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OPTIMIZATION OF CALCULATION AND SELECTION OF HEAT EXCHANGE EQUIPMENT IN THE USE OF NANOFLUID HEAT CARRIERS

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INTRODUCTION

In modern chemical, food, pharmaceutical, technologies an important place is occupied by heat exchange processes, which account for the dominant share of all energy consumption of enterprises. The efficiency of these processes largely depends on the correct choice of heat exchange equipment. Very often the heat exchange equipment is selected for maximum power and then throughout the period of its operation works with the overuse of thermal energy, which places a heavy burden on the work of the whole enterprise and entire industries as ahole. Therefore, the choice of high-efficiency thermal equipment must also be accompanied by aorrect preliminary calculation of its heat output.

To date, an important part of the energy saving of heat exchange equipment is the use of modern nanofluids coolants. However, the choice and calculation of heat exchangers with nanofluids encounters problems of correct theoretical calculation, because numerical empirical equations that describe the processes of heat exchange with classical coolants do not accurately reflect the process using nanofluids. Additional costly experiments are needed to make corrections to existing classical equations that are losing their universality.

1. The most important fragments of classical scientific research, on which our approach to the existing problem is based

1. The law of Italian scientist Torricelli E. asserts (1643) in fluid dynamics that the velocity V of aluid that flows from a reservoir under the action of gravity is proportional to the square root $V = \sqrt{2gh}$ ($g = 9.81$ – Earth)[1].

[1] Thermal Engineering / What is Torricelli's Law – Definition. URL: https:// www.thermal-engineering.org/what-is-torricellis-law-definition (date of the appeal 5.05.2022)

2. In 1848, the Irish mathematician and physicist George Gabriel Stokes derived differential equations that describe the law of variation of aluid vortex over a certain period of time. In 1851, G. Stokes obtained a formula for the resistance force of**,** which acts on aphere when it uniformly moves in aiquid medium with unlimited viscosity $F = 3\pi dV\mu$ [2].

3. In 1869, the French scientist J. Boussinesq suggested that the profile (shape and speed) of aater jet in Savard's experiments is the result of the combined effect of surface and gravitational forces on its particles[3]. He ishe founder of the term and model of turbulent viscosity of aiquid (1877). The Boussinesq's model assumes the turbulent motion of aiquid medium as the motion of Newtonian "turbulent fluid"[4].

4. British engineer – physicist Reynolds O. experimentally established (1876–1883) of the transitional number of the laminar regime of fluid motion in aylindrical pipe to turbulent mode. For the turbulent regime of motion derived differential equations of averaged fluid motion, which contain additives of irregular pulsations. He formulated criteria for the similarity of two different flows of aiscous fluid[5].

5. The German scientist V. Nusselt (1910) for the first time experimentally demonstrated the influence of turbulent gas flow on heat transfer. Further research by many scientists led to aeneralization of empirical numerical equations, which are used to date to find the heat transfer coefficients in turbulent and laminar modes of motion of coolants in pipes or channels. These equations are based on the experimental work of many independent authors and carry powerful experimental material[6].

6. The most important result of the German scientist Prandtl (1928) is the universal logarithmic law for the profile of fluid velocities in flows (channels, pipes, boundary layers)[7].

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[2] Stokes G. G.. On the theories of internal friction of fluids in motion, and of the equilibrium and motion of elastic solids. *Transactions of the Cambridge Philosophical Society*. 1845. № 8. P. 317.

[3] Boussinesq J. Théorie des expériences de Savart sur la forme que prend une veine liquide après s'être choquée contre un plan circulaire. 1869. CR 69 45–48. P. 130.

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[7] Prandtl L. Über die ausgebildete Turbulenz. *ZAMM*.1925. P. 142.

7. In 1931, the British experimenters A. Fage and V. M. Falkner experimentally proveds that the laws of temperature and velocity distribution in turbulent and laminar flows of liquids and gases are not identical[8].

8. In 1944, a German scientist H. Reichardt published an article on the distribution of temperatures and velocities in conditions of free turbulence and showed that the maximum and minimum temperatures and velocities in aurbulent flow of liquids and gases are related by relation (1.1). In formula (1.1), the recommended average ratio of impulse coefficients A_τ and heat transfer A_q in the middle of the flow is about 0.7[9].

$$\frac{T_{\min}}{T_{\max}} = \left(\frac{V_{\min}}{V_{\max}} \right)^{A_\tau / A_q} \tag{1.1}$$

where $T_{\min}$ − minimal temperature, °C; $T_{\max}$ − maximal temperature, °C; $V_{\min}$ − minimal velocity, m.s^{-1}; $V_{\max}$ −maximal velocity, m.s^{-1}; A_τ − impulse coefficients, kg.m^{-1}.s^{-1};

A_q − heat transfer coefficients, kg.m^{-1}.s^{-1}.

It should be emphasized that H. Reichardt did not rely on tony theory of turbulence, and he drew conclusions only on the basis of his experimental data. After these studies, many theoretical and experimental works were published on the laws of the distribution of velosities and temperatures in liquid and gas flows, which are compactly collected in the H. Schlichting monograph and which confirm dependence (1.1)[10].

9. In 2011, Italian scientists Quadrio M., Ricco P. in the field of fluid mechanics received differential equations that imply complete control of the laminar zone of the fluid flow over the turbulent zone, that is, in the near-wall region of the channel through which the fluid flows, the laminary border layer (LBL) fully controls its seemingly main turbulent part[11].

[8] Taylor G. I. The transport of velocity and heat through fluids in turbulent motion. Note on Experiments on the Temperature and Velocity in the Wake of Heated Cylindrical Obstacle (by A. Fage and V. M. Falkner) *Proc. Soc*. 1932. № A 135. P. 905.

[9] Reichardt H. Impuls – und Wärmeaustausch bei freier Turbulenz. *ZAMM.* 1944. № 24. P. 271. DOI: https://doi.org/10.1002/zamm.19440240515

[10] Шлихтинг Г. Теория пограничного слоя М. 1974. С. 672.

[11] Quadrio M., Ricco P. The laminar generalized Stokes layer and turbulent drag reduction. *Journal of Fluid Mechanics.* 2011. № 667, P. 156. http://dx.doi.org/10.1017/S0022112010004398

1.1. The derivation of the classical numerical equation for calculating heat transfer coefficients by the method of dimensional analysis

Convection coefficient h [W.m^{-2}.K^{-1}] during turbulent movement of liquid or gaseous heat carrier in pipes or channels will depend on the following main factors:

– the speed of movement of the coolant V [m.s^{-1}];
– density of the liquid ρ [kg.m^{-3}];
– specific heat capacity of the liquid heat carrier C_P [J.kg^{-1} K^{-1}];
– coefficient of dynamic viscosity of the liquid μ [Pa s];
– coefficient of thermal conductivity k [W.m^{-1} K^{-1}];
– main linear dimension (pipe diameter or equivalent channel diameter) D [m].

We express these dimensions in terms of basic (kg, m, s, K):

h – [W.m^{-2} K^{-1} = N.m.s^{-1}.m^{-2} K^{-1}= kg s^{-3} K^{-1}]; V – [m.s^{-1}]; ρ – [kg.m^{-3}];

C_P – [J.kg^{-1} K^{-1}= N.m.kg^{-1} K^{-1}= kg.m^2.s^{-2}.kg^{-1} K^{-1} = m^2.s^{-2}.K^{-1}];

k – [W.m^{-1} K^{-1} = N.m.s^{-1}.m^{-1}.K^{-1} = kg.m.s^{-3}.K^{-1}];

μ – [Pa s = N.s.m^{-2} = kg.m.s^{-1}.m^{-2} = kg.s^{-1}.m^{-1}]; D – [m];

To simplify calculations, we introduce the value of the mass flow rate of the liquid heat carrier, that is, the product of the speed and the flow density

$V.\rho$ – [m.s^{-1}.kg.m^{-3} = kg.m^{-2}s^{-1}].

Since any functional dependence can be represented in the form of aower function, such a power-law equation can be written (1.2).

$$h = B \cdot \left((V \cdot \rho)^X \cdot C_P^{\,Y} \cdot k^Z \cdot \mu^M \cdot D^N \right); \tag{1.2}$$

By expressing the unknown powers in terms of the powers of X and Y, we arrive at the well-known classical numerical equation (1.3):

$$h = B \cdot \left((V \cdot \rho)^X \cdot C_P^{\,Y} \cdot k^{(1-Y)} \cdot \mu^{(Y-X)} \cdot D^{(X-1)} \right); \tag{1.3}$$

or

$$\left(\frac{h \cdot D}{k} \right)^1 = B \cdot \left(\frac{V \cdot D \cdot \rho}{\mu} \right)^X \left(\frac{C_p \cdot \mu}{k} \right)^Y;$$

or

$$Nu = B(Re)^X.(Pr)^Y, \tag{1.4}$$

where in the formulas (1.2) − (1.4) h − heat transfer coefficient, W.m^{-2}.K^{-1}; B − dimensionless constant; V − velocity, m.s^{-1}; ρ − density, kg.m$^{-3;}$ C_p − specific heat capacity, J.kg^{-1}. K^{-1}; k − thermal conductivity, W. m^{-1}. K^{-1}; μ − coefficient viscosity of coolant, kg.m^{-1}.s^{-1}; D − diameter, m; X, Y, Z, M, N − unknown exponents; Nu −Nusselt number; Re − Reynolds number; Pr − Prandtl number.

Equation (1.4) is alassic numerical equation that calculates heat transfer coefficients for different types of heat exchangers and different liquid and gas heat transfer fluids. Constant B and indicators X, Y are found experimentally and are in noay interconnected. We present two well-known classical equations of turbulent motion of aoolant in pipes and spaces of shell-and-tube (a, b) and plate (c) heat exchangers (1.5)[12].

$$\text{a) } Nu = 0.023 Re^{0.8} Pr^{0.43}; \quad \text{b) } Nu = 0.24 Re^{0.6} Pr^{0.36}; \quad \text{c) } Nu = 0.135 Re^{0.73} Pr^{0.33}. \tag{1.5}$$

Constants B and indicators X, Y are empirical values. With the use of liquid coolants with different nanoparticles, ie nanofluids, for the possibility of analytical calculation of heat exchangers, constants B and exponents X, Y in equations of type (1.4) completely change and can not be calculated analytically. They do not become universal. It should also be noted that equations of type (1.4), such as the Nusselt equation[13], were obtained on the basis of experimental data and not by aniversal method of dimensional analysis.

These and other equations can be found, for example, in numerous reviews and dissertations for the calculation of shell-and-tube and plate heat exchangers are presented in[14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27,28, 29, 30, 31, 32, 33, 34, 35, 36].

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To date, these equations are widely used to calculate heat exchange equipment, as the heat carriers were “classic” liquids (water, various solutions) and so on. However, with the use of nanofluids, the exponents *X* and *Y*, and the constant *B* are also determined by costly experiments, which makes the calculation of heat exchangers difficult and time consuming. The structure of equations of type (1.5) does not change, but these equations, as aule, acquire a rather complex form and are problematic for fast engineering calculations. Some numerical empirical equations proposed by aumber of independent authors, compiled on the basis of classical type (1.5) to determine the convection coefficients in shell-and-tube heat exchangers using nanofluids with TiO_2 nanoparticles in turbulent flow are presented in table 1.

As we can see, these equations are similar in structure to equations of type (1.5), where there are classical similarities between Nusselt, Reynolds, and Prandtl. In fact, the authors hope to improve the classical empirical equations with various corrections of exponents *X*, *Y*, constant *B*, as well as take into account the fractionality and concentration of nanoparticles in the nanofluid (Table 1).

Our previous studies indicate that the classical numerical equations presented in table 1, as well as others, are correct, but they do not give a sufficiently accurate result when using nanofluids, especially at temperatures above 50 °C[37]. The results presented by usn show that our computer calculation of heat transfer coefficients by equations (1.6–1.11) differs from the experimental results presented in[38] by (5–30)%. Moreover,

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the higher the temperature (50 °C), the greater the difference between the experiment and computer calculation.

