performing typical algebraic transformations: substitutions in expressions, simplification of expressions, operations on power polynomials, solution of linear and non-linear equations and their systems, and finding their roots. This makes it possible to obtain analytical (symbolic) results in all operations, where possible.

According to the interpretation⁵ of CMS is a type of software tool designed to manipulate mathematical formulas. Systems of computer algebra (CMS) or symbolic computation system is understood as a powerful software product capable of solving a wide range of problems with different levels of computational complexity, ranging from simple transformations of expressions: polynomials, series, rational functions and formulas, and up to the solution of various systems of equations.

In mathematics, computer algebra, also called symbolic computation or algebraic computation, is a scientific field that refers to the study and development of algorithms and software for managing mathematical expressions and other mathematical objects. Although computer algebra can be viewed as a subfield of scientific computing, they are usually treated as separate fields because scientific computing is usually based on numerical calculations with approximate floating-point numbers, while symbolic computing emphasizes exact calculations with variable expressions. which have no set value and are treated as symbols⁶.

⁴ Кудрявцев Л. Д. О реформах образования в России. Образование, которое мы можем потерять . Москва : МГУ, 2002. 45–70 с.

⁵ Аладьев В. З. Программирование в пакетах Maple и Mathematica: Сравнительный аспект : монография: Гродно 2011. 517 с.

⁶ Триус Ю. В. Комп'ютерно-орієнтовані методичні системи навчання математичних дисциплін у вищих навчальних закладах: дис. ... доктора пед. наук : 13.00.02 Київ, 2005. 649 с.

We believe that a «computerized mathematics course» is comparable to a «computerized drawing course.» For many recent years while working on term and diploma works students make drawings using exclusively the appropriate software systems of computer-aided design and calculation intended for automation of works of the industrial enterprise at stages of design and technological preparation of manufacture: ArchiCAD, AutoCAD, Autodesk Inventor, CorelCAD, nanoCAD, OrCAD, P-CAD, Pro/ENGINEER, Solid Edge, SolidWorks, Spectra, SprutCAM, T-FLEX CAD, Compass etc. Nowadays students are getting acquainted with the basics of working in the environment of these and similar applications in their first year of studies at university. However, 5-10 years ago, when most students were working on their theses and projects exclusively with the use of software applications of this type, some teachers, remaining in captivity of traditional ideas, designed the training activities on the subject «Engineering and computer graphics» with regard to the mandatory use of pencil, ruler and other drawing tools by students. However, computer drawing tools were not allowed to be used! Teaching assignments that were done in this way simply did not count against the student.

As for drawing activities, the creation of computerized courses occurred relatively quickly in this area. As for the teaching process in higher mathematics, we can see that the processes of restructuring have been much more difficult, slower, and more painful. Nevertheless, let us assume that the characteristic feature of a computerized mathematics course is to ensure that the student works one hundred percent in an appropriate electronic environment with the practical elimination of manual symbolic or numerical calculations and charting.

In our opinion, in the end we should come to the creation of new computerized mathematics courses, but this should be preceded by a long period of searching for the methods of using CMS through the creation on their basis of software tools for educational purposes (STEP) with a high level of intellectual component.

The main goal of CMS is to automate cumbersome and complex symbolic transformations. This is the main difference between CMSs and other systems of computer mathematics, the main task of which is to provide solutions to problems with high-performance tools – numerical methods. It should be noted that both goals and capabilities of CMSs change in a fairly wide range, but their main goal remains the same, namely, the processing of symbolic expressions.

A prime example is the Maple system, which was created and developed for many years as a CMS. But since version Maple 6, a package

NAG (Numeric Algorithms Group) – known among specialists in numerous methods as a package of highly efficient matrix calculations – has been implanted into the computational environment of the system. Another, no less striking example: the environment of the system MATLAB, which is a leader among systems for numerical calculations, is currently implanted core SCA MuPAD, which operates as a separate component⁷.

In the last decade, Mathematica and Maple SCMs have developed somewhat at the artificial intelligence level. In Maple SCM the already existing means of pattern recognition were developed, in particular, match, patmatch, functions of function continuity recognition on a segment appeared – iscont, discount, which are currently absent in other systems. But otherwise, these systems remained almost at the same level of intelligence of the input languages as in 2000.

V. M. Mykhalevich has shown that the Excel-VBA-Maple integrated environment is a convenient tool for creating a block for generating tasks on the typical problems of higher mathematics. The ideology of program realization of generation block is most effective. According to this ideology all necessary mathematical operations are controlled from VBA environment by using Maple-functions, converting results to needed form and putting them to Excel worksheets for further printing. It is by such technology generated hundreds of problems from different areas of higher mathematics, including linear programming⁸.

Taking into account the need to take into account the mentioned influence and modern tendencies of development of CMS, the refined interpretation of such systems is offered. Computer mathematics system is a universal system capable of solving a wide range of problems and, first and foremost, solving mathematical problems by numerical and analytical methods, the main task of which is to automate a fairly cumbersome and complex symbolic transformations.

It should also be noted that various methodological materials on how to use Excel to solve FGD(LPP)s contain learning errors. One of the common errors of this type is that it specifies a way to solve LPPs using the simplex method with MS Excel, but it actually describes a way to find a solution using an add-in for the program. You can find solutions for LPP

⁷ Клочко В. І. Застосування нових інформаційних технологій навчання при вивченні курсу вищої математики в технічному вузі. Вінниця, 1997. 216 с

⁸ Михалевич В. М. Excel-VBA-Maple програма генерації задач з дисциплін математичного спрямування. Інформаційні технології та комп'ютерна інженерія. 2005. № 2. С. 74–83.

using various software packages, in particular MathCad, MS Excel Solution Finder, SPSS, SAS, but these programs give us results without alternative solutions and results of the last iteration.

It seems more promising to use MS Excel program, without involving «solution search» add-on. Studies of scientist V.V. Lystopad are of considerable interest, where a software basis for covering all stages of solution process of typical LPP is created.

In order to obtain all stages of the solution process V.V. Lystopad⁹ suggests using MS Excel in classes on higher mathematics and linear programming, the advantages of which include:

1) in comparison with manual calculation the time to solve the problem is reduced by several times;

2) the possibility of achieving a parallel assimilation of theoretical material of the topic;

3) are developed:

– skills of implementation of algorithmic procedures,

– ability to formulate a learning task, to plan activities for its solution,

– possibility to choose and use ready-made software (mathematical packages of applications),

– ability to write programs that solve typical learning tasks, – skills to master the basics of logic programming,

– skills of working with MS Excel; – ability to choose effective methods of solving problems;

4) ability to link the simplex method of solving FPGA on the Jordan-Gauss method;

5) ability to formulate control tasks of the system for the subject and final control in a relatively short period of time;

6) conveniently obtain and analyze solutions of direct and dual problems of linear programming;

7) wide application in methods of solution of linear programming problems.

In terms of reducing problem-solving time, the emphasis is on the fact that the solution process takes a few minutes, which is especially important for saving class time in practice, which became insufficient with the transition to the Bologna system.

⁹ Листопад В.В. Реалізація методу Жордана-Гаусса з допомогою MS Excel. *Науковий часопис Національного педагогічного університету ім. М. П. Драгоманова. Комп'ютерно-орієнтовні системи навчання*. Київ. 2011. № 12(19). С. 91-102

You can only write down the raw data for a typical LPP in MS Excel in a minute. The process of solving these problems is quite cumbersome, so there is a certain contradiction between the suggested and the consequences. After all, the implementation of the entire process of solving a typical LPP cannot be completed in a short period of time, even if the entire solution process is received by the students in ready-made form. In addition, the emphasis on saving classroom time also seems unreasonable, since the techniques developed using the recommendations during classroom and extracurricular independent work are not disclosed¹⁰.

We agree that the suggested methodology allows for the parallel assimilation of theoretical material and stated skills, but it does not disclose how these opportunities are realized in these works, which reduces their value. And this is also the main drawback of V.V. Lystopad's works, and its potential is actually quite powerful and vividly demonstrates the real competitiveness of using the MS Excel environment to create STEPs.

The problem that limits the widespread and effective use of CMS in teaching advanced mathematics is manifested in the creation of a situation accurately described by V. P. Dyakonov¹¹, about the danger of replacing the teaching of basic mathematics by teaching the basics of use of appropriate mathematical systems. This fully applies to linear programming, which is one part of higher mathematics. An important part and basic unit of learning activity is the training task. Therefore, the problem of designing training tasks on linear programming today is particularly relevant, given the need to use CMS to solve the problem. Currently, a large body of theoretical and applied work on the application of CMS in teaching higher mathematics has been accumulated. The most valuable materials are those related to the improvement of visualization, also known as «interactive visualization», and the implementation of the modern concept of CMS for adaptation to teaching mathematics to students through the creation of training simulators for automated reproduction of step-by-step solutions of typical mathematical problems.

At the same time, not enough attention is paid to the development of new learning tasks to be solved by students with the help of SCM. The use of

¹⁰ Листопад В. В. Реалізація методу штучного базису для розв'язування екстремальних задач лінійного програмування засобами Microsoft Excel. *Науковий часопис Національного педагогічного університету ім. М. П. Драгоманова. Комп'ютерно-орієнтовані системи навчання*. Київ. 2011. № 10 (17). С. 130-135.

¹¹ Дьяконов В. П. Компьютерная математика. Теория и практика. Москва: Нолидж, 2001. 1296 с.

simplex tables is the traditional way of solving LPP by the simplex method. Formation of knowledge, skills and abilities to address LPP by means of simplex transformations, reflected in the sequence of simplex tables, is one of the main traditional goals of linear programming as the most important subject of the content module «Mathematical programming». During many years of experiments and surveys, we found that students tend to master the skills of filling in the simplex tables when solving a typical LPP, but they know very little about the nature of their operations. A total of 527 students were surveyed. Only 54% of them had a high level of skill in solving with methods based on the use of simplex tables.

80% of the students (of those who formed a high level of skills in solving LPP using the method based on the use of simplex tables) found it difficult to answer the question: why do we divide all the coefficients of the solver row by the solver element?

60% of students failed to answer the question: what formulas are used to recalculate coefficients and free terms when moving to the next reference solution.

