

THE USE OF COMPUTER-INTEGRATED TECHNOLOGIES IN THE PROCESS OF TEACHING PHYSICS

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Abstract. Currently, there is a need to design an open informational and educational environment that will ensure a decent level of organization of the educational process in the mixed and distance forms of learning, make it flexible and personalized. The use of the Mathcad mathematical package when solving problems is an effective tool for improving the quality of the professional education in general and physics in particular. Facilitating learning due to the automation of calculations increases the level of students' mastery of the physical essence of the tasks, the level of students' educational achievements, and their motivation for the learning process.

The purpose of the work is the development of methodical material for optimizing the process of learning physics, which should help students overcome barriers associated with the imperfect educational skills and skills in the field of mathematics, simplify mathematical calculations and focus all attention on the physical essence of the task. Calculations of the electrical circuits of the alternating current are quite typical tasks. Branched electrical circuits of the alternating current are described by cumbersome mathematical functions of a complex variable. They are well suited to illustrate the benefits of the Mathcad system. The algorithm for solving the problem in this case consists of three quarters of the analysis of the electrical circuit, and the mathematical calculation is reduced to the solution of the system of equations [14]. Their exact solution is quite a difficult task, however, to solve most engineering problems, it is enough to construct their approximate solutions. When calculating real electrical

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circuits, it is necessary to take into account voltage fluctuations, changes in the parameters of electrical circuit elements caused by heating during the passage of current, accuracy classes of resistors, inductors, and capacitors. Therefore, such calculations are performed with approximate parameter values, so it is necessary to estimate the limit value of the calculation error. But with the general simplicity of setting the problem of error estimation, it is the mathematical calculation that causes significant difficulties, which is connected with the differentiation of cumbersome functions of a complex variable. A set of differential calculus functions is built into the Mathcad package, using which the student will successfully cope with the task, despite the complexity of its mathematical component.

A method of researching the frequency characteristics of the alternating current electric circuits is proposed. Its use makes it possible to determine the main frequency characteristics of an electric circuit quite effectively and to estimate their errors.

As a result of research, there is no need to explicitly find the roots of complex equations in this method. The resonant frequencies of the oscillating circuit are determined by the method of optimizing the mathematical model. The initial approximations of resonance frequencies are determined from the resonance curve and refined to the required accuracy by means of the Mathcad system. The limit values of the oscillating contour parameter errors are estimated by the classical method of logarithmization, which has been modernized for the function of a complex variable.

It was found that at resonant frequencies of the circuit, the value of the relative error of the current increases sharply. It is shown that for the function of the relative error of the current force in the electric circuit from the frequency $\varepsilon = f1(\nu)$ at the resonance frequencies of the circuit, a series of peaks is observed, which correlate with the change in the current strength on the resonance curve $I = f2(\nu)$.

Practical implications. In most cases, the problems of electromagnetism can be solved accurately using a wide range of analytical methods. However, to simplify calculations when investigating the frequency characteristics of the branched electric circuits and the oscillating circuits, it is advisable to use approximate numerical methods of the Mathcad system. *Value/originality.* The proposed method can be useful in solving a number of applied problems in physics, radio engineering, and electrical engineering.

1. Introduction

The full-scale war in Ukraine and the introduction of martial law had a detrimental effect on the field of education. Every citizen's right to education is under threat. Teachers once again faced the question of improving the teaching of educational material, expanding the possibilities of its acquisition and processing by students of higher education, regardless of the form of education. An increase in the number of hours for independent processing of educational material by students leads to a review of the methodical support of the educational process, modernization of practical classes with the active introduction of information technologies.

Computer mathematics systems include Mathematica, Derive, Mathcad, MatLab, Maple, and others.

Consider the Mathcad system. The Mathcad mathematical package works under the Windows operating system. The Mathcad work window is similar to the Word work window. This allows a novice user to quickly learn how to work on a computer, implement various complex mathematical models, without delving into the intricacies of programming as in the traditional programming languages (Pascal, Python, C++, etc.). The main advantages of the Mathcad system over other systems [2; 6; 8; 12; 18]:

- generally accepted form of writing mathematical symbols, operations, formulas and graphs is used;
- there is no hidden information, everything is displayed on the screen, the output results have the same appearance as on the display screen;
- to enter simple mathematical expressions, it is enough to simply print them on the worksheet of the system. Writing equations is simplified with the help of special toolbars that contain various mathematical operators, operands and templates;
- Mathcad makes it possible to build graphs, calculate derivatives and integrals or operate with other mathematical expressions, by filling in only the free fields in the proposed templates (special forms);
- computational algorithms have a modular structure;
- a library of mathematical functions and its own programming language are built into Mathcad;
- Mathcad numerical algorithms use standard and well-studied reliable methods;
- performs calculations with functions of a real and complex variable;

- Mathcad numerical algorithms use standard and well-studied reliable methods;
- Mathcad has its own reference systems with detailed descriptions of all topics. With the help of reference books, you can study any topic, they are equipped with various illustrative materials;
- a set of standard procedures and programs that are most often used in Mathcad is designed in the form of a set of easily accessible documents – “cheat sheets” (QuickSheet). To solve many specific problems, it is possible to find a suitable blank in the “cheat sheets” and transfer it to your working document;
- Mathcad has its own Electronic Books, which contain a variety of information, a large number of useful formulas, constants and graphs, which are easily transferred to a working document;
- carrying out mathematical calculations in numerical form (the result of calculations is a number) and analytical transformations over mathematical expressions (the result of transformations is an analytical expression);
- the package has the ability to perform calculations with dimensional physical units in the international system of units (SI).