Table 1

Convective heat transfer correlations of TiO_2 nanofluids

Numerical equation	Relevant information
1	2
$Nu = 0.021 \cdot \mathrm{Re}^{0.8} \cdot \mathrm{Pr}^{0.5}$ (1.6)[39]	Experimental study Turbulent flow Al_2O_3/water nanofluids; TiO_2/water nanofluids $(0.0 \le \phi \le 3.0)$; $(10^4 \le \mathrm{Re} \le 10^5)$
$Nu = 0.067 \cdot \mathrm{Re}^{0.71} \cdot \mathrm{Pr}^{0.35} + 0.00050\,\mathrm{Re}$ (1.7)[40]	Experimental study Turbulent flow TiO_2/water nanofluids: $(0.2 \le \phi \le 0.25)$; $(5 \cdot 10^3 \le \mathrm{Re} \le 3 \cdot 10^4)$;
$Nu = 0.074 \cdot \mathrm{Re}^{0.707}\,\mathrm{Pr}^{0.385} \cdot \varphi^{0.074}$ (1.8)[41]	Experimental study Turbulent flow TiO_2/water nanofluids $(0.2 \le \phi \le 2.0)$; $(3 \cdot 10^3 \le \mathrm{Re} \le 1.8 \cdot 10^4)$;
$Nu = \frac{(0.125f)(\mathrm{Re}-1000)\mathrm{Pr}}{1+12.7(0.125f)^{0.5}(\mathrm{Pr}^{2/3}-1)}$ (1.9)[42]	Numerical study Fully developed turbulent flow: $(3 \cdot 10^3 \le \mathrm{Re} \le 5 \cdot 10^6)$; $(0.5 \le \mathrm{Pr} \le 2000)$;
$Nu = \frac{(0.125f)\mathrm{Re} \cdot \mathrm{Pr}}{1.07+12.7(0.125f)^{0.5}(\mathrm{Pr}^{2/3}-1)}$ (1.10)[43]	Numerical study Fully developed turbulent flow: $(5 \cdot 10^5 \le \mathrm{Re} \le 5 \cdot 10^6)$; $(0.5 \le \mathrm{Pr} \le 2000)$;
$Nu_{nf} = 0.0059(1+7.6286\varphi^{0.6886} Pe_d^{0.001}) \times \mathrm{Re}_{nf}^{0.9238}\,\mathrm{Pr}_{nf}^{0.4}$ (1.11)[44]	ϕ – volume concentration (%) $\phi = \frac{\varphi}{100}$

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Because equations of the form (1.6–1.11) for the application of nanolluids do not give accurate results, and sometimes acquire a very complex form, they become unsuitable for operational engineering calculations. To calculate the heat exchanger using a new nanofluid, you must first conduct a series of expensive experiments, derive an empirical numerical equation and only then proceed to the calculations.

1.2. Molecular dynamics method (MDM) in the study of the motion of nanoparticles in aiquid coolant

MDM is annalytical, highly efficient, and accurate method of studying the motion of nanoparticles in aiquid medium.

Nanoparticles, due to their small size, collide with the molecules of the base fluid, receive from them an equivalent momentum. Therefore, the velocity of the nanoparticle is proportional to the velocity of the molecules and the motion of the nanoparticle in the liquid medium is accompanied by the formation of microfluctuations that interact with it. Also, in microfluctuations, where there is aocal change in the concentrations of liquid molecules, the linear velocity of the nanoparticle relaxes and there is anncrease in the density of the liquid medium[45, 46].

The movement of the ball in the liquid at the macro level was described by G. G. Stokes concluded it inhe well-known formula $F = 3\pi dV\mu$[47]. In this case, the conclusions of athe nanolevel correlate with the known conclusions of G. G. Stokes at the macro level.

In[48], MDM calculated that the resistance of aiquid medium to moving nanoparticles is initially 2–3 times greater than the Stokes force, and in the process of further motion is approximately equal to the Stokes force.

After relaxation, the resistance force is completely determined by the hydrostatic pressure, and its vector projection is greater than zero at $\theta < 90^0$

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and less than zero for $\theta > 90^0$. This force is anisotropic and decreases with increasing angle θ. This work does not analyze what is the nature of this force, how does it arise?

Based on our studies[49, 50, 51, 52], the resistance force of the base fluid of nanoparticle motion is the surface tension force at the interface between solid and liquid phases, which depends on hydrophilicity (cosine of contact angle, $\cos 0^0 = 1$ and $\cos 90^0 = 0$). Surface forces have a molecular nature and this conclusion is based.

As an example, which is normalized to the average concentration of the distribution of the medium's molecules near the nanoparticle at aixed point in time $t = 2\tau$ after the start of motion (Figure 1).

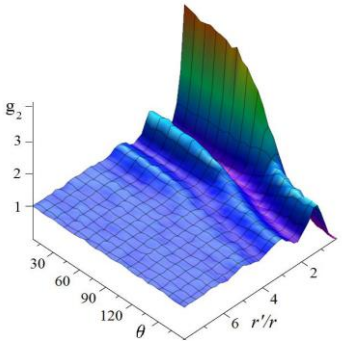

Fig. 1. The radial function of carrier molecules near a moving nanoparticle, M/m = 100, R/r = 3, ε_f = 0.088[53]

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[50] Bilonoga Y., Maksysko O. Modeling the interaction of coolant flows at the liquid-solid boundary with allowance for the laminar boundary layer. *International Journal of Heat and Technology*. 2017. № 35(3). P. 679. DOI: http://dx.doi.org/10.18280/ijht.350329

[51] Bilonoga Y., Maksysko O. Specific features of heat exchangers calculation considering the laminar boundary layer, the transitional and turbulent thermal conductivity of heat carriers. *International Journal of Heat and Technology*. 2018. № 36(1). P. 14. DOI: https://doi.org/10.18280/ijht.360102

[52] Bilonoga Y., Maksysko O. The laws of distribution of the values of turbulent thermo-physical characteristics in the volume of the flows of heat carriers taking into account the surface forces. *International Journal of Heat and Technology*. 2019. № 36(1). P. 7. DOI: https://doi.org/10.18280/ijht.370101

[53] Рудяк В. Я., Белкин А. А. Моделирование коэффициентов переноса наножидкостей. *Наносистемы: физика, химия, математика*. 2010. Т. 1, № 1. С. 162.

Modeling of nanoparticles and molecules implies their consideration as elastic solid spheres of different mass and radius. The following notation is used in work:

– r'- is the minimum distance between the surfaces of aolecule and a nanoparticle, θ is the angle between the axis connecting their centers and the velocity vector of the nanoparticle;

– r – the average free path of molecules. When the nanoparticle is not balanced, the function $g_2(r/r, , t)$ is isotropic and then becomes anisotropic.

The first maximum in the frontal region (at $\theta \leq 40^{)}$ increases by 30–50 % compared to the equilibrium value and is 3–5 times higher than the average concentration value of molekules. In aiquid, the relaxation process nanoparticle velocity is accompanied by anncrease in density in its frontal region (Figure 1)[54].

2. Problem formulation

The overwhelming majority of authors state a fairly significant increase in the overall heat transfer coefficients in heat exchange equipment using nanoparticles of various concentrations and dispersions. However, they speak of aignificant difference between the calculated and experimental data on this issue and a possible more complex mechanism of heat exchange in nanofluids. Based on this, we set ourselves the following tasks in this work:

– choose from literature sources and analyze the numerical equations by which convection coefficients can be calculated, that is, by which the optimal heat exchange equipment can be calculated and selected when using nanofluids, for example, with TiO_2 nanoparticles;

– show the consistency of the experimental data of aumber of authors on increasing heat transfer coefficients when using nanofluids in heat exchangers with calculated data based on the use of classical numerical equations using the Nusselt Reynolds and Prandtl numbers;

– show the possibility of calculating shell-and-tube, plate and other types of heat exchangers taking into account the turbulent viscosity and thermal conductivity, as well as the specific heat capacity and the surface tension coefficient of the heat-transfer fluid with nanoparticles;

– to show the compliance of the new method of calculation of heat exchange equipment based on the use of values of turbulent viscosity and thermal conductivity of nanofluid coolants, as well as new similarities

[54] Rudyak V., Belkin A., Tomilina E. The force acting on aanoparticle in aiquid. *Technical Physics Letters.* 2008. № 34(1). P. 78. DOI: https://doi.org/10.1134/S1063785008010239

numbers *Bl* and $Bl_{turb.}$ with experimental data of aumber of independent authors.

– present examples of calculation of shell-and-tube and plate heat exchangers using nanofluids with TiO_2 particles.

3. Modeling of nanofluid heaters and analytical rapid calculation of heat exchangers

3.1. Modeling of motion of solid nanoparticles in turbulent coolant flow taking into account similarity theory.

The results of aeview by S. Mishra et al. in[55] indicate that nanofluids as thermal agents require new models and approaches, since the classical ones do not sufficiently explain the increase in thermal conductivity in these systems.

H. Jiang et al., showed that the presence of annterfacial layer is the main factor in increasing the thermal conductivity of nanofluids. This model takes into account some effects – the volume fraction, thickness and thermal conductivity of the interfacial layer, as well as the average size of nanoparticles. According to this model, this nanolayer behaves as ahermal bridge between the nanoparticle and the base liquid[56].

As an example, we can see a metal nanoparticle, which randomly moves in heat transfer in aurbulent (T) mode (Fig. 2).

In front of the nanoparticle there is anxcess pressure P2, and behind the pressure decreases P1[57, 58, 59]. The surface force field forms a subsurface layer of liquid molecules LBL around the nanoparticle. In the solid state, the surface energy of the surfaces of metal nanoparticles or their oxides is $\sigma \approx (0.5–3)$ J/m^2 [60, 61, 62], and the surface energy of base liquids is

[55] Mishra S., Nayak M. K., Misra A. Thermal conductivity of nanofluids – A comprehensive review. *International Journal of Thermofluid Science and Technology. 2020.* № 7(3). P. 37. DOI: ttps://doi.org/10.36963/IJTST.2020070301

[56] Jiang H., Xu Q., Huang C., Shi L. The role of interfacial nanolayer in the enhanced thermal conductivity of carbon nanotube-based nanofluids, *Appl. Phys.* 2015. № A, 118(1). P. 203.

[57] Рудяк В. Я., Белкин А. А. Моделирование коэффициентов переноса наножидкостей. *Наносистемы: физика, химия, математика*. 2010. Т. 1, № 1. С. 162.

[58] Рудяк В. Я., Белкин А. А., Краснолуцкий С. Л. К статистической теории процессов переноса наночастиц в газах и жидкостях. *Теплофизика и аэромеханика.* 2005. № 4. С. 162.

[59] Rudyak V., Belkin A., Tomilina E. The force acting on aanoparticle in aiquid. *Technical Physics Letters.* 2008. № 34(1). P. 78. DOI: https://doi.org/10.1134/S1063785008010239

[60] Pokhmurskii V., Sirak Y., Bilonoga Y. Influence of the surface energy and of the energy of the bond of the contacting metals on the fretting fatigue life of the joints of machine parts. Soviet Materials Science. 1984. № 20(4). P. 358. DOI: https://doi.org/10.1007/BF01199367

σ ≈ (20–72). 10^{-3} J/m^2. The surface energy of the metal surface is (25–40) times greater than the surface energy of liquids. MDM shows that the dynamics of nanoparticles is similar to the movement of aacrosphere in aiquid. The average velocity of the elementary layers of the liquid relative to the nanoparticles is very low. LBL seems to "stick" around the nanoparticles in aurbulent liquid medium (Fig. 2). The average thickness of LBL δ is hypothetically several orders of magnitude larger than the nanoparticle itself. This layer ends where surface forces no longer act.

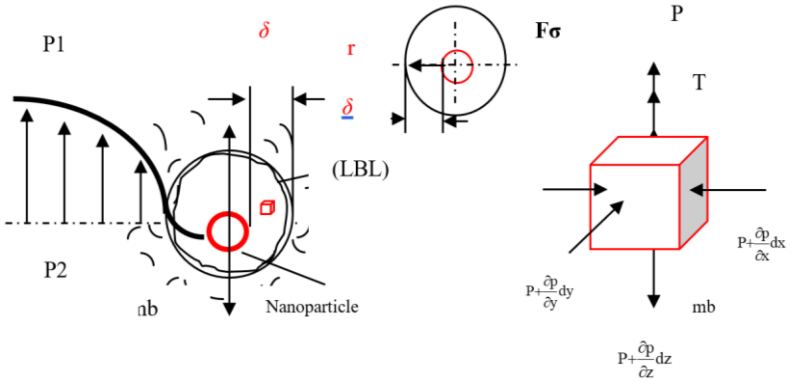

Fig. 2. Scheme of LBL formation around a mobile nanoparticle in aiquid coolant with a T – regime

The elementary volume of fluid in LBL is affected by:

1. Surface tension of the liquid force: $F_\sigma = \pi\,(dx)\cdot\sigma\cdot\cos\theta$ (3.1)
2. The gravity force: $mg = \rho\cdot g(dxdydz)$ (3.2)
3. The friction force: $T = m\frac{d^2V_z}{dx^2}(dxdydz)$ (3.3)
4. The inertia force: $F_i = mb = \rho\cdot\frac{dV_z}{d\tau}(dxdydz)$ (3.4)
5. The pressure force: $P = -\frac{dp}{dz}(dxdydz)$ (3.5)

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[61] Bilonoga Y., Pokhmurs'kii V. A Connection between the fretting-fatigue endurance of steels and the surface energy of the abradant metal. *Soviet Materials Science.* 1991. № 26(6). P. 629. DOI: DOI: https://doi.org/10.1007/BF00723647

[62] Олешко В., Пиговкин И. Оперативное определение поверхностной энергии металлических деталей авиационной техники. *Интернет-журнал «НАУКОВЕДЕНИЕ». 2006.* № 8(3). С. 12. DOI: http://naukovedenie.ru/PDF/131EVN316

According to the De Alambert principle, the algebraic sum of forces acting on the elementary volume of aluid in LBL is 0. We obtain equality (3.6), shortening by *(dxdydz):*

$$\pi\cdot(dx)\cdot\sigma\cdot\cos\theta\ /(dxdydz)+\rho\cdot g-\frac{dP}{dz}+\mu\frac{d^2V_z}{dx^2}=\frac{\rho\cdot dV_z}{d\tau} \quad (3.6)$$

where in the formulas (3.1) − (3.6) F_σ − surface tension force of the liquid, N; σ − surface tension of the liquid, N.m^{-1}; cosθ − cosine of wetting angle; ρ − fluid density, kg.m^{-3}; T − friction force, N; μ − dynamic viscosity coefficient, N.s.m^{-2}; V_z − average fluid velocity in LBL, m.s^{-1}; F_i − force of inertia, N; b − acceleration of inertia force, m.s^{-2}; P − force of pressure, N; (*dx, dy, dz*) – parameters of elementary volume on the X, Y, Z axes.

