A PRIORI RESEARCH RELATED TO THE CALCULATION OF THE REGIONAL ELLIPSOID FOR UKRAINE AND ITS EFFECTIVENESS

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Abstract. Despite the high accuracy of global geodetic reference systems and their widespread use in GPS measurements, regional (local) geodetic systems are becoming more widely used. For example, the World Geodetic System 1984 (WGS84) has 83 such local systems. The emergence of the latter is caused by the emergence of new problems of physical geodesy. These are the so-called regional problems, which make it possible to study in more detail both the geometric and gravimetric (physical) properties of the studied region (territory). For example, the tasks of constructing a high-precision regional geoid (quasi-geoid), regional ellipsoid, determining the regional normal formula of gravity, and others are becoming increasingly important.

That is why at present both national and regional reference ellipsoids are accepted for processing geodetic data on a regional scale (for example, for a specific country), and for global research – a general terrestrial reference ellipsoid GRS80 or, when processing GPS data – a general terrestrial reference ellipsoid WGS84.

In principle, any reference ellipsoid that represents a generalized figure of the Earth with appropriate accuracy can be used to process geodetic information. The deviations of the geoid from such an ellipsoid can determine the corrections that must be made in the results of geodetic measurements to bring the latter to the surface of this ellipsoid.

However, with large deviations of the geoid from the reference ellipsoid, there are large corresponding reductions of geodetic measurements, which are burdened with significant errors due to the linearization of the main problem of geodesy and, consequently, the problem of bringing geodetic measurements to the ellipsoid. Therefore, from a practical point of view, to reduce the impact of these linearization errors and obtain methodologically

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optimal results of geodetic data processing, it is expedient and even necessary to use a reference ellipsoid that best describes the generalized geoid surface in the region of specific geodetic works.

Given the above, the question arose about the national reference coordinate system, as such a system has some advantages over the national system in the process of practical processing of mass geodetic measurements, especially linear. In this regard, the issues of building a national reference system, namely, the definition of a regional ellipsoid, are very *important and relevant*.

Therefore, the scope of our research is the construction of a national reference system based on data on the regional gravitational field of Ukraine.

The *methodology* of such research is that the task of determining the regional ellipsoid is practically reduced to finding some corrections to the known, accepted by us, the general terrestrial ellipsoid GRS80. The regional ellipsoid for the territory of Ukraine should be the one that would best represent the geoid (quasi-geoid) of the region. That is, the heights of the geoid relative to the regional ellipsoid within the territory of Ukraine should be as small as possible.

These questions are reflected in this monograph, the purpose of which is to investigate a priori calculations to determine the parameters of the internal orientation of the regional ellipsoid according to its gravitational field in Ukraine.

Thus, based on the results of the above a priori studies, the following can be noted. Determining all five parameters of a regional ellipsoid leads to a strong functional dependence of the parameters. This dependence (correlation) is quite well demonstrated on the values of root mean square errors, which are proportional to the obtained parameters and even exceed the latter. Taking into account these remarks, we can conclude that the joint calculation of all five parameters by the method of least squares on the territory of Ukraine does not give us the expected good results. This is well seen from a priori calculations based on the heights of the geoid, presented in the form of a spheroidal trapezoid, which describes the territory of Ukraine. In contrast to this solution, studies to determine only the parameters of the internal orientation of the ellipsoid at a given major half-axis and compression of this ellipsoid, make it possible to choose a terrestrial regional ellipsoid that would best represent a geoid (quasi-geoid) built in Ukraine.

1. Introduction

As you know, the results of geodetic measurements carried out on the Earth's surface to determine the relative position of points, ie angles and distances between these points, primarily relate to different level surfaces of the Earth. The same can be said for the results of observations at different points on the Earth's surface, namely: finding their astronomical latitude, longitude and azimuth, which give the direction of the line is difficult as normal to level surfaces at these points. Therefore, these results of geodetic and astronomical determinations must be reduced to one level surface of the Earth, ie to the surface of the geoid [1; 5; 8].

However, this surface of the geoid has a rather complex shape. It is clear that a complex surface cannot serve as a coordinate surface for finding the relative position of geodetic points. The *novelty* of this topic is that in mathematical processing of astronomical and geodetic measurements, the surface of the geoid is usually replaced by a known and simpler surface of relativity, namely: the surface of an ellipsoid that has the appropriate size and position in the Earth's body. Such an ellipsoid of rotation is called a reference ellipsoid. The dimensions of the reference ellipsoid and its position or orientation in the Earth's body must be set so that its surface is more or less close to the surface of the geoid [4].

The dimensions of the reference ellipsoid are characterized, as a rule, by its magnitude and polar compression, and its position in the Earth's body is mainly determined by the components of the deviation of the heavy in the plane of the meridian and the first vertical from the normal to its surface is taken as the starting (initial) point of geodetic measurements. The direction of the line is difficult at the starting point relative to the main coordinate planes (ie the planes of the Earth's equator and the prime meridian) is established by astronomical determinations of its latitude and longitude. By correcting the astronomical latitude and longitude of the starting point for the deviation of the line from normal to the surface of the reference ellipsoid at the same point determine its geodetic latitude and longitude, which together with the height of the geoid at this starting point serve as so-called geodetic reference dates for processing geodetic measurements on the surface of the received reference ellipsoid [1; 3; 4].

Since such orientation of the ellipsoid requires knowledge of astronomical observations, and the elements of orientation depend on the choice of starting point on the Earth's surface (location, for example, in Ukraine is not yet definitively determined), this paper focuses on finding so-called elements of internal orientation, by means of which, according to the appropriate communication formulas, it is always possible to proceed to the above-described elements of external orientation at the starting point with any coordinates [3; 4].

From the theoretical point of view for processing of geodetic measurements and definition on these data of position of geodetic points it is possible to accept any reference ellipsoid which in one way or another characterizes a figure of the Earth. Knowing the deviation of the geoid from the accepted ellipsoid, it is always possible to determine quite accurately the corrections that must be made in the results of geodetic measurements to bring the latter to the surface of this ellipsoid.

However, if the deviations of the geoid are significant enough, then due to errors in the obtained dimensions and orientation of the received reference ellipsoid, the corresponding reductions in geodetic measurements will also be significant. In this case, the results of geodetic measurements, especially linear elements of triangulation, after bringing them to the surface of the reference ellipsoid will be very different from their values obtained on the surface of the geoid. In addition, from the point of view of their practical use, it is necessary that they do not undergo major changes and distortions. Therefore, for the processing of geodetic measurements it is necessary to use such a reference ellipsoid, which in size and orientation would be closest to the figure of the geoid in a particular region under study [2].

Thus, the main requirement for mathematical elaboration of triangulation networks, as well as for all geodetic measurements performed on the Earth's surface, is the establishment of such a reference ellipsoid, the surface of which best fits or is close to the surface of the geoid, which is relevant to scientific research.