67% of students failed to answer the following questions:

1. What is the essence of the operations of determining a solving line?

1.1. Why is the smaller of the positive values of a known ratio chosen?

Why is the absence of a positive value for a known relation in any of the lines a proof of the absence of solution of the LPP because of the unboundedness of the field of admissible values?

48% of students failed to answer the questions:

1. What is the essence of checking the current reference solution for optimality? What are the numbers in the line (often called marks) whose signs are used to conclude that the current reference solution is optimal?

1.1 Justify the thesis «However, the signs before the coefficients in the corresponding row are a sign of optimality of the current reference solution».

1.2 Justify the thesis «A necessary and sufficient sign of optimality of the current reference solution is the absence of positive coefficients in the corresponding row».

1.3 Justify the thesis «A necessary and sufficient sign of suboptimality of the current reference solution is the presence of coefficients of different signs in the corresponding row».

100% of students failed to answer the questions:

1. What is the essence of simplex transformations using Gaussian formulas and what does it mean in this context to change the places of free and basic variables?

2. What is the fundamental difference between solving SLAR(СЛАР) by the Jordana-Gauss schemes and single division (hint: the formulas for enumerating coefficients and free terms are the same)?

The previous question is related to the fact that in a linear algebra course, students often master the Gaussian method using only the single division scheme. The transformation of simplex tables is more exactly like the Jordaan-Gauss scheme.

As an indirect confirmation of the fact that the described situation is typical and nonlocal, we cite the statements of professor O.V. Spivakovskiy¹²: «Today the application of modern information technologies in those spheres of mental activity, which are the most difficult for perception, is urgent because learning is conditioned by a considerable amount of routine work. A significant number of calculations that accompany the search for the solution of this or that problem does not allow the student to grasp the essence of the processes and phenomena under study and, as a consequence, does not form the necessary knowledge and skills”.

In addition, in the course of many years of survey we have revealed a tendency to decrease students' motivation to master the simplex method of LPP solution by mastering the rules of filling in simplex tables. This can be explained by the decrease in students' interest in performing cumbersome actions based on manual arithmetic calculations and one-type entries, which is natural in the conditions of availability of software tools for carrying out calculations.

There is one area of CMS application that is supported by almost all educators in schools and HEIs. It is a graphic visualization of the solution of mathematical problems thanks to such a possibility in a few seconds without cumbersome and lengthy calculations it is possible to understand the essence of problem solution, to realize multivariate calculations, which simply do not have enough time with traditional approaches».

However, as noted, the potential of CMS, in terms of improving the efficiency of the training process in higher mathematics, incomparably more powerful. However, finding possible ways to use this potential is quite a challenge. To reveal the essence of the problems arising to find methods of using CMS in the teaching of various sections of higher mathematics, in particular, linear programming, it is necessary to consider the concept of a training problem.

Based on the basics of the theory of learning tasks interpreted by D.B. Elkonin, Yu.I. Mashbitz, G.O. Ball, let us consider the main key concepts of this theory.

¹² Співаковський О. В. Основні задачі проектування комп'ютерних систем підтримки практичної навчальної математичної діяльності. *Нові технології навчання*. 2002. Вип. 33. С. 24–28.

The object of the student's actions, solving a practical task, is the specific single objects with which the student works and in which he/she makes changes. The object of the actions of the student solving a learning problem is not the specific single items with which the student works, but how such changes should be made to the items.

When solving a practical or training task the student achieves a change in the object of his action, but the change also takes place in the student himself.

The goal and result of a student's solution to a learning problem is the realization of a change in the student himself, the side effect is to obtain a solution to the problem (an answer).

At the same time, the goal and result of a student's solution to a practical task is some changed object, and the side effect is the realization of change in the student himself.

The given interpretations of training and practical tasks allowed D.B. Elkonin¹³ to raise the question of the purpose as an essential element of a task, the substance of which is a way of action.

D.B. Elkonin understands the way of action as a specific action with material which consists in such a partitioning of it that determines not only all subsequent individual techniques, but also the stage of their implementation.

Following Yu.I. Mashbitz¹⁴, we will understand the way of action as a system of operations providing the solution of training tasks of a certain type. With reference to L.V. Bertsfaev it is emphasized that a learning task takes place where mastering the necessary way of action is the main goal of students. When solving a practical task, the main result is the result, and the way of action is only a by-product of the students' activity.

It is this interpretation of the way of action adheres to a number of scientists studying training tasks in higher mathematics and computer science.

We propose to clarify the notions of «method of problem solution», «way of problem solution» and «ways of action in problem solution» on the example of a typical NLPP: among all inalienable solutions of a system of linear inequalities.

¹³ Избранные психологические труды / Д. Б. Эльконин ; ред. В. В. Давыдов, В. П. Зинченко ; акад. педагогических наук СССР. Москва 1989, 560 с.

¹⁴ Mashbitz E. I. Психолого-педагогические проблемы компьютеризации обучения : Наука – реформе школы. Москва : Педагогика, 1988, 192 с.

$$\begin{cases} x_1 + x_2 \geq 3, \\ -x_1 + 2x_2 \leq 6, \\ 3x_1 + 2x_2 \geq 5, \\ x_1 \leq 8, \\ 3x_1 + 2x_2 \geq 0, \\ x_1 \geq 0, \quad x_2 \geq 0 \end{cases}$$

you have to find the one for which the linear function

$$z = 9x_1 + 12x_2$$

has the largest value.

There are several ways to solve the above LPP, in particular: the graphical method; the simplex method; the method based on the use of software applications.

Yes, you can specify a method for solving the LPP by choosing Excel, based on the use of software applications. Thus, the subset of solutions defined by selecting Excel will be selected from the general subset of solutions corresponding to LPP solutions, according to the use of a software application. Further instructions on how to solve the LPP problem may or may not involve the need to use the Excel Add-in Search Solution, etc.

The approach chosen identified a number of ways to solve the LPP problem, which in turn influenced the way students performed the tasks in solving the problem, but they were not clearly defined. In fact, the way students solved the LPP problem determined the appropriate subset of actions. Another important factor that affects the course of action is the choice of the STEP because the same solution method will correspond to different action methods according to the specific implementation of the STEP.

As a result of selecting Excel, you will select a subset of solutions from which you can choose whether or not to apply the Excel Add-in «Solution Finder». Each of these solutions will correspond to a subset of action methods. A particular procedure will also depend on the student's preference, the same typical operations such as copy, paste, store, etc., they can perform different operations.

The choice of a simplex-based LPP solution method can be determined by additional conditions:

- a) manually filling a sequence of simplex tables;
- b) by semi-automatic filling of a sequence of simplex tables with the help of CMS or spreadsheet environment;
- c) by using CMS or spreadsheet replication of the simplex method without applying ideas such as simplex tables.

Obviously, each of these options corresponds to a different subset of ways of doing things, which, in turn, can again be restricted by introducing additional conditions.

Therefore, it is especially important to determine how students solve problems when developing training tasks in higher mathematics using ICT and CMS. It is the *modus operandi* that allows us to determine which subject a particular learning task belongs to.

Consequently, in this case replacing one concept with another does not solve the problem of replacing higher mathematics learning tasks with learning tasks from one of the other subjects: computer science, mathematical modeling, optimization methods, automatic control theory, etc.

The principles of using CMS to develop a new type of LPP include the following main provisions:

1. Normative principles of structure and content of advanced mathematics. Given the canonical structure and content of linear programming courses need to move away from the traditional mathematical problems underlying the creation of new learning tasks.

2. Principles of gradual and non-competitive integration of ICTs into existing mathematics curricula. Given the need to ensure gradual and non-antagonistic integration of ICTs into the existing didactic systems, traditional methods of solving LPP remain without disruptive restructuring and reforms.

3. A principle of reorganization of learning tasks. Given the need to free students from monotonous and tedious calculations and notes not directly related to the nature of the method used, but occupying up to 90% of in-class time, it certainly prevents students from better understanding ideas, conceptual approaches or problem formulation, it is necessary to radically change the way they act.

Let's consider applying these provisions to new LPPs solved with the help of CMS.

One of the training tasks of traditional courses of linear programming is to consolidate the knowledge, skills and abilities to solve mathematical problems of arbitrary dimensionality by the simplex method. Of course, considering the canonical structure and content of linear programming courses, we should leave the mathematical problems, which serve as a basis for the construction of the training problem.

Of course, given the canonical structure and content of linear programming courses, we should leave the mathematical problems that serve as the basis for constructing the learning problem.

Given the principles of gradual and non-competitive incorporation of ICT into the existing training systems without destructive reorganization and reform, we consider it necessary to abandon the simplex approach to LPP. However, the use of these methods is known to result in exhausting cumbersome arithmetic calculations. The method based on completing a

simplex table is actually reduced to multiple calculations of the second-order determinant defined by the following relation:

$$\begin{cases} a'_{kl} = \begin{vmatrix} 1 & a_{il} \\ a_{kj} & a_{kl} \end{vmatrix} = a_{kl} - a_{kj} \cdot a_{il}, \\ b'_k = \begin{vmatrix} 1 & b_i \\ a_{kj} & b_k \end{vmatrix} = b_k - a_{kj} \cdot b_i, \end{cases} \quad (k \neq j)$$

where i, j are, respectively, the numbers of the solving row and column; – respectively, the previous and new value of the coefficient before the variable located in the k -th row and the l -th column; – respectively, the previous and new value of the free term in the k -th row.

M. Zhaldak¹⁵ noted: «Computers are designed to free people from routine operations. Routine operations include complex calculations and graphs. It is the informatization of the learning process that significantly frees students and teachers from the need for cumbersome routine technical manipulations, and creates the conditions and provides them with all the opportunities to solve cognitive and creative problems. Using ICTs, students do not have to perform intermediate actions. Such actions can be automated, but they require some time and mental effort, which can and should be used more efficiently.

2. Criteria for selecting CMS to create software tools for training purposes

It should be noted that not all scholars agree with the concept of using CMS, especially in in-depth mathematics instruction for students of non-mathematical specialties. According to M. I. Zhaldak¹⁶, CMS s such as MatLab, MathCad, Mathematica, Maple, etc., have wide possibilities and are excellent tools for research, but too quickly lead to the result, to the answer, often hiding the algorithm of its obtaining from the user does not allow to achieve good mastering of algorithms of mathematical problems. Therefore, CMSs are considered uncomfortable for the learning process and the use of Excel application is proposed.