According to the similarity theory, the parameters (*dx, dy, dz*) are replaced by *l*. $\frac{dV}{dl}\approx\frac{V}{l}$; $\frac{dV}{d\tau}\approx\frac{V}{\tau}$; Divide the terms of equation (3.6) by the value and obtain similarity numbers in LBL (3.7–3.10).

In these similarity numbers we substitute approximate numerical values for water under normal conditions. Other values are selected as:

- average diameter of nanoparticles ($d\approx 20.10^{-9}$ m);
- density of nanoparticles ($\rho_{np}=2100$ kg.m^{-3});
- water density ($\rho=1000$ kg.m^{-3});
- surface tension of water ($\sigma=0{,}0725$ N/m);
- average thickness of LBL ($\delta\approx 0.4.10^{-6}$ m);
- hydrophilicity of the nanoparticle surface ($\cos\theta\approx 0.8$).
- Re ≈ 100000, which corresponds to the developed T mode.

The speed of movement of water layers in LBL is anrder of magnitude less than the linear speed. We believe that the nanoparticle moves uniformly accelerated,

$$l=V_z.\tau;\ \ V_z\approx\frac{V}{N}\approx\frac{V}{10}\approx\frac{0.1}{10}\approx 0.01\ m.s^{-1};N=\frac{\rho\cdot V_z\cdot l}{\mu}\approx 10.47-11.5^{63}.$$

1. Euler number (*Eu*) in LBL:

$$Eu=\frac{P\cdot\tau}{\rho\cdot V\cdot l}=\frac{\Delta p}{\rho\cdot V_z^2}=\frac{\frac{\pi d^3/6\cdot\rho}{\pi d^2/2}\cdot\mathrm{Re}}{\rho\cdot V_z^2}=\frac{20\cdot10^{-9}\cdot2100\cdot100000}{3\cdot1000\cdot0.01^2}\approx 14; \quad (3.7)$$

[63] Bilonoga Y., Maksysko O. Modeling the interaction of coolant flows at the liquid-solid boundary with allowance for the laminar boundary layer. *International Journal of Heat and Technology*. 2017. № 35(3). P. 679. DOI: http://dx.doi.org/10.18280/ijht.350329

The pressure difference Δp in the Euler number (3.7) in front of and behind the nanoparticle[64, 65, 66] is created by the force of inertia. Instead of the pressure difference, we replace the force of gravity acting on the area of the hemisphere in front of the nanoparticle. Turbulence of the coolant is created not due to the movement of the nanoparticle under the action of non gravity, but due to the movement of the coolant in the developed turbulent mode ($Re \geq 20000 – 100000$ and more). Therefore, we multiply the Euler number by $Re \approx 100000$.

2. Surface number (Su) in LBL:

$$Su = \frac{1}{N} \cdot \frac{2 \cdot \pi \cdot \sigma \cdot \cos\theta}{\mu \cdot V_z} = \frac{1}{10} \cdot \frac{2 \cdot 3.14 \cdot 72.5 \cdot 10^{-3} \cdot 0.8}{1 \cdot 10^{-3} \cdot 0.01} \approx 3642; \qquad (3.8)$$

3. Reynolds number in (Re) in LBL:

$$\frac{1}{\mathrm{Re}} = \frac{\mu \cdot V \cdot \tau}{l^2 \cdot \rho \cdot V} = \frac{\mu}{\delta \cdot \rho \cdot V_z} = \frac{1 \cdot 10^{-3}}{0.4 \cdot 10^{-6} \cdot 1000 \cdot 0.01} \approx 250; \mathrm{Re} \approx 0.004; \qquad (3.9)$$

4. Froude number (Fr) in LBL:

$$Fr = \frac{\rho \cdot g \cdot \tau}{\rho \cdot V} = \frac{g \cdot \tau \cdot \delta}{V \cdot l} = \frac{g \cdot \delta}{V_z^2} = \frac{9.8 \cdot 0.4 \cdot 10^{-6}}{0.01^2} \approx 0.0392; \qquad (3.10)$$

where in the formulas (3.7)–(3.10) P − force of pressure, N; Δp – the pressure difference in front of and behind the particle, Pa; N − modified Reynolds number LBL; σ − surface tension coefficient of coolant, $N.m^{-1}$; μ − coefficient viscosity of coolant, $kg.m^{-1}.\ s^{-1}$; $cos\theta$ − cosine of the contact angle; τ − time, ρ − fluid density, $kg.m^{-3}$; V − velocity, $m.s^{-1}$; V_z − average fluid velocity in LBL, $m.s^{-1}$; d − diameter, m.

The numbers responsible for the forces of friction and gravity in LBL are several orders of magnitude smaller than the forces of surface tension, which allows them to beeglected except for surface and inertial forces, which create

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[64] Рудяк В. Я., Белкин А. А Моделирование коэффициентов переноса наножидкостей. *Наносистемы: физика, химия, математика*. 2010. Т. 1, № 1. С. 162.

[65] Рудяк В. Я., Белкин А. А., Краснолуцкий С. Л. К статистической теории процессов переноса наночастиц в газах и жидкостях. *Теплофизика и аэромеханика*. 2005. № 4. С. 162.

[66] Rudyak V., Belkin A., Tomilina E. The force acting on aanoparticle in aiquid. *Technical Physics Letters.* 2008. № 34(1). P. 78. DOI: https://doi.org/10.1134/S1063785008010239

excess pressure at the front to thin behind the particle. The force of inertia is one of Euler's numbers in the LBL.

We equate the forces of pressure and surface tension in LBL, acting on the elementary ring of fluid (Fig. 2).

1. The area of the elementary ring in LBL:

$$dS = 2\pi \cdot r \cdot dr \cdot \tag{3.11}$$

2. Pressure force acting on the elementary ring in LBL:

$$dP = p \cdot 2\pi \cdot r \cdot dr \tag{3.12}$$

3. The force of surface tension of aiquid acting in the LBL:

$$dF_{\tau} = 2\pi \cdot \sigma \cdot cos\theta \cdot dr. \tag{3.13}$$

We equate the forces of pressure and surface tension in LBL, acting on the elementary ring of fluid (Fig. 2). (3.14):

$$dP = \Delta p \, 2\pi \int_0^{\delta} r dr = \Delta p \pi \delta^2 \quad dF_{\Pi} = 2\pi\sigma\cos\theta \int_{R_0}^{R} dr$$

$$\Delta p \pi \delta^2 = \pi\sigma cos\theta d \tag{3.14}$$

The pressure difference Δp is created by aorce of inertia equal to mb. Instead of the pressure difference, we substitute the force of inertia acting on the area of half of the sphere in front of the nanoparticle (3.15). Instead of the difference in pressure, we substitute the force of inertia that acts on the hemisphere area in front of the nanoparticle (3.15) and by shortening the values of *d*nd *π*, we obtain the mean thickness of the LBL (3.16):

$$\frac{\pi d^3/6 \cdot \rho \cdot b \cdot \mathrm{Re}}{\pi d^2/2} \cdot \pi \cdot \delta^2 = \pi \cdot \sigma \cos\theta \cdot d \tag{3.15}$$

$$\delta = 1.732 \sqrt{\frac{\sigma \cdot \cos\theta}{b \cdot \rho \cdot \mathrm{Re}}}; \tag{3.16}$$

where in the formulas (3.11)−(3.16) δ − average thickness of the LBL, m; *d* − diameter, m; *Re* − Reynolds number; ρ − fluid density, kg.m^{-3}; μ − dynamic viscosity coefficient, kg.m^{-1}s^{-1}; σ − surface tension coefficient of coolant, N.m^{-1}; *cos*θ − cosine of the contact angle.

Formula (3.16), as well as the numbers in LBL (3.7–3.10), taking into account the works[67, 68, 69], correlate with the following statements of aumber of independent authors:

1. Surface forces in LBL prevail and they are several orders of magnitude larger than the others;

2. The increase in the heat transfer coefficient of the nanofluid is proportional to the increase in the average diameter of the nanoparticles[70].

3. If the base fluid is glycols, then this increase for glycol solutions is much more noticeable. The surface energy of glycols is about 50.10^{-3} J/m^2, and water – about

70.10^{-3} J/m^2. Glycols (formula 3.16) have (20–30)% lower surface energy compared to water.

4. With increasing diameter of nanoparticles and, accordingly, their mass, the acceleration of inertia increases with decreasing LBL thickness (formula 3.16), which increases the heat transfer coefficient, (Laminar (L) − mode slows down heat transfer compared to T – mode).

5. The addition of surfactants to nanofluids reduces their surface tension and reduces the average thickness of LBL.

6. Higher density of nanoparticles (CuO – 6510 kg.m^{-3}) than (ZnO – 5606 kg.m^{-3}) (formula 3.16) reduces the average thickness of LBL and this increases the overall thermal conductivity of the nanofluid[71].

In milk, the average diameter of fat globules is about $d \approx (2.5–5).10^{-6}$ m. The surface energy of milk fat is 35.10^{-3} J/m^2, and milk plasma is 50.10^{-3} J/m^2, which is approximately 1.4 times more. In aetal nanoparticle in aiquid, the opposite is true. The surface energy of aetal nanoparticle

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[67] Рудяк В. Я., Белкин А. А., Моделирование коэффициентов переноса наножидкостей. *Наносистемы: физика, химия, математика*. 2010. Т. 1, № 1. С. 162.

[68] Рудяк В. Я., Белкин А. А., Краснолуцкий С. Л. К статистической теории процессов переноса наночастиц в газах и жидкостях. *Теплофизика и аэромеханика*. 2005. № 4. С. 535.

[69] Rudyak V., Belkin A., Tomilina E. The force acting on aanoparticle in aiquid. *Technical Physics Letters.* 2008. № 34 (1). P. 78. DOI: https://doi.org/10.1134/S1063785008010239)

[70] Timofeeva E. V., Wenhua Yu., France D. M., Singh D., Routbort J. L. Base fluid and temperature effects on the heat transfer characteristics of SiC in ethylene-glycol/H_2O and H_2O nanofluids. *J. Appl. Phys.* 2011. № 109. 014914. DOI: https://doi.org/10.1063/1.3524274

[71] Bayareh M., Nourbakhsh A. Numerical Simulation and Analysis of Heat Transfer for Different Geometries of Corrugated Tubes in aouble Pipe Heat Exchanger. *Journal of Thermal Engineering* 2019. № 5(4). P. 299. DOI: 10.18186/thermal.581775

is approximately (0.5–3) J/m^2 [72, 73, 74], and the surface energy of liquids is approximately (20–70).10^{-3} J/m^2, which is approximately 40 times less. The surface energy of milk plasma suppresses the surface energy of fat with the formation of fat globules with an average diameter $d \approx 3.10^{-6}$ m, then the surface of the metal nanoparticle creates a field of action of surface forces at aistance $\delta \approx 100.10^{-9}$ m. T – mode of motion of nanoparticles with an average diameter $d \approx 20.10^{-9}$ m. creates LBL $\delta \approx 100.10^{-9}$ m, That is, the average length of the field of action of surface forces is approximately (5–10) times greater than the average diameter of the nanoparticle.