Based on the above, the purpose of our research is to conduct a priori calculations of the parameters of the regional ellipsoid for the territory of Ukraine and to assess the effectiveness of such a reference system in solving some practical and scientific problems of geodesy.

To achieve this goal in this scientific work the following tasks are set and solved:

– Determination of linear elements of orientation of the ellipsoid $\varnothing x$, $\varnothing y$, $\varnothing z$.

– Calculation of parameters of the reference ellipsoid $\varnothing x$, $\varnothing y$, $\varnothing z$, $\varnothing a$.

– Finding the values of the regional ellipsoid $\varnothing x$, $\varnothing y$.

– Calculation of unknown parameters of the reference ellipsoid $\varnothing x$ and $\varnothing a$.

– Finding the values of the regional ellipsoid $\varnothing y$ and $\varnothing a$.

– Determination of ellipsoid parameters $\varnothing z$, $\varnothing a$.

2. Derivation of parametric equations

Consider the results of the measured values, the true values of which are equal to $X_i \left(i=\overline{1,n}\right)$. Assume that their measured values $x_i \left(i=\overline{1,n}\right)$ are obtained with weights $p_i \left(i=\overline{1,n}\right)$, and among them are k necessary and r redundant. Choose k independent parameters (as parameters you can take the required measured or unmeasured values), the true values of which are denoted by $T_j \left(j=\overline{1,k}\right)$. The true values of the measured values X_i $\left(i=\overline{1,n}\right)$ are functionally given through the true values of the parameters $T_j \left(j=\overline{1,k}\right)$ in the form

$$X_i = f_i\left(T_1, T_2, \ldots, T_k\right),\ i=\overline{1,n}.$$

Consider the balanced values $t_j \left(j=\overline{1,k}\right)$ of unknown parameters $T_j \left(j=\overline{1,k}\right)$, the right and left parts of which are written for balanced values $x_i + v_i \left(i=\overline{1,n}\right)$ for both quantities $X_i \left(i=\overline{1,n}\right)$ and balanced values $t_j \left(j=\overline{1,k}\right)$ of parameters $T_j \left(j=\overline{1,k}\right)$

$$x_i + v_i = f_i\left(t_1, t_2, \ldots, t_k\right) \quad \left(i=\overline{1,n}\right),$$

where we get the parametric equations of corrections

$$v_i = f_i\left(t_1, t_2, \ldots, t_k\right) - x_i,\ i=\overline{1,n}.$$

For this case, the condition of least squares is written in the form

$$F = \sum_{i=1}^{n} p_i \left[f_i\left(t_1, t_2, \ldots, t_k\right) - x_i\right]^2 = \left[p \cdot \left(f - x\right)^2\right] = \left[p \cdot v^2\right] \to \min.$$

A function F is a function of parameters $t_j \left(j=\overline{1,k}\right)$, and in order to find the minimum of a function F, it is necessary to have its partial derivatives of the first order by arguments $t_j \left(j=\overline{1,k}\right)$. We differentiate the function F by variables $t_j \left(j=\overline{1,k}\right)$ and equate the obtained relations to zero (a necessary condition for the extremum of the function). As a result, we obtain a system of k equations with k unknowns $t_j \left(j=\overline{1,k}\right)$

$$\frac{\partial F}{\partial t_j} = 2\sum_{i=1}^{n} p_i v_i \frac{\partial v_i}{\partial t_j} = \left[pv\frac{\partial v}{\partial t} \right] = 0 \,,\;\; j = \overline{1,k} \,.$$

This system of equations is called a system of normal equations in general. Solving it, we obtain balanced values of the parameters $t_j \left(j = \overline{1,k} \right)$.

However, in the general case, the system of parametric equations of corrections is nonlinear and it is difficult to obtain its solution. Therefore, the above system of nonlinear equations should be linearized. To do this, you need to somehow find the approximate values $t_j^\circ \left(j = \overline{1,k} \right)$ of the parameters $t_j \left(j = \overline{1,k} \right)$ and give the balanced values as follows:

$$t_j = t_j^\circ + \tau_j \left(j = \overline{1,k} \right),$$

where $\tau_j \left(j = \overline{1,k} \right)$ are corrections to approximate values $t_j^\circ \left(j = \overline{1,k} \right)$ of parameters $t_j \left(j = \overline{1,k} \right)$.

The approximate values $t_j^\circ \left(j = \overline{1,k} \right)$ are determined as accurately as possible so that the corrections $\tau_j \left(j = \overline{1,k} \right)$ are small enough.

Therefore, we rewrite the system of parametric equations in the form

$$v_i = f_i \left(t_1^\circ + \tau_1, t_2^\circ + \tau_2, \ldots, t_k^\circ + \tau_k, \right) - x_i \,,\;\; i = \overline{1,n} \,.$$

Assume that the functions $f_i \left(i = \overline{1,n} \right)$ are such that they can be decomposed into a Taylor series around a point $\left(t_1^\circ, t_2^\circ, \ldots, t_k^\circ \right)$. Since the amendments $\tau_j \left(j = \overline{1,k} \right)$ are small, we will obtain only the linear members of the schedule

$$v_i = f_i \left(t_1^\circ, t_2^\circ, \ldots, t_k^\circ \right) + \sum_{j=1}^{k} \left(\frac{\partial f_i}{\partial t_j} \right)_0 \cdot \tau_j - x_i \quad \left(i = \overline{1,n} \right).$$

or by entering a notation

$$\left(\frac{\partial f_i}{\partial t_1} \right)_0 = a_i \,,\; \left(\frac{\partial f_i}{\partial t_2} \right)_0 = b_i \,, \ldots \; \left(\frac{\partial f_i}{\partial t_k} \right)_0 = u_i \,,\; f_i \left(t_1^\circ, t_2^\circ, \ldots, t_k^\circ \right) - x_i = l_i \,,\; (i = \overline{1,n}),$$

we obtain a system of parametric equations in linear form

$$v_i = a_i \tau_1 + b_i \tau_2 + \ldots + u_i \tau_k + l_i \,,\; i = \overline{1,n} \,.$$

Corrections v_i to the measured values $x_i \left(i = \overline{1,n} \right)$ and corrections τ_j to the approximate values of the parameters $t_j^\circ \left(j = \overline{1,k} \right)$ are unknown in these equations. Therefore, the given system of parametric equations of corrections is uncertain, because the number of unknowns is equal to $n + k$, and the number of equations is equal to n.