To eliminate this disadvantage of modern CMS, a method of automatic replication of step-by-step solutions of mathematical problems by creating

¹⁵ Жалдак М. І. Використання комп'ютера в навчальному процесі має бути педагогічно виваженим і доцільним. *Комп'ютер у школі та сім'ї*. 2011. № 3. С. 3–12.

¹⁶ Жалдак М. І. Комп'ютерно-орієнтовані системи навчання – становлення і розвиток. *Науковий часопис Національного педагогічного університету ім. М. П. Драгоманова. Серія 2, Комп'ютерно-орієнтовані системи навчання* : збірник / М-во освіти і науки України, Нац. пед. ун-т ім. М. П. Драгоманова. Київ, 2010. Вип. 9 (16). С. 3–9.

a training simulator using Maple CMS for teaching students the concepts of advanced mathematics was suggested. The studies have shown that the use of the suggested method is an effective modern means of forming skills and abilities to solve typical problems of higher mathematics as an element of procedural competence of future professionals.

In the absence of a detailed justification of the theoretical and methodological use of CMS, there is a danger of replacing the mathematical foundations of learning the basics of using mathematical systems, which most warned about many years ago. In the language of learning problem theory, there is a danger of replacing learning assignments in one subject (e.g., higher mathematics) with learning assignments in other subjects (computer science, physics, multiple methods, mathematical modeling).

One of the problems arising in the use of CMS in the learning process is the choice of the working environment. Commercial systems and freeware systems differ significantly in functionality, interface, size, built-in programming language, etc. Familiarity with only one computer mathematical system will inevitably affect further professional activity, limiting its possibilities.

A large number of existing CMS s creates the problem of rational choice of specific systems for the development of educational software (STEP) on their basis. Unfortunately, there is no exhaustive work in the literature on the analysis and comparison of modern CMS s in terms of evaluating the environment in which modern CMSs create STEPs, especially through training simulators that allow automatic repetition of step-by-step solutions typical of higher mathematics problems.

Obviously, the definition of the principles of CMS selection as an environment for creating STEPs of the specified type should be based on the requirements of their creation and use.

The author¹⁷ under a typical problem in higher mathematics (TPHM) means mathematical problems, the ability to solve which is provided by the mastered material by students at the skill level in accordance with the curriculum of higher mathematics subject for students of the relevant specialty. By training simulators of TPHM solution we mean STEP, designed for automated reproduction of the step-by-step solution of TPHM with the presence of text commentary, and under the Maple-

¹⁷ Михалевич В. М., Крупський Я. В. Розвиток системи Maple у навчанні вищої математики майбутніх інженерів-механіків: монографія . Вінниця: ВНТУ, 2013. 236 с. ISBN. – 978-966-641-539-7

trainer – training simulators of TPHM solution, developed and functioning in the CMS environment.

Tasks and requirements on creation, implementation and use of training Maple-simulators in educational process of HEI are formulated. We highlight those provisions that, in our opinion, can be taken into account in determining the principles of the selection of CMS as an environment for creating training simulators of this type.

Among the tasks of creating training Maple-simulators, we single out the following:

- 1) ensuring the possibility of use in the training process regardless of the form (full-time, part-time, remote learning);
- 2) securing the strengthening of interdisciplinary links;
- 3) provision of preparation of game forms of classes;
- 4) improving the efficiency of students' independent work in mastering the training material and providing the possibility of self-monitoring of the obtained skills, abilities and knowledge;
- 5) creation of conditions for intellectual development of students and revealing of their creative potential;
- 6) increase the level of information culture and information and computer training of students.

Among the requirements for creating training Maple-simulators we will highlight the following:

- 1) methods of solving TPHM should correspond to the established traditions in domestic pedagogical practice;
- 2) obligatory presence of textual comments in Ukrainian for all steps of solution;
- 3) provision of the possibility of implementation of different degrees of detail;
- 4) the possibility of modification of learning Maple-simulators – an open program code for making changes for further improvement not only the authors, but also users – teachers and students;
- 5) considering the peculiarities of attention (stability, concentration, switching ability, distribution and volume);
- 6) understandable interface.

Among the requirements for the introduction and use of Maple-learning simulators in the educational process of HEI we will highlight the following:

- 1) creation of appropriate infrastructure and training-scientific information environment – a complex of technical, software, information

and methodological support of communication tools for the participants of the educational process.

2) high level of appropriate information and computer training of teachers and students;

3) leading role of instructor during classes with the use of Maple-simulators

4) ensuring a high degree of individualization of learning;

5) provision of stable feedback during learning;

6) providing departments of universities with appropriate hardware and software, as well as having a computer lab in the department for classroom training of students and their independent work with the use of Maple-simulators;

7) availability of support staff who have completed internships or advanced training in computer-oriented and distance learning;

8) provision of reliable and free access for students to Maple-training simulators on educational and scientific information resources of HEI.

Taking into account the above factors, analysis of numerous literature on the use of ICT in the teaching of higher mathematics to students of non-mathematical specialties, as well as our own experience in the development and use of STEP of the above type in order to enable the above comparison, the following criteria for the selection of CMS were determined:

1. The presence in SCM tools to implement an automated representation of mathematical expressions and the sequence of mathematical transformations in accordance with the traditional view, as presented in domestic textbooks and collections of problems in mathematics;

2. provision of reference and educational and methodological literature on the use of CMS;

3. tools for the creation of didactic materials with a high degree of visibility;

4. ability to create software applications, the functioning of which does not depend on the environment in which they are implemented;

5. experience of teachers-developers in the appropriate environment;

6. availability of the software application for wide use;

7. compatibility of different versions of the software application.

In our opinion, the representation of mathematical expressions and sequences of mathematical transformations in the traditional form, introduced in domestic textbooks and problem books is one of the main obstacles to the effective use of CMS as a medium for creating STEPs. For

example, if in Maple we will try to display on the monitor the expression entered in the command line as `40`, we get the result 40. In the more general case, if we replace one variable in a particular variable on the command line with an expression with the corresponding numeric value, there is no convenient way to keep the structure of the expression in the output area, because in the standard version the system just gives the result of the calculation.

A large number of such problems requires considerable additional time for exploratory experimentation and program implementation, which often leads to finding quite large solutions involving non-standard use of commands and system operators.

Let's consider separate fragments of the program in the CMS Maple environment for learning to solve systems of linear equations by the matrix method. The traditional expanded form of recording a system of linear algebraic equations in mathematical notation looks like this:

$$\begin{cases} 7 \cdot x_1 + 6 \cdot x_2 - 4 \cdot x_3 = 44 \\ 4 \cdot x_1 - x_2 + 2 \cdot x_3 = -1 \\ -6 \cdot x_2 + 7 \cdot x_3 = -39 \end{cases}$$

In Maple, the equations of the system are usually given by a set of elements as a sequence of these equations, which are written in curly brackets, separated by commas:

```
sys_eq:={7*x1+6*x2-4*x3=44, 4*x1-x2+2*x3=-1, 6*x2+7*x3=-39};
sys_eq := { 7 x1 + 6 x2 - 4 x3 = 44, 4 x1 - x2 + 2 x3 = -1, 6 x2 + 7 x3 = -39 }
```

In the area of the output of the calculation results we have also inconvenient and uninformative form of the system as a sequence of equations, which are written separated by a comma. Improving informativeness of mathematical expression in the output area concerns only the image of indexed variables.

Convenient automated writing of equations in a form closer to the traditional form can be realized by organizing a loop

```
for i to nops(sys_eq) for sys_eq [i]end do;
```

In the area of calculation results output we also have an inconvenient and uninformative form of the system as a sequence of equations, which are written separated by a comma. Improvement of informativeness of mathematical expression in the derivation area concerns only the image of indexed variables.

Convenient automated writing of equations in a form closer to the traditional form can be realized by organizing a loop

```
for i to nops(sys_eq) for sys_eq [i]end do;
```

$$\begin{aligned} 7x_1 + 6x_2 - 4x_3 &= 44 \\ 4x_1 - x_2 + 2x_3 &= -1 \\ 6x_2 + 7x_3 &= -39 \end{aligned}$$

You can get an image of a system of equations in the usual form with a curly bracket to the left by a non-standard application of the piecewise continuous functions command:

```
piecewise(`System of equations"[1],`,`System of
equations"[2],`,`System of equations"[3],`);
```

$$\left\{ \begin{aligned} 7x_1 + 6x_2 - 4x_3 &= 44 \\ 4x_1 - x_2 + 2x_3 &= -1 \\ 6x_2 + 7x_3 &= -39 \end{aligned} \right.$$

In the expression of equations of arbitrary order, the programmatic application of setting piecewise continuous functional commands can be implemented simply by creating a special program. Thus, in the context of Maple system, the problem of visual representation of calculation results in traditional forms, to which students are accustomed, is comparable to the problem of using symbolic computation to automate the process of step-by-step visualization for solving TPHM.