Why are LBL thermal supports not taken into account in such systems? This is similar to the situation with a floating fish in anquarium, when we watch it closely, but do not notice the aquarium itself. The average thickness of LBL and its other properties play a key role in changing the thermophysical properties of nanofluids. Chaotically moving metal nanoparticle promotes heat transfer to the base fluid, and LBL around it creates additional thermal resistance. The change in the average thickness of the LBL when changing the average diameter of nanoparticles, changes in their density, their surface energy, as well as the surface energy of the base fluid controls the heat transfer coefficients of nanofluids.

The most important mistake, in our opinion, in studying the properties of nanofluid coolants is that this system is considered in static conditions, which use classical, molecular coefficients of viscosity and thermal conductivity and do not take into account the surface forces that are dominant.

3.2. Derivation of the molecular number *Bl* byhe method of dimensional analysis

The process of heat transfer will be considered on the example of heating cold milk with hot water in ahell-and-tube heat exchanger. We will assume that in all cases of heat transfer in separate zones, heat is transferred by heat conduction in the LBL zone, transitional and turbulent zones (Figure 3).

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[72] Pokhmurskii V., Sirak Y., Bilonoga Y. Influence of the surface energy and of the energy of the bond of the contacting metals on the fretting fatigue life of the joints of machine parts. *Soviet Materials Science.* 1984. № 20(4). P. 358. DOI: https://doi.org/10.1007/BF01199367

[73] Bilonoga Y., Pokhmurs'kii V. A connection between the fretting-fatigue endurance of steels and the surface energy of the abradant metal. *Soviet Materials Science.* 1991. № 26(6). P. 629. DOI: https://doi.org/10.1007/BF00723647

[74] Олешко В., Пиговкин И. Оперативное определение поверхностной энергии металлических деталей авиационной техники. *Интернет-журнал «НАУКОВЕДЕНИЕ». 2006.* № 8(3). С. 12. URL: http://naukovedenie.ru/PDF/131EVN316

The heat moves along the chain, passing respectively all the zones of the of coolants hot water (h) and cold milk (c) and is transmitted by means of the thermal conductivities of the corresponding zones ($k_{turb.h}$, $k_{trans.h}$, k_w, $k_{trans.c}$, $k_{turb.c}$) (Figure 3).

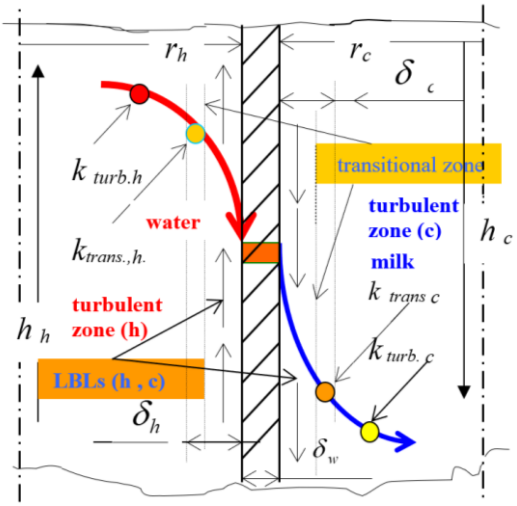

Figure 3. The scheme for transferring the heat through the metal wall from hot water (h) to cold milk (c)

Based on the results of our previous research, the heat transfer process should take into account the surface forces arising at the boundary of the heat exchanger wall – fluid flow, where they are dominant in LBL[75], and especially when using nanofluid coolants, where they also predominate contact of the nanoparticle surface with the base fluid. Equations (1.4) and (1.5) in the Nusselt and Prandtl numbers use the values of molecular viscosity and thermal conductivity of static fluid, which can be more than two orders of magnitude less than the turbulent characteristics of the same fluid. In addition, the classical Nusselt and Prandtl similarity numbers in the classical numerical equations of the type (1.4, 1.5) do not contain the surface characteristics of the heat exchange medium. It turns out a paradox – surface forces are absolutely dominant in the border layers of the LBL, and the

[75] Bilonoga Y., Maksysko O. The laws of distribution of the values of turbulent thermo-physical characteristics in the volume of the flows of heat carriers taking into account the surface forces. *International Journal of Heat and Technology.* 2019. № 36(1). P. 7. DOI: https://doi.org/10.18280/ijht.370101

similarity numbers do not contain these surface characteristics?! The thermal resistances of individual zones (Fig. 3) depend on the respective thermal conductivities and average thicknesses of these zones. Then, the ratio for determining the total heat transfer coefficient of this system (Fig. 3) is presented as (3.17). In formula (3.17), the convection coefficients h_h and h_c are replaced by the thermal conductivity coefficients and the average thicknesses of the respective zones (Fig. 3).

$$U_{\delta} = \frac{1}{\frac{r_h - \delta_h}{k_{turb.h}} + \frac{\delta_h}{k_{trans.h}} + \frac{\delta_w}{k_w} + \frac{\delta_c}{k_{trans.c}} + \frac{r_c - \delta_c}{k_{turb.c}}} \qquad (3.17)$$

where in formulas and figures 3 U_{δ} − the overall heat transfer coefficient at the averge thickness of the LBL, W.m^{-2}.K^{-1}; r_h – radius of the "live section" of the hot carrier stream, m; r_c − radius of the "live section" of the cold carrier stream, m; δ_h − average thickness LBL of the hot coolant, m; δ_c− average thickness LBL of cold coolant, m; δ_w – thickness of the tube wall, m; $k_{turb.h}$ − coefficient of average turbulent thermal conductivity of hot coolant (water), W.m^{-1}.K^{-1}; $k_{turb.c}$ − coefficient of average turbulent thermal conductivity of cold coolant (milk), W.m^{-1}.K^{-1}; $k_{trans.h}$ − transitional thermal conductivity in the LBL transitional zone hot coolant, W.m^{-1}.K^{-1}; $k_{trans.c}$ − transitional thermal conductivity in the LBL transitional zone cold coolant, W.m^{-1}.K^{-1}; k_w − thermal conductivity wally, W.m^{-1}.K^{-1}.

Characteristics of liquid-phase flows – turbulent viscosity and thermal conductivity were presented by Joseph Boussinesq, for example in[76, 77].

3.3. Numerical equation for calculating the coefficients of convective (turbulent) thermal conductivity

We replace the convection coefficient h [W.m^{-2}.K^{-1}] in the Nusselt number (equations 1.4, 1.5) with the convective (turbulent) thermal conductivity coefficient k_{turb}. (3.17) [W.m^{-1}.K^{-1}]. According to the theory of J. Boussinesq, we introduce the concept of turbulent viscosity μ_{turb}. [kg.s^{-1}.m^{-1}] (3.17) of aluid heat carrier, since a turbulent fluid is considered as Newtonian.

It also seems to ushat the parameter of the linear velocity of the movement of the coolant V [m.s^{-1}], as well as mass velocity $V.\rho$ [kg.m^{-2}s^{-1}]

[76] Boussinesq J. Théorie des expériences de Savart sur la forme que prend une veine liquide après s'être choquée contre un plan circulaire. 1869. CR 69 45–48 P. 130.

[77] Boussinesq J. Théorie des ondes et des remous qui se propagent le long d'un canal rectangulaire horizontal, en communiquant au liquide contenu dans ce canal des vitesses sensiblement pareilles de laurface au fond. JMPA. 1872. 17. P. 90

in the equation (1.4) not fully reflect the flow characteristic. We propose to introduce the parameter of the volumetric flow rate of the coolant W[m^3.s^{-1}] in the equation (3.17) which characterizes the linear velocity and the parameter of the "free area" of the flow. And most importantly, we propose to introduce into the calculation the surface parameter responsible for the surface tension forces – the surface tension coefficient of the fluid σ [N.m^{-1}].

The coefficient of convective (turbulent) thermal conductivity k_{turb}. [W.m^{-1}.K^{-1}] with turbulent movement of aiquid or gas heat carrier will depend on the following main factors:

- turbulent dynamic viscosity of the liquid μ_{turb}. [Pa. s];
- volumetric flow rate of the heat carrier W [m^3.s^{-1}];
- coefficient of surface tension of the liquid σ [N.m^{-1}];
- specific heat capacity of liquid heat carrier C_P [J.kg^{-1}.K^{-1}];
- linear dimension, e.g. pipe radius R [m].

3.3.1 The concept of transitional viscosity and thermal conductivity in LBL

The equation for determining the transitional viscosity and thermal conductivity in the LBL zone was derived by the method of analysis of the dimensions of the main thermophysical quantities. LBL has a strong field of surface forces, so the transient thermal conductivity in the LBL zone will depend on the following main factors, coefficient of surface tension[78] and dynamic viscosity, specific heat of the coolant. Let us represent this dependence in the form of aower function (3.18):

$$k_{trans.} = f(\sigma, \mu_{trans}, c_p); \; k_{trans.} = B(\sigma^X, \mu^Y{}_{trans}, c_p{}^Z) \qquad (3.18)$$

All these thermophysical quantities are represented by the main dimensions – mass, linear size, time [kg, m, s].

$$k_{trans} - [W.m^{-1}.K^{-1} = N.m.s^{-1}.m^{-1}.K^{-1} = kg.m.s^{-3}.K^{-1}];$$
$$\sigma - [N.m^{-1} = kg.m.s^{-2}.m^{-1} = kg.s^{-2}];$$
$$\mu - [N.s.m^{-2} = kg.m.s^{-1}.m^{-2} = kg.s^{-1}.m^{-1}];$$
$$c_p - [N.m.kg^{-1}.K^{-1} = kg.m^2.s^{-2}kg^{-1}.K^{-1} = m^2.s^{-2}.K^{-1}].$$

[78] Bilonoga Y., Maksysko O. Modeling the interaction of coolant flows at the liquid-solid boundary with allowance for the laminar boundary layer. *International Journal of Heat and Technology*. 2017. № 35(3). P. 679. DOI: http://dx.doi.org/10.18280/ijht.350329

The power equation is given as the equation of basic dimensions.

$$[kg^{1}.m^{1}.s^{-3}.K^{-1}] = B\left[kg.s^{-2}\right]^{X} \cdot \left[kg.s^{-1}.m^{-1}\right]^{Y} \cdot \left[m^{2}.s^{-2}.K^{-1}\right]^{Z};$$

Starting from the equation of dimensions, we can write such a system of equations for exponents:

$$X + Y = 1; \quad Y = 1 - X; \quad -Y + 2Z = 1;$$

$$Z = 1 - 0.5\mathrm{X}; \quad -2X - Y \ -2Z \ = -3$$

$$k_{trans} = B(\sigma^{X}\mu_{trans}{}^{1-X}c_{p}{}^{1-0.5X}); k_{trans} = \mu_{trans} \cdot c_{p}(\sigma/\mu_{trans} \cdot c_{p}{}^{0.5})^{X};$$

The relation is represented by one unknown degree X (3.19):

$$\frac{k_{trans}}{c_{p}\mu_{trans}} = B\left(\frac{\sigma}{\mu_{trans}\sqrt{c_{p} \cdot 1^{0} K}}\right)^{X} \tag{3.19}$$

In the left part of equation (3.19) we have the inverse Prandtl number in the LBL zone, and in the right part we have the new similarity number, which we denoted by *Bl*[79].