To solve the system of parametric equations in linear form, we use the method of least squares. That is

$$\left[pv^2\right]=\sum_{i=1}^{n} p_i\left(a_i\tau_1+b_i\tau_2+\ldots+u_i\tau_k+l_i\right)^2=F\left(\tau_1,\tau_2,\ldots\tau_k\right)\rightarrow \min .$$

To find the minimum of the function F, we find its partial derivatives over the variables $\tau_j\left(j=\overline{1,k}\right)$ and equate them to zero. As a result, we obtain a normal system of k linear equations with k unknowns $\tau_j\left(j=\overline{1,k}\right)$

$$\begin{cases}\left[pa^2\right]\tau_1+\left[pab\right]\tau_2+\ldots+\left[pau\right]\tau_k+\left[pal\right]=0\\ \left[pab\right]\tau_1+\left[pb^2\right]\tau_2+\ldots+\left[pbu\right]\tau_k+\left[pbl\right]=0\\ \vdots\\ \left[pau\right]\tau_1+\left[pbu\right]\tau_2+\ldots+\left[pu^2\right]\tau_k+\left[pul\right]=0.\end{cases}$$

For isosceles measurements, the normal system of linear algebraic equations is written as follows

$$\begin{cases}\left[a^2\right]\tau_1+\left[ab\right]\tau_2+\ldots+\left[au\right]\tau_k+\left[al\right]=0\\ \left[ab\right]\tau_1+\left[b^2\right]\tau_2+\ldots+\left[bu\right]\tau_k+\left[bl\right]=0\\ \vdots\\ \left[au\right]\tau_1+\left[bu\right]\tau_2+\ldots+\left[u^2\right]\tau_k+\left[ul\right]=0.\end{cases}$$

Consider a system of parametric equations of corrections in matrix form. We introduce a matrix A of coefficients of this dimension system $n\times k$

$$A=\begin{pmatrix}a_1b_1\ldots u_1\\ a_2b_2\ldots u_2\\ \vdots\\ a_nb_n\ldots u_n\end{pmatrix},$$

matrix-column τ of the dimension $k\times 1$ of unknown corrections τ_i, called the vector of corrections to the approximate values of the parameters $t_i\left(i=\overline{1,k}\right)$

$$\tau=\begin{pmatrix}\tau_1\\ \tau_2\\ \vdots\\ \tau_k\end{pmatrix},$$

matrix-column L of the dimension $n\times1$ of free members $l_i\left(i=\overline{1,n}\right)$ or vector of free members of the system

$$L=\begin{pmatrix} l_1 \\ l_2 \\ \vdots \\ l_n \end{pmatrix},$$

and a matrix-column V of the dimension $n\times1$ or vector of corrections $v_i\left(i=\overline{1,n}\right)$ to the measurement results

$$V=\begin{pmatrix} v_1 \\ v_2 \\ \vdots \\ v_n \end{pmatrix}.$$

As a result, the system of parametric equations of corrections in matrix form can be written as follows

$$A\cdot\tau+L=V.$$

The condition $\left[pv^2\right]\to\min$ will look like

$$V^T PV\to\min.$$

where V^T is transposed matrix-column V of corrections to the measurement results;

$$P=\begin{pmatrix} p_1 & 0 & \cdots & 0 \\ 0 & p_2 & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & \cdots & p_n \end{pmatrix}$$ is diagonal matrix of weights.

The function F will then take the form

$$F=\left(A\tau+L\right)^T\cdot P\cdot\left(A\tau+L\right),$$

and the required minimum condition for the function F is written as follows

$$A^T PV=0.$$

As a result, we obtain the matrix normal equation

$$A^T PV=A^T P\cdot(A\tau+L)=A^T PA\tau+A^T PL=0.$$

We show that it is equivalent to the normal system of linear algebraic equations. To do this, we describe the terms that are included in the matrix normal equation

$$A^T PA\tau = \begin{pmatrix} a_1\, a_2 \ldots a_n \\ b_1\, b_2 \ldots b_n \\ \vdots \\ u_1\, u_2 \ldots u_n \end{pmatrix} \cdot \begin{pmatrix} p_1\, 0 \ldots 0 \\ 0\;\, p_2 \ldots 0 \\ \vdots \\ 0\, 0 \ldots p_n \end{pmatrix} \cdot \begin{pmatrix} a_1 b_1 \ldots u_1 \\ a_2 b_2 \ldots u_2 \\ \vdots \\ a_n b_n \ldots u_n \end{pmatrix} \cdot \begin{pmatrix} \tau_1 \\ \tau_2 \\ \vdots \\ \tau_k \end{pmatrix} =$$

$$= \begin{pmatrix} a_1 p_1\;\, a_2 p_2 \ldots a_n p_n \\ b_1 p_1\;\, b_2 p_2 \ldots b_n p_n \\ \vdots \\ u_1 p_1\;\, u_2 p_2 \ldots u_n p_n \end{pmatrix} \cdot \begin{pmatrix} a_1 b_1 \ldots u_1 \\ a_2 b_2 \ldots u_2 \\ \vdots \\ a_n b_n \ldots u_n \end{pmatrix} \cdot \begin{pmatrix} \tau_1 \\ \tau_2 \\ \vdots \\ \tau_k \end{pmatrix} =$$

$$= \begin{pmatrix} [a^2 p]\, [abp] \ldots [aup] \\ [abp]\, [b^2 p] \ldots [bup] \\ \vdots \\ [aup]\, [bup] \ldots [u^2 p] \end{pmatrix} \cdot \begin{pmatrix} \tau_1 \\ \tau_2 \\ \vdots \\ \tau_k \end{pmatrix} = \begin{pmatrix} [a^2 p]\tau_1 + [abp]\tau_2 + \ldots + [aup]\tau_k \\ [abp]\tau_1 + [b^2 p]\tau_2 + \ldots + [bup]\tau_k \\ \vdots \\ [aup]\tau_1 + [bup]\tau_2 + \ldots + [u^2 p]\tau_k \end{pmatrix}$$

$$A^T PL = \begin{pmatrix} a_1 p_1\;\, a_2 p_2 \ldots a_n p_n \\ b_1 p_1\;\, b_2 p_2 \ldots b_n p_n \\ \vdots \\ u_1 p_1\;\, u_2 p_2 \ldots u_n p_n \end{pmatrix} \begin{pmatrix} l_1 \\ l_2 \\ \vdots \\ l_n \end{pmatrix} = \begin{pmatrix} [apl] \\ [bpl] \\ \vdots \\ [upl] \end{pmatrix};$$

$$A^T PA\tau + A^T PL = \begin{pmatrix} [a^2 p]\tau_1 + [abp]\tau_2 + \ldots + [aup]\tau_k + [apl] \\ [abp]\tau_1 + [b^2 p]\tau_2 + \ldots + [bup]\tau_k + [bpl] \\ \vdots \\ [aup]\tau_1 + [bup]\tau_2 + \ldots + [u^2 p]\tau_k + [upl] \end{pmatrix}.$$

As can be seen from the obtained equality, its right-hand side is equivalent to the normal system of linear algebraic equations.