Complicating the problem described is the violation of one of the cornerstone principles of quality software, namely the principle of bottom-up compatibility of releases. Let us demonstrate this by means of the following fragment of the program code, which is a part of the process of learning to solve systems of linear equations by the matrix method:

```
A:=matrix(3,3,[[7,6,-4],[4,-1,2],[0,-6,7]]);
B:=matrix(3,1,[44,-1,-39]);
A=evalm(A),B=evalm(B);
print(`n`);
'X'=evaln(A)^-1*B';
print(`n`);
for i to 3 do
seq("A"[j,i]=[-1]^cat(i,`,`,j)*det'*linalg[minor](A,j,i),j=1..3)
od;
evaln(A)^-
I^-1=1/linalg[det](A)*evalm(linalg[inverse](A)*linalg[det](A));
```

'X'=(1/det(A))*evalm(linalg[inverse](A)*linalg[det](A))*evalm(B);

In Maple 7, in the output area we get the result shown in Fig. 1.

$$A = \begin{bmatrix} 7 & 6 & -4 \\ 4 & -1 & 2 \\ 0 & -6 & 7 \end{bmatrix}, B = \begin{bmatrix} 44 \\ -1 \\ -39 \end{bmatrix}$$

$$X = A^{-1} B$$

$$'A'_{1,1} = [-1]^{1+1} \det \begin{bmatrix} -1 & 2 \\ -6 & 7 \end{bmatrix}, 'A'_{2,1} = [-1]^{1+2} \det \begin{bmatrix} 6 & -4 \\ -6 & 7 \end{bmatrix}, 'A'_{3,1} = [-1]^{1+3} \det \begin{bmatrix} 6 & -4 \\ -1 & 2 \end{bmatrix}$$

$$'A'_{1,2} = [-1]^{2+1} \det \begin{bmatrix} 4 & 2 \\ 0 & 7 \end{bmatrix}, 'A'_{2,2} = [-1]^{2+2} \det \begin{bmatrix} 7 & -4 \\ 0 & 7 \end{bmatrix}, 'A'_{3,2} = [-1]^{2+3} \det \begin{bmatrix} 7 & -4 \\ 4 & 2 \end{bmatrix}$$

$$'A'_{1,3} = [-1]^{3+1} \det \begin{bmatrix} 4 & -1 \\ 0 & -6 \end{bmatrix}, 'A'_{2,3} = [-1]^{3+2} \det \begin{bmatrix} 7 & 6 \\ 0 & -6 \end{bmatrix}, 'A'_{3,3} = [-1]^{3+3} \det \begin{bmatrix} 7 & 6 \\ 4 & -1 \end{bmatrix}$$

$$A^{-1} = -\frac{1}{37} \begin{bmatrix} 5 & -18 & 8 \\ -28 & 49 & -30 \\ -24 & 42 & -31 \end{bmatrix}$$

$$X = \frac{\begin{bmatrix} 5 & -18 & 8 \\ -28 & 49 & -30 \\ -24 & 42 & -31 \end{bmatrix} * \begin{bmatrix} 44 \\ -1 \\ -39 \end{bmatrix}}{\det(A)}$$

Fig. 1. The output area of the program fragment of learning to solve systems of linear equations by the matrix method – Maple 7

The same program code in the Maple 9.5 environment already "doesn't work". The system gives a text message about an error situation:

"Error, (in LinearAlgebra:-Multiply) definition of its 1st argument, MV1, to be type {Matrix,Vector,scalar} but 1/linalg:-det(A)**".

The rest of the computer mathematical systems have the same shortcomings. In the analysis of modern CMS it is proposed to assign the following sets of tools:

- ways of naming and describing object parameters;
- property validation procedures;
- transformation program;
- transformation management process.

It should be noted that these sets of tools, with the exception of transformation management procedures, have developed significantly in all CMSs over the last decade. The transformation apparatus consists of programs that automatically execute only a certain set of specifications, and programs that provide for interactive management of the decision

process, where the recognition function is delegated to human beings. Using a single expression as the main object of the language leads to the fact that the most important tool for substitution control in complex analytic transformations does not really exist. All possible substitutions are made by default. Management is simplified for local or global undoing of all possible substitutions¹⁸.

The ability to manage the transformation is an extremely important feature of SCM as the environment in which the STEP is created. Unfortunately, the existing and available SCMs have such a possibility contradicts the conceptual principles for which they were developed.

At the same time we should understand the real objectivity of this disadvantage. Using Maple to solve mathematical problems, there are few sources of more or less complete description of programming in the freely distributed system Maxima.

Note that T. Shamilev refers to the most important general characteristics of software tools, convenient for teaching how to solve mathematical problems, as the presence of clear and intuitive tools for presenting algorithms for solutions. We absolutely agree with this, but these characteristics lack, in our opinion, the main requirement for a software tool of this type – support for symbolic calculations.

The absence of this requirement was the key factor that led to the choice of Excel spreadsheet processors based on their criteria. However, the vast majority of solutions for training mathematical tasks involve symbolic calculations.

Among other desirable features of the above software Shamilev T.¹⁹ notes: convenient graphical interface; possibility to display all intermediate calculations in the form of a table illustrating the idea of approach; Note that the requirement for convenient graphical interface is a general requirement for any software applicable to a wide range of users.

The requirement of being able to display all intermediate calculations in the form of a table – to illustrate the idea of the method is to some extent a narrowing of the principle of providing the environment with tools for creating didactic materials with a high degree of clarity.

¹⁸ Ключко В. І., Бондаренко З.В. Вища математика. Звичайні диференціальні рівняння (з комп'ютерною підтримкою): навчальний посібник Вінниця : ВНТУ, 2013. 248 с.

¹⁹ Шамілев Т. М., Сухтаєва А.М. Практикум із математики для інженерів-педагогів із використанням пакету Excel. *Проблеми інженерно-педагогічної освіти*: збірник наукових праць Української інженерно-педагогічної академії. Харків : УІПА, 2012. № 37. С. 268–273.

As to the requirement of possibility of automatic listing of all calculations, in work Tiutiunnyk O.²⁰ such possibility it is offered to name technology of live pages and the comparative analysis of realization of this technology in applications Maple, MathCAD, Excel is spent. It is noted that realization of this technology in Excel application has certain advantages, but only in Maple "live page technology" is supported for tasks which algorithm of solving assumes symbolical transformations.

The following SCMs can be distinguished according to the proposed criteria: Maple, Mathematica, MuPAD and Maxima.

Aldeev V.²¹ notes that modern SCMs are used by mathematicians, researchers, and engineers to improve the teaching of mathematically-oriented courses. The current leaders among SCMs include Maple, Mathematica, and to some extent also MuPAD. All of these applications are commercial. Due to organizational and financial problems, MuPAD has now ceased to exist as an independent project. Based on a detailed comparison, it was concluded that Mathematica users have slightly more problems than Maple users. A significant advantage of Maxima's CMS is its free distribution and there is a rapid growth in popularity of its use. We chose Maple as a medium for creating STEP as a result of fifteen-year experience of work with this system accumulated at the Department of Higher Mathematics, VNTU, and low level of availability of Maxima system instructional and methodical literature.

3. Training simulators for solving linear programming problems

In methodical systems of training in linear programming, as well as in other sections of mathematics, the big role is played by practical aspects – cycles of practical tasks, laboratory works and organization of independent work of students on the solution of training tasks. Formation of practical skills and abilities is achieved here, and this part of the curriculum is certainly central.

Particular attention should be paid to the fact that crippling tasks can undermine students' confidence in their strength and not give a positive effect. Therefore, the teacher's work should be based on the gradual and

²⁰ Тютюнник О. І. Крупський Я. В., Михалевич В. М. Математичне програмування в Maple. Курс для дистанційного навчання: навчальний посібник. Вінниця: ВНТУ, 2012. URL: http://elearn.vntu.edu.ua/subject/index/card/subject_id/728

²¹ Аладьев В. З. Программирование в пакетах Maple и Mathematica: Сравнительный аспект : монография: ГрГУ. Гродно. 2011. 517 с.

purposeful development of students' creative cognitive abilities, development of their thinking.

According to scientists, one of the main principles of introducing CMS in the training process is the principle of new tasks, which consists in the fact that traditionally formed techniques and methods are not transferred to the computer, but are restructured in accordance with new opportunities opened up by using CMS in the learning process. In practice, this means that there is no need to spend class time on acquiring calculation skills, which can be done by a computer.

In our opinion, the last sentence should be clarified: there is no need to traditionally spend a significant portion of training hours and, first and foremost, classroom time to acquire the skills of cumbersome routine calculations that can be performed with the help of a computer.

From the results of the work of authors such as: Mykhalevich V., Krupsky Y.²² It follows that "the training Maple-simulators are one of effective modern means of formation of abilities and skills of solving TPHM as elements of procedural competence of the future mechanical engineers. The application of Maple simulators in modern conditions significantly changes the role and functions of a teacher and students, significantly influences all components of the training process: the nature, place and methods of joint activities of a teacher and students change; the ratio of didactic functions realized in the system "teacher – learning Maple-simulator – student"; methods and forms of training sessions are changed. Introduction of learning Maple-simulators inevitably leads to significant changes in the structure of the whole pedagogical system of higher mathematics teaching. It is shown that the use of learning Maple-simulators in the organization of students' independent work causes activation of cognitive activity of students, develops their cognitive motivation; creates prerequisites for reducing the time of the traditional teaching information; stimulates problem-solving and independent learning and professional activity of students; initiates self-organization of practical and research activity; gives an opportunity to exercise self-control. All this contributes to the effectiveness of learning.

Pedagogically balanced and expedient use of modern CMS in teaching linear programming to students of higher education institutions includes

²² Михалевич В. М., Крупський. Я. В Проблеми ефективної роботи студентів під час використання навчальних Maple тренажерів з покрокового розв'язання типових задач вищої математики: Збірник наукових праць Уманського держ. пед. ун-ту імені Павла Тичини. Умань. 2010. Ч. 3. С. 130–136.

development of a number of training tasks of a new type. These tasks include the following typical PLLs:

1. Solving general systems of linear algebraic equations;
2. Solving two-dimensional LPP by the graphical method;
3. The solution of two-dimensional LPP by the simplex method with clear geometric interpretation of the main concepts at all key stages of the solution process;
4. The simplex-method solution of LPP of any dimension with geometrical interpretation of the basic concepts of all key stages of the solution process;
5. The substantive interpretation of all stages of the solution process of LPP, which can be provided in the form of an initial condition or obtained automatically by using the author's Maple-training simulators;
6. Composing and solving by the simplex method of dual LPPs;
7. Finding a solution of the dual LPP on the basis of a known solution of the initial problem with the use of duality theorems;
8. Solving LPP by the dual simplex method;
9. The graphical method of solving an integer LPP. 10;
10. The solution of an integer 3JIP by the Gomori method.

At the Department of Higher Mathematics VNTU in the study of linear programming the bulk of practical training is conducted in a computer lab with the division of the academic group of students into two subgroups. Students solve LPP in the CMS Maple environment. But they use non-standard commands of this system, designed to get the solution of the problem (answers), but use their knowledge to reproduce the corresponding methods and apply commands, which allow to deprive the student of the need to perform routine calculations at certain stages of problem solution. In fact, this form of study session is more like a laboratory class. Each student is given a different version of the task. Different variants of tasks are generated with the help of the methodology reflected in the works.