Based on the work of Reichardt H., Ludwig H. and others, the Prandtl number in the LBL transitional zone is one. The index $X = 1$ and the constant $B = 1$. Taking into account the hydrophilicity of the surface, relation (3.19) has the form (3.20).

$$\frac{\mu_{trans}\sqrt{c_{p} \cdot 1^{0} K}}{\sigma \cdot \cos\theta} = 1 \tag{3.20}$$

Near the wall of the pipe, where the flow velocity goes to zero, the viscosity of the liquid is the dynamic viscosity to which we are accustomed. When moving away from the wall, the surface forces decrease and in the LBL transitional zone they are approximately equal to the product of the friction forces in the LBL and the cohesion forces. Farther from the wall outside the LBL zone, the transitional viscosity becomes turbulent and

[79] Bilonoga Y., Maksysko O. Specific features of heat exchangers calculation considering the laminar boundary layer, the transitional and turbulent thermal conductivity of heat carriers. *International Journal of Heat and Technology*. 2018. № 36(1). P. 14. DOI: https://doi.org/10.18280/ijht.360102

increases significantly. The viscosity in the LBL transitional zone, given (3.20), can be found from relation (3.21).

$$\mu_{trans} = \frac{\sigma \cdot \cos\theta}{\sqrt{C_p \cdot 1^0 K}} \tag{3.21}$$

Accordingly, the transient thermal conductivity, ie the thermal conductivity in the transition zone of the LBL can be calculated from equation (3.22)

$$k_{tranc} = \frac{\sigma \cdot \cos\theta \cdot C_p}{\sqrt{C_p \cdot 1^0 K}} \tag{3.22}$$

Consider the physical meaning of the number (3.20) *Bl*[80, 81]. The molecular number *Bl* ishe ratio of the product of internal friction forces and cohesion forces to the surface forces in the liquid coolant (3.23):

$$Bl = \frac{\mu \cdot \sqrt{C_p \cdot 1^0 K}}{\sigma \cdot \cos\theta} = \frac{\text{Friction internal force} \cdot \text{Cohesion force}}{\text{Interfacial Surface tension force}} \tag{3.23}$$

Simultaneously, it ishe ratio of molecular viscosity to the transitional viscosity in LBL (3.24) or the ratio of the rate of thermal motion of liquid molecules to the average relaxation rate of these molecules (3.25):

$$Bl = \frac{\mu}{\frac{\sigma \times \cos\theta}{\sqrt{C_p \times 1^0 K}}} = \frac{\text{molecular viscosity in LBL}}{\text{transitional viscosity in LBL}} = \frac{Pa \cdot s}{Pa \cdot s} \tag{3.24}$$

$$Bl = \frac{\sqrt{C_p \cdot 1^0 K}}{\frac{\sigma \cdot \cos\theta}{\mu}} = \frac{\text{the thermal rate of the molecules}}{\text{the relaxation rate of the molecules}} = \frac{m \cdot s^{-1}}{m \cdot s^{-1}} \tag{3.25}$$

where in the formulas (3.18)–(3.25) $k_{trans.}$ – transitional thermal conductivity in the LBL transitional zone, W.m^{-1}.K^{-1}; σ – surface tension coefficient of coolant, N.m^{-1}; cosθ – cosine of the contact angle; μ_{trans} – coefficient of transitional viscosity of coolant, kg.m^{-1} s $^{-1}$; μ – coefficient viscosity of coolant, kg.m^{-1}.s^{-1}; C_p – specific heat capacity, J.kg^{-1}.K^{-1}; *B, X, Y, Z* – dimensionless constants.

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[80] Bilonoga Y., Maksysko O. Specific features of heat exchangers calculation considering the laminar boundary layer, the transitional and turbulent thermal conductivity of heat carriers. *International Journal of Heat and Technology.* 2018. № 36(1). P. 14. DOI: https://doi.org/10.18280/ijht.360102

[81] Bilonoga Y., Stybel V., Maksysko O., Drachuk U. Substantiation of aew Calculation and Selection Algorithm of optimal Heat Exchangers with Nanofluid Heat Carriers taking into Account surface Forces. *International Journal of Heat and Technology.* 2021. № 39 (6). P. 1706, DOI: https://doi.org/10.18280/ijht.390602

The product of viscosity and heat capacity gives the value of thermal conductivity. Representing the number *Bl* as (3.24), it isimilar to the left part of equation (1.1), and (3.25), it isimilar to the right part (1.1). The number *Bl* derived by usas a simultaneous distribution of temperatures and velocities in fluid flows. At normal temperature (about 20 °C) for water, the molecular number of *Bl*s approximately of one[82], which indicates the balance of friction, cohesion and surface forces under normal terrestrial conditions.

3.4. Bl_{turb} – the number of distribution of temperature and velocity fields in turbulent flows of liquid coolants

The dimensionless number Bl_{turb} is (3.26)[83]. When we consider the turbulent part of the flow of the heat carrier, the linear velocity makes the prevailing value in the formation of its structure. The speed is greater – the turbulization is more intense.

$$Bl_{turb.} = \frac{\mu_{turb.} \cdot \sqrt{C_p \cdot 1^0 K}}{\sigma \cdot \cos\theta} = \frac{\text{Friction force in turbulent flow} \cdot \text{Cohesion force}}{\text{Interfacial Surface tension force}} \quad (3.26)$$

3.4.1. The dominant factors affecting the turbulent thermal conductivity of the turbulent zone of the coolant flow

The function of turbulent thermal conductivity of the coolant flow from the dominant factors (3.27):

$$k_{turb.} = f\left(\mu_{turb.}, V, \sigma, C_P, (r-\Delta)\right) \quad k_{turb.} = B\left(\mu_{turb.}^{X} \cdot V^{Y} \cdot \sigma^{Z} \cdot C_P^{M} \cdot (r-\Delta)^{E}\right) \quad (3.27)$$

Dimensional analysis of function (3.27) looks like this: The dimension of all thermophysical quantities through the main – mass, linear size, time [kg, m, s].

$$k_{turb.} - \left[W.m^{-1}.K^{-1} = N.m.s^{-1}.m^{-1}.K^{-1} = kg.m.s^{-3}.K^{-1}\right];$$

$$\mu_{turb.} - \left[\text{Pa.s} = N.s.m^{-2} = kg.m^{-1}.s^{-1}\right];$$

$$V - \left[m^{3}.s^{-1}\right]; \sigma - \left[N.m^{-1} = kg.m.s^{-2}.m^{-1} = kg.s^{-2}\right];$$

$$C_P - \left[N.m.kg.^{-1}K^{-1} = kg.m^{2}s^{-2}.kg^{-1}.K^{-1} = m^{2}.s^{-2}.K^{-1}\right]; \ (r-\Delta) - [m];$$

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[82] Bilonoga Y., Maksysko O. Specific features of heat exchangers calculation considering the laminar boundary layer, the transitional and turbulent thermal conductivity of heat carriers. *International Journal of Heat and Technology*. 2018. № 36(1). P. 14. DOI: https://doi.org/10.18280/ijht.360102

[83] Bilonoga Y., Maksysko O. The laws of distribution of the values of turbulent thermo-physical characteristics in the volume of the flows of heat carriers taking into account the surface forces. *International Journal of Heat and Technology*. 2019. № 36(1). P. 10. DOI: https://doi.org/10.18280/ijht.370101

We will represent the power equation with the help of several unknown powers.

$$\left[kg.m.s^{-3}.K^{-1}\right] = B\left[kg.m^{-1}.s^{-1}\right]^{X}\left[m^{3}.s^{-1}\right]^{Y}\left[kg.s^{-2}\right]^{Z} \quad \left[m^{2}.s^{-2}.K^{-1}\right]^{M}[m]^{E}$$

The system of equations for unknown indicators looks like this:

$$X+Z=1 \qquad -X+3Y+2M+E=1 \qquad -X-Y-2Z-2M=-3$$

$$Z=1-X; \quad M=0.5X+0.5; \qquad E=0; \quad Y=0;$$

Select all unknown degrees through one unknown X:

$$k_{turb.} = B(\mu_{turb.}^{X} \times V \times \sigma^{1-X} \times C_P^{\,0,5+0,5X}) : \sigma.\sqrt{C_P \times 1^0 K}$$

Grouping all terms, we obtain equation (3.28):

$$\frac{k_{turb.}}{\sigma \cdot \sqrt{C_p \cdot 1^0 K}_{.}} = B\left(\frac{\mu_{turb.}\sqrt{C_p \cdot 1^0 K}}{\sigma}\right)^{X} \tag{3.28}$$

$\sigma \cdot \sqrt{C_p \cdot 1^0 K} = k_{trans.}$ – transitional thermal conductivity in the transitional zone LBL[84] (3.22).

Given the hydrophilicity of the metal wall surface, (3.28) is represented as (3.29):

$$\frac{k_{turb.}}{\sigma\cos\theta\sqrt{C_p \cdot 1^0 K}_{.}} = B\left(\frac{\mu_{turb.}\sqrt{C_p \cdot 1^0 K}}{\sigma\cos\theta}\right)^{X} \tag{3.29}$$

On the left (3.29) this is the ratio of turbulent to transient thermal conductivity. On the right is (3.29) the dimensionless number Blturb. which we obtained in [85] . From equation (3.29) using equations (3.30 and 3. 31) we can obtain turbulent thermal conductivity in the central part of the coolant flow.

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[84] Bilonoga Y., Maksysko O. The laws of distribution of the values of turbulent thermo-physical characteristics in the volume of the flows of heat carriers taking into account the surface forces. *International Journal of Heat and Technology.* 2019. № 36(1). P. 7. DOI: https://doi.org/10.18280/ijht.370101

[85] Ibid.

$$k_{turb.} = B\left(\frac{\mu_{turb.}\sqrt{C_p \cdot 1^0 K}}{\sigma\cos\theta}\right)^{-X} \sigma\cos\theta\sqrt{C_p \cdot 1^0 K} \quad (3.30)$$

$$Bl_{turb.} = \frac{k_{turb.}}{k_{trans.}} = \frac{\mu_{turb}}{\mu_{trans}} = B\left(\frac{\mu_{turb.}\sqrt{C_p \cdot 1^0 K}}{\sigma\cos\theta}\right)^{-X} \quad (3.31)$$

$$\frac{k_{turb.}}{C_P \cdot \mu} = \frac{\sigma \cdot \cos\theta}{\mu \cdot \sqrt{C_P \cdot 1^0 K}}\left(\frac{\sqrt{C_P \cdot 1^0 K}}{V}\right)^{-X} \quad ; \quad \text{or} \quad \frac{\mu_{turb.}}{\mu} = Bl_{turb.,}/Bl;$$

$$\text{or} \quad \frac{k_{turb.}}{k} = Bl_{turb.,}/Bl;$$

where in the formulas(3.26)–(3.31) σ – surface tension coefficient of coolant, N.m^{-1}; cosθ – cosine of the contact angle; μ – coefficient viscosity of coolant; kg.m^{-1}.s^{-1}; μ_{trans} – coefficient of transitional viscosity of coolant, kg.m^{-1}s^{-1}; μ_{turb} – coefficient of turbulent viscosity of coolant, kg.m^{-1}s^{-1}; $k_{trans.}$– transitional thermal conductivity in the transitional zone, W.m^{-1}.K^{-1} ; $k_{turb.}$ – coefficient of turbulent thermal conductivity of coolant , W.m^{-1}.K^{-1}; C_p – specific heat capacity, J.kg^{-1}. K^{-1}; *B, X, Y, Z, M, E* – dimensionless constants; *V* – velocity, m.s^{-1}; *r* – pipe radius, *m*; Δ – step along the radius of the pipe from the center to the wall, m.

3.4.2 The main factors affecting the turbulent viscosity of the flow of cooling liquid in the zone of free turbulence

Function of turbulent viscosity of coolant flow from dominant factors (3.32):

$$\mu_{turb.} = f\left(V, \sigma, C_p, (r-\Delta)\right) \quad \mu_{turb.} = B\left(V^Y \cdot \sigma^Z \cdot C_p^{\ M} \cdot (r-\Delta)^E\right) \quad (3.32)$$

Dimensional analysis of this function: All thermophysical quantities are given through the main dimensions – mass, linear size, time [kg, m, s].

$$\mu_{turb.} - \left[Pa.s = N.s.m^{-2} = kg.m^{-1}.s^{-1}\right]; \; V - \left[m^3.s^{-1}\right];$$

$$\sigma - \left[N.m^{-1} = kg.m.s^{-2}.m^{-1} = kg.s^{-2}\right];$$

$$C_p - \left[N.m.kg.^{-1}K^{-1} = kg.m^2s^{-2}.kg^{-1}.K^{-1} = m^2.s^{-2}.K^{-1}\right]; \; (r-\Delta) - [m].$$

Dimensional equation:

$$\left[kg.m^{-1}.s^{-1}\right] = B\left[m^3.s^{-1}\right]^Y \left[kg.s^{-2}\right]^Z \left[m^2.s^{-2}.K^{-1}\right]^M [m]^E;$$

System of equations for unknown powers:

$$Z = 1;\ 3Y + 2M + E = -1;\quad -Y - 2Z - 2M = -1;$$
$$2M = -1 - Y;\quad E = -2Y$$

$$\mu_{turb.} = B(V^Y \cdot \sigma^1 \cdot C_p^{-0.5-0.5Y} \cdot (r-\Delta)^{-2Y}) : \frac{\sigma}{\sqrt{C_p \cdot 1^0 K}}$$

All degrees are expressed in one unknown X, grouped, we obtain (3.33):

$$\frac{\mu_{turb.}\sqrt{C_p \cdot 1^0 K}}{\sigma \cos} = B\left(\frac{V}{(r-\Delta)^2\sqrt{C_p \cdot 1^0 K}}\right)^X, \qquad (3.33)$$

$\frac{\sigma \cos}{\sqrt{C_p \cdot 1^0 K}} = \mu_{trans.}$ – coefficient of transitional viscosity in the transitional zone LBL.