Now we turn to the problem of determining the parameters of the regional ellipsoid relative to the known accepted total terrestrial ellipsoid

GRS80 on the basis of our formulas of the parametric method of compatible balancing of measured quantities.

Under the definition of a regional ellipsoid here we understand the finding of its parameters: the major semi-axis a, the polar compression α and the rectangular coordinates of its center in the Earth's body: x_0, y_0, z_0. The relationship between these quantities can be represented as an abbreviated formula of the Molodensky transformation for geodetic height [4]

$$\Delta H = \Delta x \cos \bar{B} \cos \bar{L} + \Delta y \cos \bar{B} \sin \bar{L} + \Delta z \sin \bar{B} + \left(\bar{a}\Delta\alpha + \bar{\alpha}\Delta a\right)\sin^2 \bar{B} + \Delta a. \quad (1)$$

Formula (1), as it is easy to see, gives a connection not of the parameters themselves, but of some parameter shifts $\Delta a, \Delta\alpha, \Delta x, \Delta y, \Delta z$, ie corrections, which represent the differences between the parameters of some two ellipsoids. In this formula: $\varnothing H$ is the difference in geodetic heights of a point on the Earth's surface relative to each of the two ellipsoids. In practice, this value can be written as [2]

$$\Delta H = N - \bar{N}, \quad (2)$$

where N and $\bar{N}$ are the height of the geoid (quasi-geoid) relative to each of the ellipsoids.

So, to find the necessary parameters a, α, x_0, y_0, z_0 of some ellipsoid E, you need to have an ellipsoid $\bar{E}$ with known parameters $\bar{a}, \bar{\alpha}, \bar{x}_0, \bar{y}_0, \bar{z}_0$.

Then

$$\left.\begin{aligned} a &= \bar{a} + \Delta a; \\ \alpha &= \bar{\alpha} + \Delta\alpha; \\ x_0 &= \bar{x}_0 + \Delta x; \\ y_0 &= \bar{y}_0 + \Delta y; \\ z_0 &= \bar{z}_0 + \Delta z. \end{aligned}\right\} \quad (3)$$

In principle, you can use any known ellipsoid $\bar{E}\left(\bar{a}, \bar{\alpha}, \bar{x}_0, \bar{y}_0, \bar{z}_0\right)$. But, as we will see below, it is best to adopt a geocentric ellipsoid. Such an ellipsoid may be, for example, the well-known general terrestrial ellipsoid GRS80. Its parameters are as follows [7]

$$\left.\begin{aligned} \bar{a} &= 6378137\ m; \\ \bar{\alpha} &= 1/298.257222101; \\ \bar{x}_0 &= 0\ m; \\ \bar{y}_0 &= 0\ m; \\ \bar{z}_0 &= 0\ m. \end{aligned}\right\} \quad (4)$$

Then formulas (3) can be rewritten as

$$\left.\begin{aligned} a &= \overline{a} + \Delta a\,; \\ \alpha &= \overline{\alpha} + \Delta\alpha\,; \\ x_0 &= \Delta x\,; \\ y_0 &= \Delta y\,; \\ z_0 &= \Delta z\,. \end{aligned}\right\} \tag{5}$$

It is easy to note that all values in formula (1) with a dash above must be known and assigned to the GRS80 system, and values without a dash will be unknown.

Therefore, the task of determining the regional ellipsoid is practically reduced to finding some corrections $\Delta a, \Delta\alpha, \Delta x, \Delta y, \Delta z$ to the known, accepted by us, the general terrestrial ellipsoid GRS80.

The regional ellipsoid for the territory of Ukraine should be the one that would best represent the geoid (quasi-geoid) of the region. That is, the heights of geoid N relative to the regional ellipsoid within the territory of Ukraine should be as small as possible. Given this basic requirement, we will perform a priori research to determine the regional ellipsoid for Ukraine.

Since we need to identify five unknown amendments $\Delta a, \Delta\alpha, \Delta x, \Delta y, \Delta z$, we need to have at least five points within the territory of Ukraine with known geodetic coordinates $\overline{B}, \overline{L}, \overline{H}$.

For the planned coordinates $\overline{B}$ and $\overline{L}$ take the approximate geodetic coordinates of the vertices of the spheroidal trapezoid *ABCD* and its center *O*, in which (trapezoid) fits the territory of Ukraine.

That is

$$\left.\begin{matrix} A\begin{Bmatrix} B_A = 52.5^0 \\ L_A = 21.6^0 \end{Bmatrix} & & B\begin{Bmatrix} B_B = 52.5^0 \\ L_B = 40.0^0 \end{Bmatrix} \\ & O\begin{Bmatrix} B_O = 48.3^0 \\ L_O = 30.8^0 \end{Bmatrix} & \\ D\begin{Bmatrix} B_D = 44.1^0 \\ L_D = 21.6^0 \end{Bmatrix} & & C\begin{Bmatrix} B_C = 44.1^0 \\ L_C = 40.0^0 \end{Bmatrix} \end{matrix}\right\} \tag{6}$$

Note that although the geodetic coordinates of points *A, B, C, D, O* are given in the system of the Krasovsky reference ellipsoid, they are approximate, so they can be considered as known in the GRS80 system.

The heights of the geoid $\bar{N}$ of the corresponding five points of the geoid can be taken, for example, from the decomposition of the potential of gravity into a number of spherical functions. The known model GEMT1 (with $n = m = 36$) gives us the following values $\bar{N}$ [9]

$$\left.\begin{array}{ccc} \bar{N}_A = 30.7\ m & & \bar{N}_B = 9.8\ m \\ & \bar{N}_O = 25.9\ m & \\ \bar{N}_D = 43.7\ m & & \bar{N}_C = 16.5\ m \end{array}\right\} \tag{7}$$

Having the necessary initial data and using Molodensky's formula (1), in which the unknown corrections $\Delta a, \Delta\alpha, \Delta x, \Delta y, \Delta z$ are presented in linear form, we write the following parametric equations for the described five points (*A, B, C, D, O*)

$$\left.\begin{array}{l} a_A\Delta x + b_A\Delta y + c_A\Delta z + d_A\bar{a}\Delta\alpha + e_A\Delta a + l_A = v_A\ ; \\ a_B\Delta x + b_B\Delta y + c_B\Delta z + d_B\bar{a}\Delta\alpha + e_B\Delta a + l_B = v_B\ ; \\ a_C\Delta x + b_C\Delta y + c_C\Delta z + d_C\bar{a}\Delta\alpha + e_C\Delta a + l_C = v_C\ ; \\ a_D\Delta x + b_D\Delta y + c_D\Delta z + d_D\bar{a}\Delta\alpha + e_D\Delta a + l_D = v_D\ ; \\ a_O\Delta x + b_O\Delta y + c_O\Delta z + d_O\bar{a}\Delta\alpha + e_O\Delta a + l_O = v_O\ , \end{array}\right\} \tag{8}$$