In order to consciously reproduce the whole solution of a typical LPP the student must be well versed in the key steps of the mastering method. In case of certain difficulties the student can use the corresponding learning Maple-trainer and get the whole course of the solution of the required problem with a comment in different details.

To support lecture and practical classes in IT classrooms and regular classrooms, and to organize students' independent work, a number of learning Maple-simulators covering the solution process of such typical LPPs has been developed:

1. Solution of two-dimensional LPP using graphical method (see Appendix A.1²³, task 1 below);
2. The simplex-method solution of LPP of arbitrary dimensionality (see Appendix A. 2. Task 2);
3. Making the dual LPPs and their solution by the simplex method (Appendix A. 3, problem 3);
4. Realization of transition from the current solution to the next basic one when solving LPP by the simplex method.

Besides, templates of worksheets were developed which enable semi-automated reproduction of the step-by-step solution of NLPPs of a new type:

1. Finding a solution of the dual LPP based on the known solution of the initial problem with the use of the duality theorem;
2. Solving LPP by a dualistic simplex method;
3. The graphical method of solving a linear integrodifferential LPP;
4. Solving an integer LPP using the Homory method.

It is important that the student can change the condition of the problem and trace the changes in the course of its solution. This, in turn, opens up new opportunities for the implementation of problem-based learning, exploratory approach and involvement of game forms of learning.

Appendix A.1

Solving two-dimensional LPP using the graphical method

Problem 1.

Find the maximal value of the function

$$z = -7x_1 - 22x_2, \quad \rightarrow \max$$

if its arguments are connected by relations:

$$\begin{aligned} x_1 - 2x_2 &\leq 4 \\ -x_1 - 5x_2 &\leq -11 \\ x_1 + x_2 &\leq 31 \\ 2x_1 - x_2 &\leq 26 \\ -17x_1 + 11x_2 &\leq 5 \end{aligned}$$

Solution.

1. Construct the area of allowable values. To do this, we first take into account the signs of the equations and construct the straight lines:

²³ Тютюнник О. І., Михалеви́ч В. М. Використання систем комп'ютерної математики у процесі навчання лінійного програмування майбутніх менеджерів-адміністраторів: монографія . Вінниця : ВНТУ, 2016. 208 с.

$$\begin{cases} x_1 - 2x_2 = 4 \\ -x_1 - 5x_2 = -11 \\ x_1 + x_2 = 31 \\ 2x_1 - x_2 = 26 \\ -17x_1 + 11x_2 = 5 \end{cases}$$

and then take into account the signs of the inequalities. The area of allowable values will be a polygon ABCDE (see Fig. 2).

2. Write down the coordinates of the gradient of the function z:

$$\text{grad}(z) = [-7, -22]$$

Since we only need the direction of the gradient, the length does not matter when representing the gradient on the graph.

3. Draw the reference line MN.

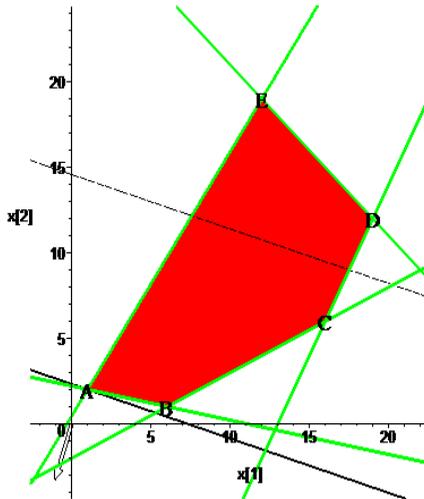


Fig. 2. The area of allowable values

Construct a table of segment names that correspond to the line equations to which these segments belong:

$$\begin{cases} AB: & x_1 + 5x_2 = 11 \\ BC: & x_1 - 2x_2 = 4 \\ CD: & 2x_1 - x_2 = 26 \\ DE: & x_1 + x_2 = 31 \\ AE: & 17x_1 - 11x_2 = -5 \end{cases}$$

4. Moving the line in the direction of the gradient parallel to itself in the domain/area ABCDE, we find the exit point of the line from the admissible values domain, point A:

$$z_{max} = z_A$$

5. Solving the system of two linear equations representing the lines intersecting at point A

$$\begin{cases} -x_1 - 5x_2 = -11 \\ -17x_1 + 11x_2 = 5 \end{cases}$$

we obtain:

$$x_1 = 1, x_2 = 2, A(1, 2)$$

Then

$$Z_{max} = -7*(1) - 22*(2) = -51$$

Remarks. We would obtain the same result by traversing all vertices of polygon ABCDE, that

Remarks. The same result would be obtained by bypassing all vertices of polygon ABCDE, which would only complicate the solution of the problem.

Appendix A. 2

The simplex solution by the LPP method of arbitrary dimension

Problem 2

Find the smallest value of the function

$$z = -7x_1 - 22x_2$$

if its arguments are connected by relations:

$$\begin{cases} x_1 - 2x_2 \leq 4 \\ -x_1 - 5x_2 \leq -11 \\ x_1 + x_2 \leq 31 \\ 2x_1 - x_2 \leq 26 \\ -17x_1 + 11x_2 \leq 5 \\ 0 \leq x_1, 0 \leq x_2 \end{cases}$$

Solution.

Introduce balance unknowns:

$$x_3, x_4, x_5, x_6, x_7$$

And reduce the system of constraints to the standard form:

$$\left\{ \begin{array}{l} x_1 - 2x_2 + x_3 = 4 \\ -x_1 - 5x_2 + x_4 = -11 \\ x_1 + x_2 + x_5 = 31 \\ 2x_1 - x_2 + x_6 = 26 \\ -17x_1 + 11x_2 + x_7 = 5 \end{array} \right.$$

Number of equations=5, total number of variables=7.

Hence, 2 variables must be chosen as free variables.

We choose as free variables

$$x_1, x_2$$

and assume that they are equal to zero:

$$x_1 = 0, x_2 = 0$$

This makes the system of canonical equations look like:

$$\left\{ \begin{array}{l} x_3 = 4 \\ x_4 = -11 \\ x_5 = 31 \\ x_6 = 26 \\ x_7 = 5 \end{array} \right.$$

And the solution of the last system is:

$$x_3 = 4$$

$$x_4 = -11$$

$$x_5 = 31$$

$$x_6 = 26$$

$$x_7 = 5$$

Solution

$$XI = [x_1 = 0, x_2 = 0, x_3 = 4, x_4 = -11, x_5 = 31, x_6 = 26, x_7 = 5]$$

Invalid since it contains a negative value:

$$x_4 = -11$$

We choose as free variables

$$x_7, x_2$$

and assume them to be equal to zero:

$$x_7 = 0, x_2 = 0$$

The system of canonical equations takes the form:

$$\left\{ \begin{array}{l} x_1 + x_3 = 4 \\ -x_1 + x_4 = -11 \\ x_1 + x_5 = 31 \\ 2x_1 + x_6 = 26 \\ -17x_1 = 5 \end{array} \right.$$

And the solution of the last system is:

$$x_1 = \frac{-5}{17}$$

$$x_5 = \frac{532}{17}$$

$$x_4 = \frac{-192}{17}$$

$$x_6 = \frac{452}{17}$$

$$x_3 = \frac{73}{17}$$

Solution

$$X2 = \left[x_1 = \frac{-5}{17}, x_2 = 0, x_3 = \frac{73}{17}, x_4 = \frac{-192}{17}, x_5 = \frac{532}{17}, x_6 = \frac{452}{17}, x_7 = 0 \right]$$

Invalid since it contains a negative value:

$$x_1 = \frac{-5}{17}$$

We choose as free variables

$$x_7, x_1$$

and assume them to be equal to zero:

$$x_7 = 0, x_1 = 0$$

The system of canonical equations takes the form:

$$\left\{ \begin{array}{l} x_2 + x_5 = 31 \\ -x_2 + x_6 = 26 \\ -2x_2 + x_3 = 4 \\ -5x_2 + x_4 = -11 \\ 11x_2 = 5 \end{array} \right.$$

And the solution of the last system is:

$$x_2 = \frac{5}{11}$$

$$x_6 = \frac{291}{11}$$

$$x_5 = \frac{336}{11}$$

$$x_3 = \frac{54}{11}$$

$$x_4 = \frac{-96}{11}$$

Solution

$$X3 = \left[x_1 = 0, x_2 = \frac{5}{11}, x_3 = \frac{54}{11}, x_4 = \frac{-96}{11}, x_5 = \frac{336}{11}, x_6 = \frac{291}{11}, x_7 = 0 \right]$$

Not allowed, since it contains a negative value:

$$x_4 = \frac{-96}{11}$$

We choose as free variables

$$x_6, x_2$$

and assume them to be equal to zero:

$$x_6 = 0, x_2 = 0$$

The system of canonical equations takes the form:

$$\left\{ \begin{array}{l} x_1 + x_3 = 4 \\ -x_1 + x_4 = -11 \\ x_1 + x_5 = 31 \\ -17x_1 + x_7 = 5 \\ 2x_1 = 26 \end{array} \right.$$

And the solution of the last system is:

$$x_5 = 18$$

$$x_1 = 13$$

$$x_4 = 2$$

$$x_3 = -9$$

$$x_7 = 226$$

Solution

$$X4 = [x_1 = 13, x_2 = 0, x_3 = -9, x_4 = 2, x_5 = 18, x_6 = 0, x_7 = 226]$$

Invalid since it contains a negative value

$$x_3 = -9$$

We choose as free variables

$$x_6, x_1$$

and assume them to be equal to zero:

$$x_6 = 0, x_1 = 0$$

The system of canonical equations takes the form:

$$\left\{ \begin{array}{l} x_2 + x_5 = 31 \\ 11x_2 + x_7 = 5 \\ -2x_2 + x_3 = 4 \\ -5x_2 + x_4 = -11 \\ -x_2 = 26 \end{array} \right.$$

And the solution of the last system is:

$$x_2 = -26$$

$$x_5 = 57$$

$$x_3 = -48$$

$$x_4 = -141$$

$$x_7 = 291$$

Solution

$$X5 = [x_1 = 0, x_2 = -26, x_3 = -48, x_4 = -141, x_5 = 57, x_6 = 0, x_7 = 291]$$