It is easy to see that in the brackets on the right-hand side of equation (3.33) we have the ratio of the average velocity of the flow (the ratio of the volume flow rate of the refrigerant to the cross-sectional area of the pipes) to the value representing the average velocity of the thermal motion of the coolant molecules (the value $\sqrt{C_p \cdot 1^0 K}$). Since the total refrigerant flow extends to all pipes, in formula (3.33) we need to enter the value *(n/z)*. Then this flow rate will correspond to the flow rate in one heat exchanger tube. In addition, in formula (3.33) it isecessary to introduce the number π, since it enters the cross-sectional area of one tube and is not fixed by the method of dimensional analysis. The value of the average velocity of thermal movement of the refrigerant molecules is much higher than the average linear velocity of the refrigerant. In the right side of equation (3.33) the numerator must be replaced by aenominator. Then the exponent of degree *(–X)* gets a minus sign. Taking into account the hydrophilicity of the surface, equation (3.33) takes the form (3.34). This equation is valid for the tubular space of the heat exchanger. For a space with a shell, equation (3.34) takes the form (3.35). In equation (3.35), the number π is not introduced, but the conditional radius is calculated from the cross-sectional area of the $A_c = 4.5.10^{-3}m^2$ pass, taking into account the total number of the pipes ($n = 206$). The value of Δ – is the step of the radius of the coolant flow in the

pipes or inhe annular space, with which we move away from the center of the coolant flow to the wall.

$$Bl_{turb.c} = \frac{\mu_{turb.c}\sqrt{C_{p.c}\cdot 1^0 K}}{\sigma_c \cos\theta} = B\left(\frac{\pi(r_c-\Delta)^2 \frac{n}{z}\sqrt{C_{p.c}\cdot 1^0 K}}{V_c}\right)^{-X_c} \quad (3.34)$$

$$Bl_{turb.h} = \frac{\mu_{turb.h}\sqrt{C_{p.h}\cdot 1^0 K}}{\sigma_h \cos\theta} = B\left(\frac{(r_h-\Delta)^2 n\sqrt{C_{p.h}\cdot 1^0 K}}{V_h}\right)^{-X_h} \quad (3.35)$$

where in the formulas (3.32) − (3.35) $Bl_{turb.c}$ turbulent dimensionless numbers for cold coolant; $Bl_{turb.h}$ turbulent dimensionless numbers for hot coolant; $\mu_{turb.h}$ − coefficient of turbulent viscosity of hot coolant, kg.m^{-1}s^{-1}; $\mu_{turb.c}$ − coefficient of turbulent viscosity of cold coolant, kg.m^{-1} s^{-1}; $C_{p.h}$ − specific heat capacity of hot coolant, J.kg^{-1}.K^{-1}; $C_{p.c}$− specific heat capacity of cold coolant, J.kg^{-1}.K^{-1}; $(-X_h),(-X_c)$ − degree indicators for hot and cold coolant, respectively (degree of flow turbulence); σ_h −surface tension coefficient of hot coolant, N.m^{-1}; σ_c − surface tension coefficient of cold coolant, N.m^{-1}; cosθ −cosine of the contact angle; r_h − radius of the "live section" of the hot carrier stream, m; r_c − radius of the "live section" of the cold carrier stream, m; Δ − step along the radius of the pipe from the center to the wall, m; n − number of the tubes; z − number of the passes. B – constant.

The turbulent zone of the liquid coolant is modeled by usaking into account the work of J. Boussinesq[86, 87]. Turbulent number Bl_{turb}. absolutely identical in shape to the molecular, but contains the value of turbulent viscosity. These two numbers confirm the conclusion of Italian scientists that the LBL is inomplete control of its turbulent zone[88].

In[89], the final formulas for calculating the turbulent number Bl_{turb}., which has the form (3.36), are presented.

[86] Boussinesq J. Théorie des expériences de Savart sur la forme que prend une veine liquide après s'être choquée contre un plan circulaire. 1869. CR 69 45–48 P. 130.

[87] Boussinesq J. Théorie des ondes et des remous qui se propagent le long d'un canal rectangulaire horizontal, en communiquant au liquide contenu dans ce canal des vitesses sensiblement pareilles de laurface au fond. JMPA. 1872. 17. P. 90.

[88] .Quadrio M., Ricco P. The laminar generalized Stokes layer and turbulent drag reduction. *Journal of Fluid Mechanics*. 2011. № 667, P. 156. DOI: http://dx.doi.org/10.1017/S0022112010004398.

[89] Bilonoga Y., Maksysko O. The laws of distribution of the values of turbulent thermo-physical characteristics in the volume of the flows of heat carriers taking into account the surface forces. *International Journal of Heat and Technology*. 2019. № 36(1). P. 7. DOI: https://doi.org/10.18280/ijht.370101

$$Bl_{turb} = \left(\frac{\sqrt{C_P \cdot 1^0 K}}{V} \right)^{-X} \text{ or } Bl_{turb.} = \frac{1}{Bl} \left(\frac{\sqrt{C_p \cdot 1^0 K}}{V} \right)^{-X} \frac{\mu \sqrt{C_p \cdot 1^0 K}}{\sigma \cdot \cos\theta}. \quad (3.36)$$

If we add the product of specific heat in the numerator and denominator of the left part of equation (3.36), then we have the ratio of the maximum turbulent thermal conductivity in the middle of the coolant flow to the minimum value (transition value) at the end of LBL (see correlates with the left-hand side of formula (1.1)). The right-hand side of equation (3.36) shows the ratio of the average thermal velocity of liquid molecules (maximum) to the linear flow rate of the coolant (minimum).

3.5. Force field acting on aanoparticle based on MDM

By comparing different similarity numbers in the LBL in Section 3.1, we proved that the Stokes forces and surface forces dominate in the force field acting on the nanoparticle in the liquid coolant. In the turbulent medium of the coolant, the nanoparticles also move chaotically with the formation of LBL (Fig. 2). The nanoparticle interacts with the molecules of the base fluid and receives from them an almost equivalent momentum[90, 91]. Based on the conclusions[92] the force of surface tension in non-stationary conditions can exceed the Stokes force by (2–3) times. In stationary conditions, these forces are almost equal. Based on the Boussinesqs principle, we introduce the value of turbulent viscosity into Stokes' formula, taking into account the hydrophilicity of the nanoparticle surface. We introduce the equation based on the previous analysis (3.37)[93]:

$$\pi d \sigma \cos\theta = (2 \div 3) \cdot 3 \cdot \pi d V \mu_{turb} \quad (3.37)$$

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[90] Рудяк В. Я. Белкин А. А. Моделирование коэффициентов переноса наножидкостей. *Наносистемы: физика, химия, математика.* 2010. Т. 1, № 1. С. 162.

[91] Рудяк В. Я., Белкин А. А., Краснолуцкий С. Л. К статистической теории процессов переноса наночастиц в газах и жидкостях. *Теплофизика и аэромеханика*. 2005. № 4. С. 535.

[92] Rudyak V., Belkin A., Tomilina E. The force acting on aanoparticle in aiquid. *Technical Physics Letters.* 2008. № 34 (1). P. 76–78. DOI: https://doi.org/10.1134/S1063785008010239(78)

[93] Bilonoga Y., Stybel V., Maksysko O., Drachuk U. A New Universal Numerical Equation and a New Method for Calculating Heat-Exchange Equipment using Nanofluids. *International Journal of Heat and Technology*. 2020. № 38 (1). С. 164. DOI: https://doi.org/10.18280/ijht.380117

Given equalites (3.36 and 3.37), we deduce the relation (3.38)[94]:

$$\left(\frac{\sqrt{C_p \cdot 1^0 K}}{(3-9)\cdot V}\right)=\left(\frac{\sqrt{C_p \cdot 1^0 K}}{V}\right)^{-X} 1/Bl; \qquad (3.38)$$

Equation (3.38) with respect to the unknown exponent (–*X*) do not have a single universal solution. Equations (Table 2) (3.39) were obtained by computer calculations[95]:

Table 2

Determination of the indicator (-*X*) (Formula 3.39)

Bl	**Dynamic viscosity**	**Degree (-X) (Formula 3.39)**
$Bl \geq 1$	$2\mu_{H_20}(20\ ^0C)\ ^3\ \mu\ ^3 1.5\times\mu_{H_20}(20\ ^0C)$ milk, oils, aqueous solutions of glycols	$-X=1-\frac{\ln(1/Bl)+\ln 6}{\ln\frac{\sqrt{C_p \cdot 1^0 K}}{V}};$
$Bl \leq 1$	$\mu \leq \mu_{H_2}0\ (20\ ^0C)$ water at high temperatures	$-X=1-\frac{\ln(1/Bl)+\ln 3}{\ln\frac{\sqrt{C_p \cdot 1^0 K}}{V}};$
$Bl \geq 1$	$1.5\mu_{H_20}(20\ ^0C) \geq \mu \geq \mu_{H_20}(20^0)$ milk, glycols at high temperatures	$-X=1-\frac{\ln(1/Bl)+\ln 4}{\ln\frac{\sqrt{C_p \cdot 1^0 K}}{V}};$
$Bl \geq 1$	$\mu \geq 2\cdot\mu_{H_20}\ (20\ ^0C)$ glycols at low temperatures, cream, butter, cooking, sugar and salt solutions	$-X=1-\frac{\ln(1/Bl)+\ln 5}{\ln\frac{\sqrt{C_p \cdot 1^0 K}}{V}};$
$Bl \leq 1$	$\mu \leq \mu_{H_20}(20\ ^0C)$ solutions of alcohols, gasolines, kerosene	$-X=1-\frac{\ln(1/Bl)+\ln 9}{\ln\frac{\sqrt{C_p \cdot 1^0 K}}{V}}$

[94] Bilonoga Yu., Stybel V., Maksysko O., Drachuk U. Substantiation of aew Calculation and Selection Algorithm of optimal Heat Exchangers with Nanofluid Heat Carriers taking into Account surface Forces. International *Journal of Heat and Technology*. 2021. № 39 (6). P. 1706, DOI: https://doi.org/10.18280/ijht.390602

[95] Bilonoga Y., Stybel V., Maksysko O., Drachuk U. A New Universal Numerical Equation and a New Method for Calculating Heat-Exchange Equipment using Nanofluids. *International Journal of Heat and Technology*. 2020. № 38(1). C. 164. DOI: https://doi.org/10.18280/ijht.380117

The use of formulas 3.39 is complicated. Therefore, we propose a different approach to this problem empirical formula (3.40) for calculating the turbulent viscosity of air flows has the form[96].

$$\mu_{turb.} = \frac{\mu \cdot a\sqrt{2\,\mathrm{Re}}}{0.769} \tag{3.40}$$

We obtain the relation (3.41), taking into account (3.36 and 3.40) to find the value of the degree (–*X*) (3.41):

$$-X = \frac{\ln \dfrac{a \cdot \sqrt{2\,\mathrm{Re}}}{0.769 \cdot 1/Bl}}{\ln \dfrac{\sqrt{C_p \cdot 1^0 K}}{V}}; \tag{3.41}$$

where a − experimental coefficient, $a = (0.05 - 0.08)$[97]; Re − Reynolds number; Bl − dimensionless number; C_p − specific heat capacity, J.kg^{-1}.K^{-1}; V − velocity, m.s^{-1}.

3.6. Express methods for evaluating the efficiency of nanofluid coolants in heat exchangers.

Classical express method for evaluating the efficiency of nanofluids in heat exchangers[98]:

– when calculating heat exchangers according to the method using classical numbers of Nusselt, Reynolds, Prandtl and classical equations of the type (1.4, 1.5), the positive effect of nanofluids occurs with increasing ratio of their thermal conductivity and viscosity;

– the increase in the convection coefficient of the liquid coolant h with the addition of nanoparticles is directly proportional to the growth of the complex. From equation type (1.5), for example, for the casing of the casing-and-tube heat exchanger, where water is supplied with the addition of the corresponding ethylene glycols:

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[96] Bilonoga, Y., Maksysko, O. Specific features of heat exchangers calculation considering the laminar boundary layer, the transitional and turbulent thermal conductivity of heat carriers. *International Journal of Heat and Technology.* 2018. № 36(1). P. 14. DOI: https://doi.org/10.18280/ijht.360102

[97] Пирашвили С. А., Поляев В. М., Сергеев М. Н. Вихревой эффект. Эксперимент, теория, технические решения. Москва : УНПУ, Энергомаш, 2000. С. 179.