where

$$\left.\begin{array}{l} a_A = \cos B_A \cos L_A;\ b_A = \cos B_A \sin L_A;\ c_A = \sin B_A;\ d_A = \sin^2 B_A;\ e_A = 1-\bar{\alpha}\sin^2 B_A\ ; \\ a_B = \cos B_B \cos L_B;\ b_B = \cos B_B \sin L_B;\ c_B = \sin B_B;\ d_B = \sin^2 B_B;\ e_B = 1-\bar{\alpha}\sin^2 B_B\ ; \\ a_C = \cos B_C \cos L_C;\ b_C = \cos B_C \sin L_C;\ c_C = \sin B_C;\ d_C = \sin^2 B_C\ ;\ e_C = 1-\bar{\alpha}\sin^2 B_C\ ; \\ a_D = \cos B_D \cos L_D;\ b_D = \cos B_D \sin L_D;\ c_D = \sin B_D;\ d_D = \sin^2 B_D\ ;\ e_D = 1-\bar{\alpha}\sin^2 B_D\ ; \\ a_O = \cos B_O \cos L_O;\ b_O = \cos B_O \sin L_O;\ c_O = \sin B_O;\ d_O = \sin^2 B_O\ ;\ e_O = 1-\bar{\alpha}\sin^2 B_O \end{array}\right\} \tag{9a}$$

and

$$\left.\begin{array}{l} v_A = N_A\ ; \\ v_B = N_B\ ; \\ v_C = N_C\ ; \\ v_D = N_D\ ; \\ v_O = N_O\ . \end{array}\right\} \tag{9b}$$

The right-hand sides of equations (8), as can be seen, will play the role of unknown corrections. Since the system (8) of five equations has ten unknown quantities $\left(\Delta a, \Delta\alpha, \Delta x, \Delta y, \Delta z, v_A, v_B, v_C, v_D, v_O\right)$, we impose on it (the system) an additional condition of the least squares method

$$\sum v_i^2 \to \min, \qquad i = A, B, C, D, O, \tag{10}$$

to get a single solution.

Using formulas (9a), we first find the coefficients of the parametric equations of corrections (8). The results are recorded in Table 1.

Table 1

Calculation of coefficients of parametric equations of corrections

Points	Formula	Value	Formula	Value	Coefficient	Value
A	$\cos B_A$	0.608761	$\mathrm{COS}B_A\mathrm{COS}L_A$	0.5660113	aA	-0.5660113
	$\mathrm{COS}L_A$	0.929776	$\cos B_A \sin L_A$	0.2241001	bA	-0.2241001
	$\sin B_A$	0.793353	$\sin^2 B_A$	0.6294089	cA	-0.793353
	$\sin L_A$	0.368125			dA	0.6294089
					eA	-0.9978897
B	$\cos B_B$	0.608761	$\mathrm{COS}B_B\mathrm{COS}L_B$	0.4663377	aB	-0.4663377
	$\cos L_B$	0.766044	$\cos B_B \sin L_B$	0.3913042	bB	-0.3913042
	$\sin B_B$	0.793353	$\sin^2 B_B$	0.6294089	cB	-0.793353
	$\sin L_B$	0.642788			dB	0.6294089
					eB	-0.9978897
C	$\cos B_C$	0.718126	$\cos B_C \cos L_C$	0.5501161	aC	-0.5501161
	$\cos L_C$	0.766044	$\cos B_C \sin L_C$	0.4616027	bC	-0.4616027
	$\sin B_C$	0.695913	$\sin^2 B_C$	0.4842949	cC	-0.695913
	$\sin L_C$	0.642788			dC	0.4842949
					eC	-0.9983762
D	$\cos B_D$	0.718126	$\mathrm{COS}B_D\mathrm{COS}L_D$	0.6676963	aD	-0.6676963
	$\mathrm{COS}L_D$	0.929776	$\cos B_D \sin L_D$	0.2643601	bD	-0.2643601
	$\sin B_D$	0.695913	$\sin^2 B_D$	0.4842949	cD	-0.6959130
	$\sin L_D$	0.368125			dD	0.4842949
					eD	-0.9983762
O	$\cos B_O$	0.665230	$\cos B_O \cos L_O$	0.5714059	aO	-0.5714059
	$\mathrm{COS}L_O$	0.858960	$\cos B_O \sin L_O$	0.3406263	bO	-0.3406263
	$\sin B_O$	0.746638	$\sin^2 B_O$	0,5574683	cO	-0.746638
	$\sin L_O$	0.512043			dO	0.5574683
					eO	-0.9981309

The free terms of the parametric equations of corrections (8) according to formula (1) can then be written

$$\left.\begin{aligned} l_A &= \bar{N}_A\,; \\ l_B &= \bar{N}_B\,; \\ l_C &= \bar{N}_C\,; \\ l_D &= \bar{N}_D\,; \\ l_O &= \bar{N}_O\,. \end{aligned}\right\} \qquad (11)$$

Substituting instead of the heights of the geoid $\bar{N}_A, \bar{N}_B, \bar{N}_C, \bar{N}_D, \bar{N}_O$ their values from expression (7), we obtain

$$\left.\begin{aligned} l_A &= 30.7\ m\,; \\ l_B &= 9.8\ m\,; \\ l_C &= 16.5\ m\,; \\ l_D &= 43.7\ m\,; \\ l_O &= 25.9\ m\,. \end{aligned}\right\} \qquad (12)$$

Therefore, taking the values of the coefficients from Table 1 and the values of the free terms from expression (12), we solve the system of parametric equations of corrections (8) under the condition of least squares (10). Then we obtain the following required parameters and their root mean square errors

$$\left.\begin{aligned} \Delta x &= \quad -5 \pm 28\ m\,; \\ \Delta y &= -136 \pm 17\ m\,; \\ \Delta z &= \quad 982 \pm 549\ m\,; \\ \bar{a}\Delta\alpha &= 783 \pm 393\ m\,; \\ \Delta a &= -222 \pm 170\ m\,. \end{aligned}\right\} \qquad (13)$$

The obtained a priori values of the parameters of the regional ellipsoid and their errors indicate a strong dependence (correlation) between the unknown values. To find out which values are most correlated with each other, you need to perform some additional research to calculate the required parameters. Consider a few partial cases.

3. Determination of parameters Δx, Δy, Δz

<u>Case 1.</u> Assume that the parameters Δa and $\Delta\alpha$ are known. Then we will look for only linear elements of orientation Δx, Δy, Δz of the regional ellipsoid.