Invalid since it contains a negative value

$$x_2 = -26$$

We choose as free variables

$$x_6, x_7$$

and assume them to be equal to zero:

$$x_6 = 0, x_7 = 0$$

The system of canonical equations takes the form:

$$\left\{ \begin{array}{l} x_1 - 2x_2 + x_3 = 4 \\ -x_1 - 5x_2 + x_4 = -11 \\ x_1 + x_2 + x_5 = 31 \\ 2x_1 - x_2 = 26 \\ -17x_1 + 11x_2 = 5 \end{array} \right.$$

And the solution of the last system is:

$$x_4 = \frac{2496}{5}$$

$$x_1 = \frac{291}{5}$$

$$x_2 = \frac{452}{5}$$

$$x_3 = \frac{633}{5}$$

$$x_5 = \frac{-588}{5}$$

Solution

$$X6 = \left[x_1 = \frac{291}{5}, x_2 = \frac{452}{5}, x_3 = \frac{633}{5}, x_4 = \frac{2496}{5}, x_5 = \frac{-588}{5}, x_6 = 0, x_7 = 0 \right]$$

Invalid since it contains a negative value:

$$x_5 = \frac{-588}{5}$$

We choose as free variables

$$x_5, x_2$$

and assume them to be equal to zero:

$$x_5 = 0, x_2 = 0$$

The system of canonical equations takes the form:

$$\left\{ \begin{array}{l} x_1 = 31 \\ x_1 + x_3 = 4 \\ -x_1 + x_4 = -11 \\ 2x_1 + x_6 = 26 \\ -17x_1 + x_7 = 5 \end{array} \right.$$

And the solution of the last system is:

$$x_1 = 31$$

$$x_3 = -27$$

$$x_4 = 20$$

$$x_6 = -36$$

$$x_7 = 532$$

Solution

$$X7 = [x_1 = 31, x_2 = 0, x_3 = -27, x_4 = 20, x_5 = 0, x_6 = -36, x_7 = 532]$$

Invalid since it contains a negative value:

$$x_3 = -27$$

We choose as free variables

$$x_5, x_1$$

and assume them to be equal to zero:

$$x_5 = 0, x_1 = 0$$

The system of canonical equations takes the form:

$$\left\{ \begin{array}{l} x_2 = 31 \\ -x_2 + x_6 = 26 \\ 11x_2 + x_7 = 5 \\ -2x_2 + x_3 = 4 \\ -5x_2 + x_4 = -11 \end{array} \right.$$

And the solution of the last system is:

$$\begin{aligned} x_2 &= 31 \\ x_6 &= 57 \\ x_7 &= -336 \\ x_3 &= 66 \\ x_4 &= 144 \end{aligned}$$

Solution

$$X8 = [x_1 = 0, x_2 = 31, x_3 = 66, x_4 = 144, x_5 = 0, x_6 = 57, x_7 = -336]$$

Invalid since it contains a negative value:

$$x_7 = -336$$

We choose as free variables

$$x_5, x_7$$

and assume them to be equal to zero:

$$x_5 = 0, x_7 = 0$$

The system of canonical equations takes the form:

$$\left\{ \begin{array}{l} x_1 - 2x_2 + x_3 = 4 \\ -x_1 - 5x_2 + x_4 = -11 \\ 2x_1 - x_2 + x_6 = 26 \\ x_1 + x_2 = 31 \\ -17x_1 + 11x_2 = 5 \end{array} \right.$$

And the solution of the last system is:

$$\begin{aligned} x_6 &= 21 \\ x_2 &= 19 \end{aligned}$$

$$x_1 = 12$$

$$x_3 = 30$$

$$x_4 = 96$$

Since the solution

$$[x_1 = 12, x_2 = 19, x_3 = 30, x_4 = 96, x_5 = 0, x_6 = 21, x_7 = 0]$$

does not have negative values, we have the first admissible solution.

Further solution algorithm consists of two repeated steps:

1. Check the admissible solution for optimality.

2. Transition to a new admissible solution (in case of a negative answer at the first step).

To check for optimality you need to express the target function in terms of free unknowns. This can be done by solving the initial system of standard equations with respect to free unknowns in algebraic form.

Let us check the obtained admissible solution for optimality.

Solve the original system with respect to free unknowns:

After substituting the arguments of the target function for their expressions through the free unknowns, the target function will have the form:

$$\left\{ \begin{array}{l} x_3 = 30 - \frac{23}{28}x_5 - \frac{3}{28}x_7 \\ x_4 = 96 - \frac{24}{7}x_5 - \frac{1}{7}x_7 \\ x_6 = 21 + \frac{5}{28}x_5 - \frac{3}{28}x_7 \\ x_2 = 19 - \frac{17}{28}x_5 - \frac{1}{28}x_7 \\ x_1 = 12 - \frac{11}{28}x_5 + \frac{1}{28}x_7 \end{array} \right.$$

In an admissible solution, the free unknowns are zero. Consequently, they can only be increased.

$$z = \frac{451}{28}x_5 - 502 + \frac{15}{28}x_7$$

Free unknowns are preceded by coefficients with the sign «+». As the free unknowns increase, the objective function will increase.

So the obtained solution

$$[x_1 = 12, x_2 = 19, x_3 = 30, x_4 = 96, x_5 = 0, x_6 = 21, x_7 = 0]$$

is optimal. Minimum value of target function $z = -502$

Appendix A. 3

Making the dual LPP and their solution by the simplex method

Problem 3.

Compose a dual problem and find its solution, if the function, according to the condition, goes to the maximum.

$$z = -7 x_1 - 22 x_2$$

$$\left\{ \begin{array}{l} x_1 - 2 x_2 \leq 4 \\ -x_1 - 5 x_2 \leq -11 \\ x_1 + x_2 \leq 31 \\ 2 x_1 - x_2 \leq 26 \\ -17 x_1 + 11 x_2 \leq 5 \end{array} \right.$$

Dual problem

$$z = 4 y_1 - 11 y_2 + 31 y_3 + 26 y_4 + 5 y_5, \quad \rightarrow \min$$

$$\left\{ \begin{array}{l} -7 \leq y_1 - y_2 + y_3 + 2 y_4 - 17 y_5 \\ -22 \leq -2 y_1 - 5 y_2 + y_3 - y_4 + 11 y_5 \\ 0 \leq y_1, 0 \leq y_2, 0 \leq y_3, 0 \leq y_4, 0 \leq y_5 \end{array} \right.$$

Introduce balance unknowns:

$$y_6, y_7$$

And reduce the constraint system to the standard form:

$$\left\{ \begin{array}{l} y_1 - y_2 + y_3 + 2 y_4 - 17 y_5 - y_6 = -7 \\ -2 y_1 - 5 y_2 + y_3 - y_4 + 11 y_5 - y_7 = -22 \end{array} \right.$$

The number of equations is 2, the total number of variables is 7.

Hence, we should choose 5 variables as free variables.

We choose as free variables

$$y_1, y_2, y_3, y_6, y_7$$

and assume that they equal zero:

$$y_6 = 0, y_7 = 0, y_1 = 0, y_2 = 0, y_3 = 0$$

The system of canonical equations takes the form:

$$\begin{cases} 2y_4 - 17y_5 = -7 \\ -y_4 + 11y_5 = -22 \end{cases}$$

And the solution of the last system is:

$$y_4 = \frac{-451}{5}$$

$$y_5 = \frac{-51}{5}$$

Solution

$$X1 = \left[y_1 = 0, y_2 = 0, y_3 = 0, y_4 = \frac{-451}{5}, y_5 = \frac{-51}{5}, y_6 = 0, y_7 = 0 \right]$$

Invalid since it contains a negative value:

$$y_5 = \frac{-51}{5}$$

We choose as free variables

$$y_1, y_2, y_4, y_6, y_7$$

and assume them to be equal to zero:

$$y_6 = 0, y_7 = 0, y_1 = 0, y_2 = 0, y_4 = 0$$

The system of canonical equations takes the form:

$$\begin{cases} y_3 - 17y_5 = -7 \\ y_3 + 11y_5 = -22 \end{cases}$$

And the solution of the last system is:

$$y_3 = \frac{-451}{28}$$

$$y_5 = \frac{-15}{28}$$

Solution

$$X2 = \left[y_1 = 0, y_2 = 0, y_3 = \frac{-451}{28}, y_4 = 0, y_5 = \frac{-15}{28}, y_6 = 0, y_7 = 0 \right]$$

Invalid since it contains a negative value:

$$y_5 = \frac{-15}{28}$$

We choose as free variables

$$y_1, y_2, y_5, y_6, y_7$$

and assume them to be equal to zero:

$$y_6 = 0, y_7 = 0, y_1 = 0, y_2 = 0, y_5 = 0$$

The system of canonical equations takes the form:

$$\begin{cases} y_3 + 2y_4 = -7 \\ y_3 - y_4 = -22 \end{cases}$$

And the solution of the last system is:

$$y_3 = -17$$

$$y_4 = 5$$

Solution

$$X^3 = [y_1 = 0, y_2 = 0, y_3 = -17, y_4 = 5, y_5 = 0, y_6 = 0, y_7 = 0]$$

Invalid, because it contains a negative value:

$$y_3 = -17$$

We choose as free variables

$$y_1, y_3, y_4, y_6, y_7$$

and assume them to be equal to zero:

$$y_6 = 0, y_7 = 0, y_1 = 0, y_3 = 0, y_4 = 0$$

The system of canonical equations takes the form:

$$\begin{cases} -y_2 - 17y_5 = -7 \\ -5y_2 + 11y_5 = -22 \end{cases}$$

And the solution of the last system is:

$$y_2 = \frac{451}{96}$$

$$y_5 = \frac{13}{96}$$

Since the solution

$$\left[y_1 = 0, y_2 = \frac{451}{96}, y_3 = 0, y_4 = 0, y_5 = \frac{13}{96}, y_6 = 0, y_7 = 0 \right]$$

does not have negative values, we have the first admissible solution

Further solution algorithm consists of two repeated steps:

1. Check the admissible solution for optimality.

2. Transition to a new admissible solution (in case of a negative result at the first step).

Check the obtained admissible solution for optimality.