[98] Rudyak V. Ya,. Minakov A. V,. Krasnolutskii S. L. Physics and mechanics of heat exchange processes in nanofluid flows. Physical Mesomechanics. 2016. № 19(3). P. 304. DOI: https://doi.org/10.1134/S1029959916030085)

$$Nu_h = 0.24 \cdot \mathrm{Re}_h^{0.6} \mathrm{Pr}_h^{0.36};$$

i.e

$$\frac{h_h \cdot D}{k} = 0.24 \cdot \left(\frac{V \cdot D \cdot \rho}{\mu} \right)^{0.6} \left(\frac{C_{P\cdot} \cdot \mu}{k} \right)^{0.36}$$

$$h_h \sim \frac{k^{(1-0..36)}}{\mu^{(0.6-0.36)}} = \frac{k^{0.64}}{\mu^{0.24}}; \quad (3.42\ a)$$

– numerical equation when using nanofluid such as TiO_2, eg. $Nu = 0.021 Re^{0.8} Pr^{0.5}$ [99], (analogous equation without using nanofluid $Nu = 0.021 Re^{0.8} Pr^{0.43}$);

– let us increase the exponent in the equation (1.5) for the shell-and-tube heat exchanger shell by 0.07, respectively, eq.:

$$Nu_h = 0.24 \cdot \mathrm{Re}_h^{0.6} \mathrm{Pr}_h^{(0.36+0.07)=0.43};$$

$$\frac{h_h \cdot D_h}{k_h} = 0.24 \cdot \left(\frac{V_h \cdot D_h \cdot \rho_h}{\mu_h} \right)^{0.6} \left(\frac{C_{P.h} \cdot \mu_h}{k_h} \right)^{0.43}$$

– complex when using nanofluid for the shell-and-tube heat exchanger shell (3.42 b):

$$h_h \sim \frac{k_h^{(1-0.43)}}{\mu_h^{(0.6-0.43)}} \sim \frac{k_h^{0.57}}{\mu_h^{0.17}}; \quad (3.42\ b)$$

– according to[100], if the complex grows, then we should get an improvement in heat transfer, and if itecreases – deterioration. Classical numerical equation for shell and tube heat exchange pipes (1.5) (3.43):

$$Nu_c = 0.023 \cdot \mathrm{Re}_c^{0.8} \mathrm{Pr}_c^{0.43};$$

$$\frac{h_c \cdot D_c}{k_c} = 0.023 \cdot \left(\frac{V_c \cdot D_c \cdot \rho_c}{\mu_c} \right)^{0.8} \left(\frac{C_{P.c} \cdot \mu_c}{k_c} \right)^{0.43}$$

$$h_{(milk)c} \sim \frac{k_c^{(1-0.43)}}{\mu_c^{(0.8-0.43)}} = \frac{k_c^{0.57}}{\mu_c^{0.37}} \quad (3.43)$$

[99] Pak B., Cho Y. I. Hydrodynamic and Heat Transfer Study of Dispersed Fluids with Submicron Metallic Oxide Particle. *Experimental Heat Transfer*. 1998. №(11) P. 168. DOI: https://doi.org/10.1080/08916159808946559

[100] Rudyak V. Ya,. Minakov A. V,. Krasnolutskii S. L. Physics and mechanics of heat exchange processes in nanofluid flows. *Physical Mesomechanics*. 2016. № 19(3). P. 304. DOI: https://doi.org/10.1134/S1029959916030085

3.6.1. Classical express – method

1. Thermophysical characteristics of hot water, mixtures of hot water with H_2O + EG (60:40) and hot water with H O + EG (60:40) + 1.5 % TiO_2 at 70 ° C Therefore, as well as complexes (3.42 a, 3.42 b) are presented in Table 3.

Table 3

Thermophysical characteristics and complexs of hot water corresponding mixtures

Volume	H_2O	H_2O+EG (60:40)	H_2O+EG (60:40) +1.5 % TiO_2
Complexs (3.42 a, 3.42 b)	**5.026** ←	**3.017** ←	**2.042** ←
Thermal conductivity turb. $k_{turb.h}$, $W.m^{-1}.K^{-1}$ (3.31)	**44.39** →	**76.24** →	**84.24** →
Density, ρ_h, $kg.m^{-3}$	970	1033	1081
Dynamic viscosity.10^3 μ_h $kg.m^{-1}.s^{-1}$	0.41	1.11	1.48
Surface tension.$10^3\sigma_h$, $N.m^{-1}$	62.25	51.19	51.00
Heat capacity: C_{ph}, $J.kg^{-1}.K^{-1}$	4198	3636	3463.3
Thermal conductivity:k_h, $W.m^{-1}.K^{-1}$	0.670	0.438	0.501
cosθ	0.850	0.795	0.880
The degree of flow turbulization, $(-X_h)$	**0.603** →	**0.823** →	**0.834** →

(arrows indicate the direction of increasing value)

– in comparison with pure water at aemperature of 70 °C, the convection coefficient h ofhe mixture H_20 + EG (60:40) decreased by (– 4.5)%, and the use of TiO_2 nanoparticles by (– 1.5)%.

– calculation of the complex by classical formulas (3.42 a and 3.42 b) showed that the use of known numerical equations for the calculation of heat exchangers with nanofluids at high temperatures (50–70 °C) is erroneous.

The complexs (3.42 a, b) in table 3 decreases, which should show a decrease in the convection coefficient using mixtures of H_2O + EG (60:40) + 1.5 % TiO_2, but experiments in[101].

So far, we have used water as aoolant with appropriate additives (ethylene- and propylene glycols with TiO_2 nanoparticles). In our work[102]

[101] Hamid, K. Abdul, Azmi, W., Mamat R., Usri, N., Najafi, G. Effect of temperature on heat transfer coefficient of titanium dioxide in ethylene glycol-based nanofluid. *Journal of Mechanical Engineering and Sciences*. 2015. № 8: P. 1372. DOI: http://dx.doi.org/10.15282/jmes.8.2015.11.0133

we considered the possibility of adding surfactants (a wide range of vegetable oils in concentrations up to 1 %) to liquid foods, such as milk. Formula (3.16) states that the introduction of surfactants into a liquid food product should increase its turbulent thermal conductivity while reducing the surface tension coefficient of the coolant.

We will conduct a rapid assessment of the effectiveness of pumpkin oil, as the most suitable for milk in terms of surfactants and trace elements. The thermal conductivity of pumpkin oil ($k_{oil} = 0.146$) is 3.9 times lower than that of milk ($k_{milk} = 0.5698$)[103]. Addition of + 0.5 % pumpkin oil reduces the thermal conductivity of milk to $k_{milk} = 0.5562$ W m^{-1}.K^{-1}. Thermophysical characteristics of milk and the corresponding mixture at 42.5 °C, as well as complexes are presented in Table 4.

Table 4

Complexs and thermophysical characteristics of cold milk and corresponding mixture

Volume	Cold milk	Cold milk + 0,5 %, pumpkin oil (SAS)
Complexs (3.42 a, 3.42 b)	**9.49** ←	**9.43** ←
Thermal conductivity turb., $k_{turb.c}$, W.m^{-1}.K^{-1}	**58.70** →	**66.38** →
Density, ρ_c, kg.m^{-3}	1020	1020
Dynamic viscosity·10^3: μ_c, kg.m^{-1}.s^{-1}	0.96	0.94
Surface tension: $\sigma_c \cdot 10^{-3}$, N.m^{-1};	47.75	35.25
Heat capacity: c_{pc}, J.kg^{-1}.K^{-1}	3914	3914
Thermal conductivity: k_c, W.m^{-1}.K^{-1};	0.569	0.556
The degree of flow turb., $(-X_c)$	**0.732** →	**0.759** →
Average linear flow rate of coolant, V_c, m.s^{-1}	0.659	0.659
cosθ	0.70	0.95

(arrows indicate the direction of increasing value)

[102] Bilonoga Y., Stybel V., Lorenzini E., Maksysko O., Drachuk U. Changes in the hydro-mechanical and thermo-physical characteristics of liquid food products (for example, milk) under the influence of natural surfactants. *Italian Journal of Engineering Science: Tecnica Italiana*, 2019. № 63(1). P. 26. DOI: https://doi.org/10.18280/ti-ijes.630103

[103] Bilonoga Y., Stybel V., Maksysko O., Drachuk U. Substantiation of aew Calculation and Selection Algorithm of optimal Heat Exchangers with Nanofluid Heat Carriers taking into Account surface Forces. International *Journal of Heat and Technology*. 2021. № 39(6). P. 1706. DOI: https://doi.org/10.18280/ijht.390602

The calculation shows that the complex does not increase, but slightly decreases.

3.6.2. New express – method

Thermophysical characteristics are the same as for the classical calculation (Tables 3, 4.). In Section 3.3, we replaced the heat transfer coefficient h with the turbulent thermal conductivity coefficient $k_{turb.}$. Since the convection coefficient does not fully reflect the turbulent and thermal characteristics of the liquid coolant[104].

From equation (3.30) the turbulent thermal conductivity is equal to (3.44):

$$k_{turb.} = (\sigma \cdot \cos\theta)\sqrt{C_P \cdot 1^0 K}\left(\frac{\sqrt{C_P \cdot 1^0 K}}{V}\right)^{-X} \qquad (3.44)$$

$$k_{turb.} = k_{trans.}\left(\frac{\sqrt{C_P \cdot 1^0 K}}{V}\right)^{-X} \quad ; \quad \frac{k_{turb.}}{k_{trans.}} = \left(\frac{\sqrt{C_P \cdot 1^0 K}}{V}\right)^{-X} ;$$

The values of turbulent thermal conductivity (3.44) for water and the corresponding mixtures are presented in table 3, as well as for the mixture of cold milk + 0.5 %, pumpkin oil (surfactant) – in table 4:

Turbulent thermal conductivity of water increases under the influence of appropriate mixtures. The same picture with milk under the influence of 0.5 % pumpkin oil.

This calculation, made by usccording to formula (3.44), fully correlates with experiments[105] and shows an increase in turbulent thermal conductivity (Table 3, 4). The same calculations performed according to the classical equation (1.4, 1.5) using complexes (3.42 a – b) show a decrease of – (4.5)%. Therefore, the classical equation is insensitive to the presence of nanoparticles in nanofluids, where surface forces are dominant. To eliminate this discrepancy, researchers make corrections to the corresponding classical equations, for example (1.6–1.11), but without taking into account the dominant surface forces. Obtaining these equations requires large expensive experiments, they acquire a complex, cumbersome

[104] Devette M. M. Heat transfer analysis of nanofluids and phase change materials. Universitat Polit'ecnica de Catalunya, 2013. 19.

[105] Hamid K., Abdul Azmi, W., Mamat R., Usri N., Najafi G. Effect of temperature on heat transfer coefficient of titanium dioxide in ethylene glycol-based nanofluid. *Journal of Mechanical Engineering and Sciences*. 2015. № 8. P. 1372. DOI: http://dx.doi.org/10.15282/jmes.8.2015.11.0133

shape and become unsuitable for engineering rapid calculations of heat exchangers.

If we take into account the turbulent viscosity and thermal conductivity of nanofluids and take into account surface forces, the effect is achieved by changing the degree of turbulence of the coolant (-*X*), which depends on aumber of factors included in formula (3.41).

Compared with water, the total heat transfer coefficient of H_20 + EG (60:40) at 70 ° C increased by (+12.86)%, and with TiO_2 nano-particles – by (+14.45)%.

In the second heat carrier (milk) with the addition of mix (0.5 % pumpkin oil), the heat transfer coefficient increases by (+16.75)%.

The use of aew express method based on the new equation (3.30) showed that the calculation of heat exchangers using nanofluids at high temperatures (70 °C) is accurate and reliable.

Examples of complete calculations of the plate heat exchanger by classical and new methods are given in our previous work[106]. In this paper, we present comparative calculations using mixtures of H_2O + EG (60:40) and H_2O + EG (60:40) +1.5 % TiO_2, which fully confirm the reliability of the new method and the new numerical equation in the calculation of heat exchange equipment at using nanofluids.

3.6.3. Comparison of the efficiency of classical and new numerical equations in the calculation of heat exchangers with nanoliquids

The classical (1.4) and new (3.30) numerical equations have the form:

$$\left(\frac{h\cdot D}{k}\right)^{1} = B\cdot\left(\frac{V\cdot D\cdot\rho}{\mu}\right)^{X}\left(\frac{C_{P.}\cdot\mu}{k}\right)^{Y};$$

$$k_{turb.h} = (\sigma_h\cdot\cos\theta)\sqrt{C_P\cdot 1^0 K}\left(\frac{\sqrt{C_P\cdot 1^0 K}}{V}\right)^{-X}$$

In the first classical numerical equation, Nusselt, Reynolds, and Prandtl numbers with empirical exponents of degrees *X* and *Y*, which are determined by careful, time-consuming experiments showing works such

[106] Bilonoga Y., Stybel V., Maksysko O., Drachuk U. A New Universal Numerical Equation and a New Method for Calculating Heat-Exchange Equipment using Nanofluids. *International Journal of Heat and Technology*. 2020. № 38(1). C. 164. DOI: https://doi.org/10.18280/ijht.380117

as[107, 108, 109, 110, 111, 112]. With such an empirical approach without taking into account surface forces, these equations sometimes take a complex form and are problematic for rapid analytical rapid calculations of heat exchangers.