That is, the parametric equations will look like

$$\left.\begin{aligned} a_A\Delta x + b_A\Delta y + c_A\Delta z + l_A^I &= v_A\ ;\\ a_B\Delta x + b_B\Delta y + c_B\Delta z + l_B^I &= v_B\ ;\\ a_C\Delta x + b_C\Delta y + c_C\Delta z + l_C^I &= v_C\ ;\\ a_D\Delta x + b_D\Delta y + c_D\Delta z + l_D^I &= v_D\ ;\\ a_O\Delta x + b_O\Delta y + c_O\Delta z + l_O^I &= v_O\ , \end{aligned}\right\} \quad (14)$$

and free members will be recorded as follows

$$\left.\begin{aligned} l_A^I &= \left(\bar{a}\Delta\alpha + \bar{\alpha}\Delta a\right)\sin^2 B_A + \Delta a + \bar{N}_A\ ;\\ l_B^I &= \left(\bar{a}\Delta\alpha + \bar{\alpha}\Delta a\right)\sin^2 B_B + \Delta a + \bar{N}_B\ ;\\ l_C^I &= \left(\bar{a}\Delta\alpha + \bar{\alpha}\Delta a\right)\sin^2 B_C + \Delta a + \bar{N}_C\ ;\\ l_D^I &= \left(\bar{a}\Delta\alpha + \bar{\alpha}\Delta a\right)\sin^2 B_D + \Delta a + \bar{N}_D\ ;\\ l_O^I &= \left(\bar{a}\Delta\alpha + \bar{\alpha}\Delta a\right)\sin^2 B_O + \Delta a + \bar{N}_O\ . \end{aligned}\right\} \quad (15)$$

The corresponding coefficients of the system (14) are selected from Table 1. To calculate the free terms (15) you need to enter some numerical values for corrections Δa, $\Delta\alpha$.

Regarding the choice of these amendments, the following should be noted. The European Regional Geodetic System European 1950, which is used by 16 countries located in Europe, is well known to all. As Ukraine is a European state, it is quite logical to use the parameters of the European 1950 system. Therefore, the main parameters here and in our future research will be the shift of the parameters of the regional geodetic system European 1950 in the national system GRS80 [7]

$$\left.\begin{aligned} \Delta a &= 251\ m\ ;\\ \Delta\alpha &= 0.14192702\times 10^{-4}\ ;\\ \Delta x &= -87\ m\ ;\\ \Delta y &= -98\ m\ ;\\ \Delta z &= -121\ m\ . \end{aligned}\right\} \quad (16)$$

Therefore, taking the values $\varnothing a$ and $\Delta\alpha$ from expression (16), we calculate by formula (15) the values of free members. That is

$$\left.\begin{aligned} l_A^I &= -162.8\ m\,; \\ l_B^I &= -183.7\ m\,; \\ l_C^I &= -190.3\ m\,; \\ l_D^I &= -163.1\ m\,; \\ l_O^I &= -174.2\ m\,. \end{aligned}\right\} \qquad (17)$$

Then, having the corresponding values of the coefficients from Table 1 and the free terms from expression (17), we solve the system of parametric equations (14) under the condition of least squares (10). Unknown values and their root mean square errors will take the following values

$$\left.\begin{aligned} \Delta x &= -58 \pm 9\ m\,; \\ \Delta y &= -167 \pm 8\ m\,; \\ \Delta z &= -115 \pm 8\ m\,. \end{aligned}\right\} \qquad (18)$$

Let us now consider the following case.

4. Determination of parameters $\varnothing x$, $\varnothing y$, $\varnothing z$, $\varnothing a$

Case 2. Let us know only the value $\Delta\alpha$. You need to find the parameters $\varnothing x$, $\varnothing y$, $\varnothing z$, $\varnothing a$. Parametric equations in this case will take the form

$$\left.\begin{aligned} a_A\Delta x + b_A\Delta y + c_A\Delta z + e_A\Delta a + l_A^{II} &= v_A\,; \\ a_B\Delta x + b_B\Delta y + c_B\Delta z + e_B\Delta a + l_B^{II} &= v_B\,; \\ a_C\Delta x + b_C\Delta y + c_C\Delta z + e_C\Delta a + l_C^{II} &= v_C\,; \\ a_D\Delta x + b_D\Delta y + c_D\Delta z + e_D\Delta a + l_D^{II} &= v_D\,; \\ a_O\Delta x + b_O\Delta y + c_O\Delta z + e_O\Delta a + l_O^{II} &= v_O\,, \end{aligned}\right\} \qquad (19)$$

and free members are equal

$$\left.\begin{aligned} l_A^{II} &= \bar{a}\sin^2 B_A\Delta\alpha + \bar{N}_A\,; \\ l_B^{II} &= \bar{a}\sin^2 B_B\Delta\alpha + \bar{N}_B\,; \\ l_C^{II} &= \bar{a}\sin^2 B_C\Delta\alpha + \bar{N}_C\,; \\ l_D^{II} &= \bar{a}\sin^2 B_D\Delta\alpha + \bar{N}_D\,; \\ l_O^{II} &= \bar{a}\sin^2 B_O\Delta\alpha + \bar{N}_O\,. \end{aligned}\right\} \qquad (20)$$

Then the coefficients of the system (19) can be taken from Table 1. And the free terms (20), if we take the value from expression (16), after simple calculations will have values

$$\left.\begin{aligned} l_A^{II} &= 87.7\ m\,;\\ l_B^{II} &= 66.8\ m\,;\\ l_C^{II} &= 60.3\ m\,;\\ l_D^{II} &= 87.5\ m\,;\\ l_O^{II} &= 76.4\ m\,. \end{aligned}\right\} \tag{21}$$

Thus, taking into account the corresponding values from Table 1 and from expression (21), we solve the system of parametric equations (19) under condition (10). Then we will get

$$\left.\begin{aligned} \Delta x &= 110 \pm 89\ m\,;\\ \Delta y &= -67 \pm 53\ m\,;\\ \Delta z &= 101 \pm 115\ m\,;\\ \Delta a &= -39 \pm 154\ m\,. \end{aligned}\right\} \tag{22}$$

5. Determination of parameters $\varnothing x$, $\varnothing y$

<u>Case 3.</u> Let the quantities $\Delta a, \Delta\alpha, \Delta z$ be known. You need to find the values $\varnothing x$, $\varnothing y$. Parametric equations of corrections will then be written as

$$\left.\begin{aligned} a_A\Delta x + b_A\Delta y + l_A^{III} &= v_A\,;\\ a_B\Delta x + b_B\Delta y + l_B^{III} &= v_B\,;\\ a_C\Delta x + b_C\Delta y + l_C^{III} &= v_C\,;\\ a_D\Delta x + b_D\Delta y + l_D^{III} &= v_D\,;\\ a_O\Delta x + b_O\Delta y + l_O^{III} &= v_O\,. \end{aligned}\right\} \tag{23}$$

The coefficients in equations (23), as in the previous cases, can be selected from Table 1.