Solve the initial system with respect to basic unknowns:

$$\left\{ \begin{array}{l} y_2 = -\frac{23}{96}y_1 - \frac{11}{96}y_6 + \frac{7}{24}y_3 + \frac{5}{96}y_4 + \frac{451}{96} - \frac{17}{96}y_7 \\ y_5 = \frac{7}{96}y_1 - \frac{5}{96}y_6 + \frac{1}{24}y_3 + \frac{11}{96}y_4 + \frac{13}{96} + \frac{1}{96}y_7 \end{array} \right.$$

After substituting the arguments of the target function for their expressions through the free unknowns, the target function will have the form:

$$F = 7y_1 + 28y_3 + 26y_4 + y_6 + 2y_7 - 51$$

In the admissible solution, the free unknowns are zero.

Hence, they can only be increased.

Free unknowns are preceded by coefficients with the sign «+».

The target function will increase as the free unknowns increase.

So the obtained solution

$$\left[y_1 = 0, y_2 = \frac{451}{96}, y_3 = 0, y_4 = 0, y_5 = \frac{13}{96}, y_6 = 0, y_7 = 0 \right]$$

is optimal. Minimum value of target function $F = -51$.

4. Providing clear forms of calculation results output when creating Maple-learning simulators and worksheet templates

In paragraph 3 it was pointed out that the «weak link» of some technological implementations of NLPP of a new type is due to the emergence of additional information load on the students, which is associated with the need for familiarity with a number of commands of this system. Even a greater additional information load arises due to the low degree of visualization of the result of many standard commands of all popular CMS and Maple, in particular. This is due to the fact that modern CMSs lack a wide set of standard tools for presenting the results of symbolic and numerous calculations in the traditional form, as it is given in domestic textbooks and manuals on mathematics. This situation is due to the fact that all CMSs were created, first of all, for professional scientific and engineering activities. It is only over time that their developers have recognized the powerful potential of these systems in teaching.

Here is an example that illustrates the problem of ensuring the output of the results of automated calculations in the Maple environment in a form that is as close to traditional forms and has a high degree of clarity.

From our point of view, the illustration of results for a more evident explanation of the scheme of calculations to determine the solver string in simplex transforms should be presented in the following form:

$$\left\{ \begin{array}{l} x_1 = -x_2 - \frac{1}{9}x_4 + \frac{1}{6}x_5 + 8 \Rightarrow -\frac{b_1}{a_{11}} = -\frac{8}{-1} = 8, \\ x_3 = \frac{1}{4}x_2 + \frac{1}{12}x_4 - \frac{1}{4}x_5 + 18 \Rightarrow -\frac{b_2}{a_{21}} = -\frac{18}{1/4} = -72, \\ x_6 = \frac{5}{4}x_2 + \frac{11}{36}x_4 - \frac{1}{12}x_5 + 86 \Rightarrow -\frac{b_3}{a_{31}} = -\frac{86}{5/4} = -\frac{344}{5}. \end{array} \right.$$

However, it is not a trivial task to provide such a form of output of results in automated calculations in Maple. One of the simplest options for the software implementation of the output looks like this:

$$\begin{aligned} x_1 &= -x_2 - \frac{1}{9}x_4 + \frac{1}{6}x_5 + 8, -\frac{b_1}{a_{1,1}} = 8 \\ x_3 &= \frac{1}{4}x_2 + \frac{1}{12}x_4 - \frac{1}{4}x_5 + 18, -\frac{b_2}{a_{2,1}} = -72 \\ x_6 &= \frac{5}{4}x_2 + \frac{11}{36}x_4 - \frac{1}{12}x_5 + 86, -\frac{b_3}{a_{3,1}} = -\frac{344}{5} \end{aligned}$$

Obviously, the degree of visualization in the given form is low: there are no separate values of the free term and the coefficient a_{ik} ; the desired ratio is located immediately after the corresponding equation without a gap, which makes it difficult for students to recognize individual elements of the mathematical expression, causes additional visual and intellectual stress and distracts from comprehension and analysis of the essence of the presented visualization²⁴.

In order to find forms of output that have, in our opinion, a higher degree of clarity, we resorted to experiments with non-standard application of Maple system commands and operators. In this connection, we note that we are not aware of any sources that provide systematic data on the methods of output in the Maple environment of the results of

mathematical expressions in various traditional forms with a high degree of clarity²⁴.

Below we present three variants of forms of automated output of results for a clearer explanation of the calculation scheme for determining the solution string. All variants use the technique of displaying results in separate columns of a matrix, which allows for a clearer structuring of the results of the calculation.

Variant 1:

$$\left[\begin{array}{l} x_1 = -x_2 - \frac{1}{9}x_4 + \frac{1}{6}x_5 + 8 \rightarrow -\frac{b_1}{a_{1,1}} = -\frac{["8"]}{["-1"]} \rightarrow -\frac{b_1}{a_{1,1}} = 8 \\ x_3 = \frac{1}{4}x_2 + \frac{1}{12}x_4 - \frac{1}{4}x_5 + 18 \rightarrow -\frac{b_2}{a_{2,1}} = -\frac{["18"]}{"1/4"} \rightarrow -\frac{b_2}{a_{2,1}} = -72 \\ x_6 = \frac{5}{4}x_2 + \frac{11}{36}x_4 - \frac{1}{12}x_5 + 86 \rightarrow -\frac{b_3}{a_{3,1}} = -\frac{["86"]}{"5/4"} \rightarrow -\frac{b_3}{a_{3,1}} = \frac{-344}{5} \end{array} \right].$$

In order to output numerical values of the free term and coefficient a_{ik} , we used the method of converting numerical data into text data. The disadvantages of this form are quotation marks, which accompany the output of text data in Maple, and the need to use square brackets to represent fractions.

Variant 2:

$$\left[\begin{array}{l} x_1 = -x_2 - \frac{1}{9}x_4 + \frac{1}{6}x_5 + 8 \rightarrow -\left[\frac{b_1}{a_{1,1}} \right] = -\left\{ \frac{["8"]}{["-1"]} \right\} = 8 \\ x_3 = \frac{1}{4}x_2 + \frac{1}{12}x_4 - \frac{1}{4}x_5 + 18 \rightarrow -\left[\frac{b_2}{a_{2,1}} \right] = -\left\{ \frac{["18"]}{"1/4"} \right\} = -72 \\ x_6 = \frac{5}{4}x_2 + \frac{11}{36}x_4 - \frac{1}{12}x_5 + 86 \rightarrow -\left[\frac{b_3}{a_{3,1}} \right] = -\left\{ \frac{["86"]}{"5/4"} \right\} = \frac{-344}{5} \end{array} \right].$$

Additional curly braces, as compared to the previous version, allowed to get rid of the double entry in each line of the expression $-\frac{b_i}{a_{i1}}$ and to

²⁴ Тютюнник О. І., Михалевич В. М. Реалізація принципу наочності за допомогою засобів СКМ у процесі навчання лінійного програмування. *Сучасні інформаційні технології та інноваційні методики навчання у підготовці фахівців: методологія, теорія, досвід, проблеми: збірник наукових праць*. Вінниця: Планер, 2013. Випуск 36. С. 434-440.

realize in one expression two equal signs, which in standard Maple system situations is not provided. At the same time, the additional curly braces and square brackets contain unnecessary elements for the student of visual information, generating additional questions.

Variant 3:

$$\left[\begin{array}{l} x_1 = -x_2 - \frac{1}{9}x_4 + \frac{1}{6}x_5 + 8 \quad \rightarrow \quad \left[\begin{array}{l} b_1 = 8 \\ a_{1,1} = -1 \end{array} \right] \quad \rightarrow \quad -\frac{b_1}{a_{1,1}} = 8 \\ x_3 = \frac{1}{4}x_2 + \frac{1}{12}x_4 - \frac{1}{4}x_5 + 18 \quad \rightarrow \quad \left[\begin{array}{l} b_2 = 18 \\ a_{2,1} = \frac{1}{4} \end{array} \right] \quad \rightarrow \quad -\frac{b_2}{a_{2,1}} = -72 \\ x_6 = \frac{5}{4}x_2 + \frac{11}{36}x_4 - \frac{1}{12}x_5 + 86 \quad \rightarrow \quad \left[\begin{array}{l} b_3 = 86 \\ a_{3,1} = \frac{5}{4} \end{array} \right] \quad \rightarrow \quad -\frac{b_3}{a_{3,1}} = \frac{-344}{5} \end{array} \right]$$

This option seems to us to be the most successful. The numerical values of the free term and the coefficient a_{ik} are given in square brackets as a regular commentary. All individual elements are clearly structured and there are no additional unconventional for mathematical entries construction.

The technological implementation of the simplex method with the full extraction of manual calculations in the CMS Maple assumes the use of the standard solve command – to solve systems of linear equations.

The application of the solve command to solve systems of linear equations leads to a result given by the system in a non-traditional form with a low degree of clarity. Consider the result of applying the solve command to obtain a general solution to a system of linear equations, which was discussed in the paragraph:

$$\text{solve}(\{\mathbf{e1}, \mathbf{e2}, \mathbf{e3}, \mathbf{e4}, \mathbf{e5}\}, \{\mathbf{x}[1], \mathbf{x}[3], \mathbf{x}[4], \mathbf{x}[5], \mathbf{x}[7]\}):$$

$$\{x_1 = -x_6 + 8, x_3 = x_2 - x_6 + 5, x_7 = 2x_2 - 3x_6 + 24, x_4 = -2x_2 - x_6 + 14,$$

$$x_5 = 2x_2 - 3x_6 + 19\}$$

The found expressions of basic variables through free variables are fed by the system by means of a comma-separated sequence. Of course, it is inconvenient to isolate and analyze each individual equation of the system from such an entry and requires additional intellectual effort. In addition, the equations in the sequence are not sorted by the number of the basic variable, which creates additional search difficulties.

You can improve the clarity of the output of the solve command by using the following technique

```
solve({e1,e2,e3,e4,e5}, {x[1],x[3],x[4],x[5],x[7]}): Roz_1:=%:  
for i in Roz_1 do i od;
```

$$\begin{aligned}x_1 &= -x_6 + 8 \\x_3 &= x_2 - x_6 + 5 \\x_7 &= 2x_2 - 3x_6 + 24 \\x_4 &= -2x_2 - x_6 + 14 \\x_5 &= 2x_2 - 3x_6 + 19\end{aligned}$$

This method improves visualization of results, because each equation of the system is given on a separate line. However, this method causes the use of operator for repeated calculations: [for name] [from expression] [by expression] [to expression] [while boolean_expression] do sequence_operators end do. In spite of the fact that such constructions are basic for any programming language, this information distracts students from the main goal – understanding of the key ideas of the simplex method of finding NLPP solutions.