The new equation is annalytical expression of equation (1.1), which was experimentally obtained in[113] and duplicated by many authors both analytically and experimentally on the principles of fluid mechanics[114] (see pages 661, 667, 668, 672, 673).

This is the ratio of turbulent and transient thermal conductivity or viscosity of the liquid coolant and its comparison with the ratio of the average rate of thermal motion of liquid molecules to the average linear flow rate (see[115]).

The fundamental difference between equation (3.30) is the use of the coefficient of surface tension of the liquid coolant, where surface forces become dominant with the use of nanofluids, and the turbulent properties of the liquid phase coolant – turbulent viscosity and thermal conductivity.

The derivation of this equation takes into account classical and modern research:

– equation (3.30) is analytical, where new numbers of similarity *Bl* and Bl_{turb} are used obtained by usarlier;

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[107] Pak B., Cho Y. I. Hydrodynamic and Heat Transfer Study of Dispersed Fluids with Submicron Metallic Oxide Particle. *Experimental Heat Transfer*. 1998. № 11. P. 168 DOI: https://doi.org/10.1080/08916159808946559

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[114] Шлихтинг Г. Теория пограничного слоя. М., 1974. С. 672.

[115] Bilonoga Y., Maksysko O. The laws of distribution of the values of turbulent thermo-physical characteristics in the volume of the flows of heat carriers taking into account the surface forces. *International Journal of Heat and Technology.* 2019. № 36(1). P. 7. DOI: https://doi.org/10.18280/ijht.370101

– the degree (–*X*) is annalytical expression of the Prandtl turbulent number, which is the ratio of pulse coefficients $A\tau$ and heat transfer Aq in the middle of the coolant flow, which obeys the logarithmic law of distribution and correlates with its classical studies[116];

– when deriving the index of degree (–*X*) (3.39 or 3.41) used modern research in the field of MDM, which equated the Stokes force to the forces of surface tension. We used the classical Stokes formula[117], as well as the values of turbulent viscosity of the coolant and analytical approaches of J. Boussinesq[118, 119];

– in formula (3.41) contains the Reynolds number, which is close to his classical studies[120];

– equation (3.30) in form is annalytical expression of Nusselt's empirical equations[121];

– current research in the field of fluid mechanics[122] takes into account that LBL completely controls its turbulent part.

This is shown by the similarity numbers we derived earlier[123]. The molecular number *Bl* simultaneously serves as a "constant" and a number responsible for the molecular component of the flow, which is sensitive to changes in external factors (temperature, concentration of nanoparticles, etc.). $Bl_{turb.}$ number responsible for the turbulent component of the flow, which depends on the ratio of the thermal velocity of molecules to the linear velocity *V*.

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[116] Prandtl L. Über die ausgebildete Turbulenz. *ZAMM.1925*. № 5. P. 137.

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[123] Bilonoga Y., Maksysko O. The laws of distribution of the values of turbulent thermo-physical characteristics in the volume of the flows of heat carriers taking into account the surface forces. *International Journal of Heat and Technology*. 2019. № 36(1). P. 7. DOI: https://doi.org/10.18280/ijht.370101

The molecular and turbulent numbers *Bl* and Bl_{turb} are dependent on the surface tension forces that dominate in the presence of nanoparticles and which cannot be neglected in the nanofluid coolant. The Bl_{turb} number completely depends on the molecular number *Bl* due to the degree of turbulence (*–X*) of the refrigerant flow depending on external factors, such as temperature (Fig. 4).

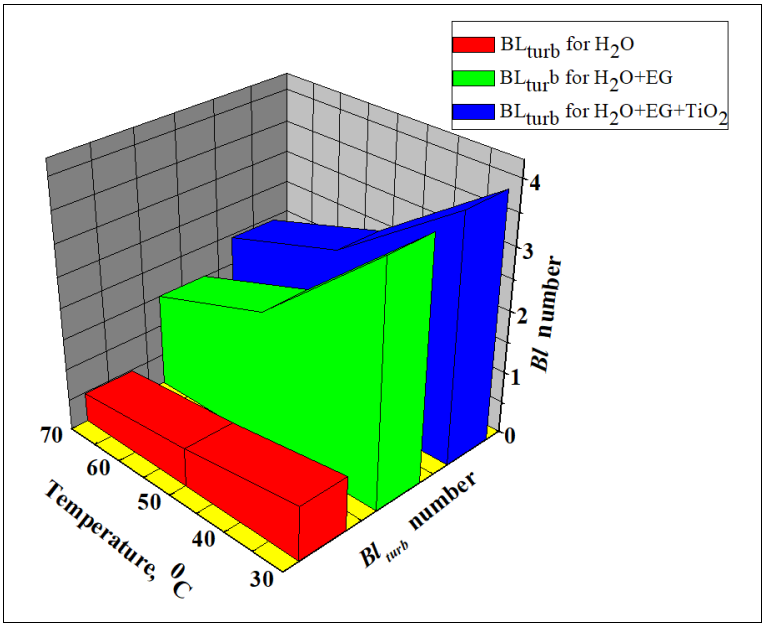

Fig. 4. Influence of the molecular *Bl* number on the turbulent Bl_{turb} number depending on temperature for various mixes of nanofluidic heat transfer fluids

When choosing coolants using nanoliquids, a new numerical equation (3.30) should be used, which takes into account the action of surface forces. But it isn the classical equations that new ones can be based, which contain a great deal of experimental material from many generations of scientists.

Our approach, which is based on the works of the classics, does not contradict but only unites them. Taking into account the influence of surface forces in heat exchange with nanoliquid heat carriers, as well as the application of the concepts of turbulent viscosity and thermal conductivity of flows allowed to obtain the corresponding result.

3.7. Algorithm for calculating a shell-and-tube heat exchanger using a new method for heating mix of (milk + 0.5 % pumpkin oil) with hot mix of (water + EG (60:40) +1.5 % TiO_2)

We present the calculation of the shell-and-tube heat exchanger according to the new method using a mixture of water + EG (60:40) + 1.5 % TiO_2. Thermophysical properties of mixtures (hot water + EG (60:40) + 1.5 % TiO_2) and (cold milk + 0.5 % pumpkin oil) are given in section 3.6.1 (Tables 3 and 4). The final results are presented in Table 5.

Table 5

Calculating a shell-and-tube heat exchanger using a new method

Parameter with formula and numbering	**Cold milk + 0.5 % pumpkin oil (c)**	**Hot mix of (water + EG (60:40) +1.5 % TiO_2) (h)**
Molecular number $Bl = \frac{\mu\sqrt{C_p \cdot 1^0 K}}{\sigma \cdot \cos\theta}$ (3.23)	1.756	1.941
The average speed, m.s^{-1} $V = \frac{\mathrm{Re}\mu}{d\rho}$	0.659	0.690
The exponent (-X) $-X = \frac{\ln \frac{a \cdot \sqrt{2\mathrm{Re}}}{0.769 \cdot 1/Bl}}{\ln \frac{\sqrt{C_p \cdot 1^0 K}}{V}}$; (3.41)	**0.759** →	**0.834** →
The turbulent number $Bl_{turb.} = \left(\frac{\sqrt{C_p \cdot 1^0 K}}{V}\right)^{-X}$ (3.36)	33.31	32.07
Turbulent thermal Conductivity, W.m^{-1}K^{-1} $k_{turb.} = (\sigma \cdot \cos\theta)\sqrt{C_P \cdot 1^0 K}\left(\frac{\sqrt{C_P \cdot 1^0 K}}{V}\right)^{-X}$ (3.30, 3.44)	**66.38** →	**84.24** →
Overall heat transfer coefficient, W.m^{-2}K^{-1} $U = \frac{1}{\frac{r_c}{k_{turb.c}} + \frac{\delta_w}{k_w} + 2\frac{1}{3000} + \frac{r_h}{k_{turb.h}}}$ (3.17)	920.81	

$2 \cdot \frac{1}{3000}$ − thermal resistance of contaminants (Table 5);

$r_c = d_c/2 = 21.10^{-3}/2 = 10.5.10^{-3}$m. (radius in the tube space of the shell-and-tube heat exchanger);

$r_h = d_h/2 = 25.10^{-3}/2 = 12.5.10^{-3}$m. (equivalent radius of the annular space).

Increase in overall heat transfer coefficient compared to pure water and pure milk: $\Delta_{\text{water + EG (60:40)+1.5 \%TiO}_2} = \frac{920.81 - 788.7}{788.7} \cdot 100\ \% = 16.75\ \%;$

$\Delta_{\text{water + EG (60:40)}} = \frac{890.6 - 788.7}{788.7} \cdot 100\ \% = 12.86\ \%;$

In work[124] we presented express calculations of the plate heat exchanger using nano-liquid mixtures. Note that the results of the full calculation of this shell-and-tube heat exchanger completely coincide with the results of our new express method in section 3.6.1.

CONCLUSIONS

1. Classical numerical equations, which are widely used for the calculation and selection of heat exchangers that work with nanofluids, give an error of (15–20)% or more. This leads to the fact that during their operation, thermal energy is partially lost with negative environmental consequences.

2. The motion of nanoparticles in the turbulent flow of liquid coolant is modeled taking into account surface forces. Froud, Reynolds, Euler numbers and the surface criterion in the laminar boundary layer (LBL) were calculated. It ishown that the surface forces are several orders of magnitude higher than the rest.

3. It is shown that the calculation of heat exchangers on the basis of classical similarity numbers, which do not contain surface characteristics, can not describe with sufficient accuracy the heat transfer process using nanofluids, because surface forces are dominant.

4. A new formula is obtained for the qualitative estimation of the average LBL thickness that occurs around a turbulently moving solid nanoparticle. A number of qualitative correlations are explained that explain the behavior of solid nanoparticles in aurbulent liquid medium.

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[124] Bilonoga Y., Stybel V., Maksysko O., Drachuk U. A New Universal Numerical Equation and a New Method for Calculating Heat-Exchange Equipment using Nanofluids. *International Journal of Heat and Technology*. 2020. № 38 (1). С. 164. DOI: https://doi.org/10.18280/ijht.380117

5. A new approach to heat transfer processes using the values of turbulent viscosity and thermal conductivity of liquid coolant is considered.

6. New formulas for analytical calculation of the degree of turbulence of the liquid coolant flow under the action of surface forces and Stokes resistance forces on nanoparticles are obtained on the basis of MDM.

7. The physical content of new numbers of similarity *Bl* and Bl_{turb}. Containing surface characteristics of heat carrier flows is considered and their effective application for express calculations of heat exchange equipment is shown.

8. A new rapid method for estimating the efficiency of nanofluids coolants in heat exchangers based on aew numerical equation using the new similarities numbers *Bl* and Bl_{turb} is presented.

9. The method presented by ushows that the mix H_2O + EG (60:40) increases the heat transfer of water by + 12.86 %, and the mix (H_2O + EG (60:40) + 1.5 % TiO_2) and (+ 0.5 % of pumpkin seed oil) – by + 16.75 %. This is confirmed by experiments and our computer simulations. The classic express method shows a negative result of – 4.5 % and – 1.2 %, respectively.

10. An example of the calculation of shell-and-tube heat exchanger for heating cold milk with hot water with the addition of mixes (H_2O + EG (60:40) + 1.5 % TiO_2) and (milk + 0.5 % pumpkin oil) completely confirms previous express calculations.

SUMMARY

The section of the monograph highlights the issues of correct calculation and selection of heat exchangers when using nanofluids for heat treatment of liquid-phase food products. The model of motion of aetal nanoparticle in aurbulent flow of aiquid heat carrier taking into account the action of surface forces is presented. The similarity numbers of Euler, Reynolds, Froud and the surface criterion (*Su*) in the LBL around the metal nanoparticle are calculated, which show the absolute dominance of surface forces. A new approach to heat transfer processes is presented, taking into account the turbulent viscosity and thermal conductivity of nanofluid coolant flows. The physical content of new numbers of similarity *Bl* and Bl_{turb}, as well as their use in the new numerical equation for the calculation of heat exchangers are presented. A new rapid method for evaluating the efficiency of nanorluids in heat exchangers, which shows that the mix H_2O + EG (60:40) increases the heat transfer coefficient by + 12.86 %, and the mix H_2O + EG (60:40) + 1. 5 % TiO_2) and (milk + 0.5 % pumpkin oil) – by + 16.75 % compared to water, which corresponds to experiments and

computer simulations, and the classic express method shows a deterioration of – 4.5 % and, accordingly, by -1.2 %, which contradicts the experiments.

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