Free members of the system (23), as you can see, will take the form

$$\left.\begin{aligned}
l_A^{III} &= \sin B_A \Delta z + \left(\bar{a}\Delta\alpha + \bar{\alpha}\Delta a\right)\sin^2 B_A + \Delta a + \bar{N}_A\ ;\\
l_B^{III} &= \sin B_B \Delta z + \left(\bar{a}\Delta\alpha + \bar{\alpha}\Delta a\right)\sin^2 B_B + \Delta a + \bar{N}_B\ ;\\
l_C^{III} &= \sin B_C \Delta z + \left(\bar{a}\Delta\alpha + \bar{\alpha}\Delta a\right)\sin^2 B_C + \Delta a + \bar{N}_C\ ;\\
l_D^{III} &= \sin B_D \Delta z + \left(\bar{a}\Delta\alpha + \bar{\alpha}\Delta a\right)\sin^2 B_D + \Delta a + \bar{N}_D\ ;\\
l_O^{III} &= \sin B_O \Delta z + \left(\bar{a}\Delta\alpha + \bar{\alpha}\Delta a\right)\sin^2 B_O + \Delta a + \bar{N}_O\ .
\end{aligned}\right\} \quad (24)$$

If we take the values of $\bar{a}$ and $\bar{\alpha}$ from expression (4), and the values $\Delta a, \Delta\alpha, \Delta z$ of expression (16), then the free terms after simple calculations will be equal

$$\left.\begin{aligned}
l_A^{III} &= -66.8\ m\ ;\\
l_B^{III} &= -87.7\ m\ ;\\
l_C^{III} &= -106.0\ m\ ;\\
l_D^{III} &= -78.8\ m\ ;\\
l_O^{III} &= -83.8\ m\ .
\end{aligned}\right\} \quad (25)$$

Then, solving the parametric equations (23) under condition (10), we obtain

$$\left.\begin{aligned}
\Delta x &= -52 \pm 4\ m\ ;\\
\Delta y &= -164 \pm 6\ m\ .
\end{aligned}\right\} \quad (26)$$

6. Determination of parameters Δx, Δa

Case 4. Let us be given the values of Δy, Δz and $\Delta\alpha$. It is necessary to find the values of Δx and Δa. Then the parametric equations (8) will look like this

$$\left.\begin{aligned}
a_A \Delta x + e_A \Delta a + l_A^{IV} &= v_A\ ;\\
a_B \Delta x + e_B \Delta a + l_B^{IV} &= v_B\ ;\\
a_C \Delta x + e_C \Delta a + l_C^{IV} &= v_C\ ;\\
a_D \Delta x + e_D \Delta a + l_D^{IV} &= v_D\ ;\\
a_O \Delta x + e_O \Delta a + l_O^{IV} &= v_O\ ,
\end{aligned}\right\} \quad (27)$$

where

$$\left.\begin{aligned}
l_A^{IV} &= \cos B_A \sin L_A \Delta y + \sin B_A \Delta z + \bar{a} \sin^2 B_A \Delta\alpha + \bar{N}_A \ ;\\
l_B^{IV} &= \cos B_B \sin L_B \Delta y + \sin B_B \Delta z + \bar{a} \sin^2 B_B \Delta\alpha + \bar{N}_B \ ;\\
l_C^{IV} &= \cos B_C \sin L_C \Delta y + \sin B_C \Delta z + \bar{a} \sin^2 B_C \Delta\alpha + \bar{N}_C \ ;\\
l_D^{IV} &= \cos B_D \sin L_D \Delta y + \sin B_D \Delta z + \bar{a} \sin^2 B_D \Delta\alpha + \bar{N}_D \ ;\\
l_O^{IV} &= \cos B_O \sin L_O \Delta y + \sin B_O \Delta z + \bar{a} \sin^2 B_O \Delta\alpha + \bar{N}_O \ .
\end{aligned}\right\} \qquad (28)$$

Substituting in formula (28) the values of the corresponding values from expressions (4), (6), (7) and (16), we obtain

$$\left.\begin{aligned}
l_A^{IV} &= 205.6\ m ;\\
l_B^{IV} &= 201.1\ m ;\\
l_C^{IV} &= 189.8\ m ;\\
l_D^{IV} &= 197.7\ m ;\\
l_O^{IV} &= 200.1\ m .
\end{aligned}\right\} \qquad (29)$$

Taking the coefficients from Table 1 and the free terms from expression (29), we solve the system of parametric equations of corrections (27) under the condition of least squares (10). Then we will get

$$\left.\begin{aligned}
\Delta x &= -10 \pm 47\ m ;\\
\Delta a &= 205 \pm 27\ m .
\end{aligned}\right\} \qquad (30)$$

Consider the following case.

7. Determination of parameters $\varnothing y$, $\varnothing a$

Case 5. Suppose we know the values of $\varnothing x$, $\varnothing$ and $\Delta\alpha$. Using the system of equations (8), we find the quantities $\varnothing y$ and $\varnothing a$. In this case, the parametric equations of corrections will take the form

$$\left.\begin{aligned}
b_A \Delta y + e_A \Delta a + l_A^V &= v_A \ ;\\
b_B \Delta y + e_B \Delta a + l_B^V &= v_B \ ;\\
b_C \Delta y + e_C \Delta a + l_C^V &= v_C \ ;\\
b_D \Delta y + e_D \Delta a + l_D^V &= v_D \ ;\\
b_O \Delta y + e_O \Delta a + l_O^V &= v_O \ ,
\end{aligned}\right\} \qquad (31)$$

and the free members of the system of equations (31) are written as

$$\left.\begin{aligned} l_A^V &= \cos B_A \cos L_A \Delta x + \sin B_A \Delta z + \bar{a} \sin^2 B_A \Delta\alpha + \bar{N}_A \ ; \\ l_B^V &= \cos B_B \cos L_B \Delta x + \sin B_B \Delta z + \bar{a} \sin^2 B_B \Delta\alpha + \bar{N}_B \ ; \\ l_C^V &= \cos B_C \cos L_C \Delta x + \sin B_C \Delta z + \bar{a} \sin^2 B_C \Delta\alpha + \bar{N}_C \ ; \\ l_D^V &= \cos B_D \cos L_D \Delta x + \sin B_D \Delta z + \bar{a} \sin^2 B_D \Delta\alpha + \bar{N}_D \ ; \\ l_O^V &= \cos B_O \cos L_O \Delta x + \sin B_O \Delta z + \bar{a} \sin^2 B_O \Delta\alpha + \bar{N}_O \ . \end{aligned}\right\} \quad (32)$$

Taking the values Δx, Δz and $\Delta\alpha$ from expression (16), we will have

$$\left.\begin{aligned} l_A^V &= 232.9\ m ; \\ l_B^V &= 203.3\ m ; \\ l_C^V &= 192.4\ m ; \\ l_D^V &= 229.8\ m ; \\ l_O^V &= 216.4\ m . \end{aligned}\right\} \quad (33)$$

Taking into account the coefficients from Table 1 and the free terms from expression (33), we solve the system of equations (31) under the condition of least squares (10). Then we will get

$$\left.\begin{aligned} \Delta y &= -179 \pm 12\ m ; \\ \Delta a &= 276 \pm 4\ m . \end{aligned}\right\} \quad (34)$$

Consider the latter case.