An even higher degree of clarity can be achieved by presenting the system of equations in the traditional form with the left curly bracket and sorting the order of the equations in the system by the number of the base variable:

```
my_solve:=proc()  
local rozv;  
solve(args);  
map(zz->lhs(zz)=sort(rhs(zz)),[op(%)])  
rozv:=my_sort(%);  
my_sys([op(%)]);print(%);  
rozv  
end proc;  
my_solve({e1,e2,e3,e4,e5},{x[1],x[3],x[4],x[5],x[7]}):
```

$$\left\{ \begin{aligned}x_1 &= -x_6 + 8 \\x_3 &= x_2 - x_6 + 5 \\x_4 &= -2x_2 - x_6 + 14 \\x_5 &= 2x_2 - 3x_6 + 19 \\x_7 &= 2x_2 - 3x_6 + 24\end{aligned}\right.$$

The program code that implements the above degree of clarity is quite cumbersome. Therefore, it was decided to «hide» this code in a custom procedure my_solve, which has the same structure of setting arguments as

the standard solve command. The custom procedure `my_solve` uses two more similar procedures, but much larger in number of program lines, which provide sorting of equations and their output together with curly brackets: `my_sort`, `my_sys`.

A certain assessment of the effectiveness of the proposed methodology of using the Maple system in teaching HEI students can be the results of a survey conducted by specially designed questionnaires.

Some of the answers are shown in the form of diagrams in Fig. 3,4.

1. Did you enjoy solving problems with the system Maple?

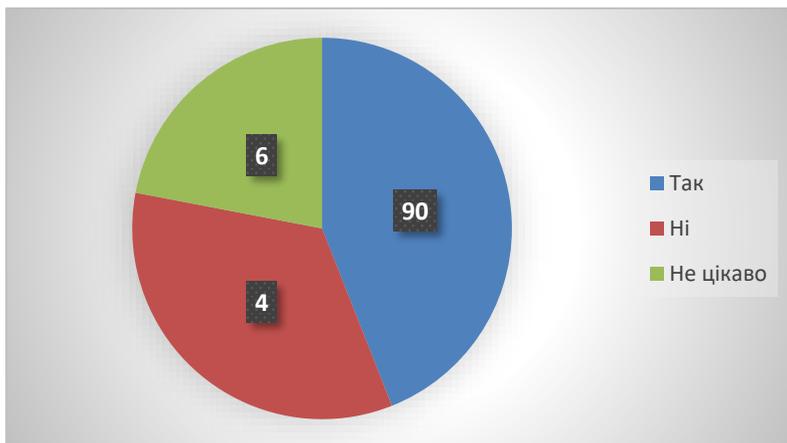


Fig. 3. The use of Maple system in teaching, the result of answering 1 question

2. Do you use the acquired skills of working with SCM while studying other disciplines of the natural cycle?

From the results obtained it follows that the majority of students believe that the use of CMS is appropriate not only for better mastering of the educational material of mathematical disciplines (75%), but also for further use in learning activities (71%).

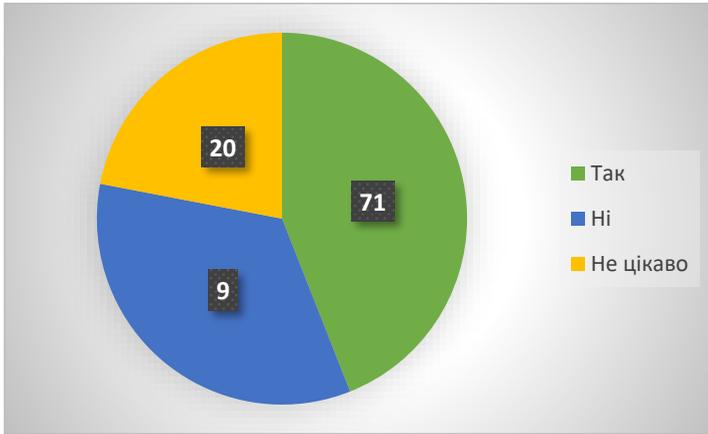


Fig. 4. The use of Maple system in teaching, the result of answering 2 question

Almost all students (94%) believe that the use of CMS in the teaching of different sections of mathematics is advisable, first of all, for the automation of routine calculations. For 45% of students it is an important opportunity to experiment with the condition of the problem and analyze changes in the step-by-step solution. When asked about the benefits of using CMS in training, 84% of respondents note the opportunity to receive a step-by-step solution with a detailed commentary, allowing you to check your solution, find and understand the errors.

The following answers prevailed among disadvantages of using CMS in training: I don't like working with computer (4%), I didn't like working with Maple CMS (4%), lack of window interface (23%).

The results of the questionnaire showed that students understand the importance and necessity of using CMS both in teaching linear programming and in teaching the mathematical subjects.

CONCLUSIONS

One of the important common shortcomings of practically all CMSs has been highlighted, which consists in the difficulty of automated presentation of mathematical expressions and the sequence of mathematical transformations in full compliance with the traditional form, as given in textbooks and collections of problems. It is noted that the creation, on the basis of CMS, STEP in the form of training simulators allows you to eliminate these and other shortcomings associated with the lack of standard means of automated reproduction of the step-by-step

solution of typical problems of linear programming with the presence of text comments.

Application of CMS allows to release students from a considerable volume of routine actions at the solution of traditional training tasks of linear programming that more often becomes an obstacle which prevents students from deeper understanding of key ideas which are put in a basis of used methods, and also their connection with such fundamental concepts, as systems of linear algebraic equations, their general and partial solution, geometrical content of solutions of linear algebraic equations and inequalities, Gauss method of system solution. At the same time, the use of CMS poses the problem of the danger of replacing teaching the basics of linear programming with teaching the basics of working with these mathematical systems.

The examples of the use of CMS in the teaching of linear programming show:

- ways to build a new type of LPP with updating the way students act while mastering the methods of solving these problems;

- ways of providing a number of didactic principles in designing new-type learning tasks, including principles: a) systematicity and consistency of learning; b) gradual and non-antagonistic incorporation of ICT into the existing didactic systems; c) IT support, according to which students in solving the learning task in question are deprived of the need for routine cumbersome calculations and notes and are able to focus on theoretical and practical aspects of the method studied; d) clarity, according to which the course of the solution is illuminated not only schematically by the values of the coefficients and free terms of the system of linear equations and the target function, which are located in separate cells of simplex tables, but also in the traditional form of recording general and partial solutions of linear systems of equations, which allows students to visually perceive the learning material, facilitates its absorption and consolidation in the mind; e) provision of intra-subject links, which creates opportunities for students not only to restore but also to deepen their knowledge, abilities and skills related to fundamental concepts of systems of linear equations; f) differentiation and individualization – each student himself determines what actions and in what quantity he needs to reproduce, in order to understand the essence of transformations at the current step of the process of solving the educational problem; g) ensuring the humanization of the learning process, which manifests itself in a significant change in the ratio of routine activities and creative processes in favor of the latter; i) providing the student with the means of step-by-step visualization of NLPP for self-testing;

– ways and methods of implementing the principle of visualization, which consist in creating according to a number of new requirements for STEP a dynamic controlled model with the provision of the possibility of involving students in the implementation of transformative activity with the models of objects of study.

It was noted that the developed NLPP of a new type compared to traditional training tasks, designed to represent the simplex method, have the advantage of greater transparency, contribute to the transfer of emphasis on the formation of students' skills of routine calculations and notes on formal rules for the acquisition of skills to consciously reproduce the key stages of the corresponding algorithm and provide the student with tools for the possibility of research and assimilation of this method.

Certain disadvantages of using CMS in the teaching of linear programming are associated with the emergence of additional information burden on students of HEI when familiarizing with the basics of working in the environment of these systems and a number of their standard commands and procedures. Therefore it is necessary to constantly improve developed techniques to find an optimal ratio between the number of commands and procedures, which should be operated by a student while solving LPP, and the degree of visualization of all intermediate results. These disadvantages are more than offset by the fact that it is an ideal way to master any such system, the idea of which will allow future specialists to independently and meaningfully continue studying both higher mathematics and CMS.

Development of methods and techniques of using CMS, in particular in the form of training simulators and templates of worksheets of LPP solution, to disclose the essence of LPP degeneracy concept, implementation of the principle of clarity constitute the essence of educationally balanced and expedient use of modern CMS in teaching linear programming to future specialists the preconditions for significant restructuring of the whole training process on mathematical programming and higher mathematics.

SUMMARY

Theoretical justification and development of methods of using computer mathematics systems in the process of teaching linear programming to students of HEI are considered. It is shown that in the process of teaching linear programming there is a danger of replacing teaching the basics of mathematics by teaching the basics of working with software tools to solve problems. The practical significance of the obtained results of the study consists in the creation of training tasks of linear programming of a new type.

The problem of application of computer mathematics systems in teaching linear programming to students of HEI has been studied. It is shown that in the process of teaching linear programming to students of HIE the use of information and communication technologies causes the danger of replacing learning the basics of mathematics learning the basics of working with software tools for solving problems. The criteria for selecting CMS as a medium for the development of software tools for educational purposes are formulated. The principles of designing training tasks of linear programming of a new type using CMS, as well as these tasks, in which the way of action to implement a certain mathematical method of solution is designed taking into account the scientific basis of ICT use in the educational process of HEI, are defined. Criteria for evaluating and selecting the CMS environment for the creation of LPP and training simulators with automated reproduction of the step-by-step course of solving typical mathematics problems are formulated.

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Information about the authors:

Tiutyunnik Oksana Ivanivna,

Candidate of Pedagogical Sciences,
Assistant Professor at the Department of Higher Mathematics
Vinnytsia National Technical University
95, Khmelnytsk Avenue, Vinnytsia, 21021, Ukraine

Klieopa Iryna Anatoliivna,

Assistant at Department of Higher Mathematics
Vinnytsa National Technical University
95, Khmelnytsk Avenue, Vinnytsia, 21021, Ukraine