8. Determination of parameters Δz, Δa

Case 6. Let the quantities $\Delta x, \Delta y, \Delta\alpha$ be known. It is necessary to calculate the values Δz, Δa. Parametric equations of corrections will then be written

$$\left.\begin{aligned} c_A \Delta z + e_A \Delta a + l_A^{VI} &= v_A \ ; \\ c_B \Delta z + e_B \Delta a + l_B^{VI} &= v_B \ ; \\ c_C \Delta z + e_C \Delta a + l_C^{VI} &= v_C \ ; \\ c_D \Delta z + e_D \Delta a + l_D^{VI} &= v_D \ ; \\ c_O \Delta z + e_O \Delta a + l_O^{VI} &= v_O \ , \end{aligned}\right\} \quad (35)$$

where

$$\left.\begin{aligned}
l_A^{VI} &= \cos B_A \cos L_A \Delta x + \cos B_A \sin L_A \Delta y + \bar{a} \sin^2 B_A \Delta\alpha + \bar{N}_A\ ;\\
l_B^{VI} &= \cos B_B \cos L_B \Delta x + \cos B_B \sin L_B \Delta y + \bar{a} \sin^2 B_B \Delta\alpha + \bar{N}_B\ ;\\
l_C^{VI} &= \cos B_C \cos L_C \Delta x + \cos B_C \sin L_C \Delta y + \bar{a} \sin^2 B_C \Delta\alpha + \bar{N}_C\ ;\\
l_D^{VI} &= \cos B_D \cos L_D \Delta x + \cos B_D \sin L_D \Delta y + \bar{a} \sin^2 B_D \Delta\alpha + \bar{N}_D\ ;\\
l_O^{VI} &= \cos B_O \cos L_O \Delta x + \cos B_O \sin L_O \Delta y + \bar{a} \sin^2 B_O \Delta\alpha + \bar{N}_O\ .
\end{aligned}\right\} \quad (36)$$

Assuming the values $\varnothing x$, $\varnothing y$ and $\Delta\alpha$ from expression (16), the free terms will accept the following results

$$\left.\begin{aligned}
l_A^{VI} &= 158.9\ m\ ;\\
l_B^{VI} &= 145.7\ m\ ;\\
l_C^{VI} &= 153.4\ m\ ;\\
l_D^{VI} &= 171.5\ m\ ;\\
l_O^{VI} &= 159.5\ m\ .
\end{aligned}\right\} \quad (37)$$

Taking into account the values of the corresponding coefficients from Table 1 and the free terms from expression (37), we solve the system of parametric equations (35) under the least squares method (10). Then we will have

$$\left.\begin{aligned}
\Delta z &= -103 \pm 94\ m\ ;\\
\Delta a &= 235 \pm 70\ m\ .
\end{aligned}\right\} \quad (38)$$

9. Conclusions

Thus, based on the results of the above a priori studies, the following can be noted.

The determination of all five parameters of the regional ellipsoid (or rather, their corrections $\Delta a, \Delta\alpha, \Delta x, \Delta y, \Delta z$), shown in the form of results (13), is due to the strong functional dependence of the parameters. This dependence (correlation) is quite well demonstrated on the values of root mean square errors, which are proportional to the obtained parameters and even exceed the latter. Moreover, according to the results (22), (30), (34) and (38), the greatest correlation occurs between the following values: a) $\varnothing a$ and $\varnothing x$; b) $\varnothing a$ and $\varnothing y$; c) $\varnothing a$ and $\varnothing z$.

Taking into account these remarks, we can conclude that the joint finding of all five parameters by the method of least squares on the territory of

Ukraine does not give us the expected good results. This is clearly seen from the a priori calculations (13) according to the geoid heights, presented in the form of a spheroidal trapezoid, which describes the territory of Ukraine. In contrast to this solution, studies to determine only three parameters $\varnothing x$, $\varnothing y$, $\varnothing$ for the given $\varnothing a$ and $\Delta\alpha$ demonstrated by the results (18), make it possible to choose a regional ellipsoid that would best represent a geoid (quasi-geoid) built on the territory of Ukraine.

Thus, we have two different tasks: a) joint determination of the parameters of the regional ellipsoid $\Delta a, \Delta\alpha, \Delta x, \Delta y, \Delta z$; b) determination of the parameters of the displacement of the regional ellipsoid $\varnothing x$, $\varnothing y$, $\varnothing$, provided that the major half-axis of the ellipsoid $\varnothing a$ and the compression of the ellipsoid $\Delta\alpha$ are given. The solution of these problems showed us very different results, which were obtained according to the same data for the same territory. This leads us to the need for additional research to obtain correct solutions to the so-called unstable or ill-defined problems [6].

References:

1. Hoffmann-Wellenhof B., Moritz H. (2007). *Fizicheskaya geodeziya* [Physical geodesy]. Moscow. (in Russian)
2. Zagrebin D. V. (1952). Teoriya regulyarizirovannogo geoida [Theory of regularized geoid]. *Trudy ITA*, no. 1, pp. 52–61.
3. Meshcheryakov G. A., TSerklevich A. L. (1987). *Gravitatsionnoye pole, figura i vnutrenneye stroyeniye Marsa* [Gravitational field, figure and internal structure of Mars]. Kyiv. (in Russian)
4. Molodenskiy M. S., Eremeyev V. F., YUrkina M. I. (1960). Metody izucheniya vneshnego gravitatsionnogo polya i figury Zemli [Methods for studying the external gravitational field and the figure of the Earth]. *Trudy TSNIIGAiK*, vol. 131, pp. 250–251.
5. Moritz H. (1983). *Sovremennaya fizicheskaya geodeziya* [Advanced physical geodesy]. Moscow. (in Russian)
6. Tikhonov A. N., Samarskiy A. A. (1966). *Uravneniya matematicheskoy fiziki* [Equations of mathematical physics]. Moscow. (in Russian)
7. Boucher C., Altamimi Z. (2001). ITRS, PZ–90 and WGS–84: Current Realizations and the Related Transformation Parameters. *Journal of Geodesy*, vol. 75, pp. 613–619.
8. Heiskanen W. and Moritz H. (1967). Physical Geodesy. W.H. Freeman and Company. San Francisco, California.
9. Lelgemann D. (1973). Spherical Approximation and the Combination of Gravimetric and Satellite Data. *Boll. Geold. Sci. Affini*, vol. 32, pp. 241–250.