The Mathcad system is a powerful and at the same time simple universal environment for solving problems in the various fields of science and technology [2; 6; 8; 9; 12; 18], mathematics and statistics [1; 5], physics and ecology [1; 7; 8; 14; 15; 17; 18], finance and economics [1; 5]. Mathcad remains one of the systems in which the description of the solution of mathematical problems is given using ordinary mathematical formulas and symbols. Mathcad allows you to perform both numerical calculations and symbolic transformations, has extremely developed graphic tools.

Therefore, it is advisable to use the Mathcad system in physics when:

- performing cumbersome calculations;
- performing analytical transformations on expressions;
- solving equations and their systems;
- solving differential equations and their systems;
- differentiation and integration of complex mathematical functions;
- processing of physical experiment results and error estimation;
- construction of graphs (two-dimensional, three-dimensional) based on the results of theoretical calculations or the results of a physical experiment;
- modeling of physical phenomena.

The use of the Mathcad software package during the study of the academic discipline “Physics” makes it possible to effectively solve problems of the various levels of complexity, not only physical, but also professional, which require a complex mathematical apparatus. Quite often, cumbersome mathematical calculations relegate the essence of physical phenomena and processes to the background. The Mathcad system allows you to simplify cumbersome mathematical calculations, and therefore does not distract from the physical content of the task and saves time for its solution. Graphs constructed using the Mathcad system contribute to reliable assimilation of educational material and a deeper understanding of physical phenomena and laws.

As an example, consider the possibilities of using Mathcad in the process of studying the section “Alternating electric current”, namely, the calculation of branched electric circuits of alternating current, the study of frequency characteristics and the estimation of calculation errors, the knowledge of which is important in the future professional activity of specialists of various specialties of the Faculty of Engineering and Technology.

In most cases, the problems of electromagnetism can be solved accurately using a wide range of analytical methods. However, to facilitate calculations in certain cases, it is advisable to use approximate numerical methods, in particular, optimization methods of the Mathcad system.

Resonance is a mode of operation of an electric circuit containing inductances and capacitances, in which the reactance or reactance of the circuit is zero.

$$X = X_L - X_C = 0. \quad (1)$$

In relation to the external circuit, the oscillating circuit acts as a purely active resistance, and therefore the voltage and current at the input of the circuit coincide in phase; accordingly, the reactive power at the circuit terminals is zero. The resonance mode can occur only when the frequency of external oscillations of the current is equal to the natural frequency of oscillations of the circuit. Therefore, the resonance frequency of the circuit is determined by the values of resistance, capacity and inductance.

Resonance of voltage and current in alternating current circuits is described by a simple mathematical apparatus [11; 13; 16]. The calculation of branched circuits of the alternating current is conveniently carried out in a complex form. But the situation becomes much more complicated when analyzing the phenomenon of resonance in the alternating current circuits with mixed connections of inductors, capacitors and active resistances.

Calculations become cumbersome, therefore they require effective methods and it is advisable to carry them out on a computer.

The ratings of the elements of electrical circuits (sources, resistors, inductors, capacitors) have certain tolerances, therefore, when examining the frequency characteristics of the alternating current circuits, in addition to the values of the resonance frequency, the current in the branches of the electrical circuit, and the voltage on the circuit elements, it is necessary to estimate the errors of these physical quantities [3].

A number of software have been developed to simulate the operation of electric circuits: Electronics Workbench, Multisim, Micro-Cap, Circuit Magic [6; 8]. They make it possible to obtain general frequency characteristics of electrical circuits, but when it is necessary to obtain detailed characteristics, they build mathematical models of the operation of electrical circuits and examine them on a computer. Matlab or Mathcad are most often used to do this. Simulink subroutine is built into Matlab for developing models. Elements of the SimPowerSystems built-in library are used to model electrical circuits in the Simulink routine. The system is quite a powerful software product, but it requires programming skills. Unlike Matlab, the Mathcad system has much less capabilities, but it has a graphical interface for entering information and does not require programming skills. In the Mathcad environment, the entries of mathematical expressions are close to the entries in ordinary mathematics, and the use of constants, variables and functions does not require additional explanations. The system has a large set of built-in functions, wide graphic capabilities, and enables working with complex variable functions. Therefore, the Mathcad environment has all the necessary set of mathematical tools for the study and calculation of electrical circuits of alternating current.

2. Resonance in the simple electric circuits

Before analyzing the resonance of the complex electric circuits with a mixed connection of elements, we will first consider cases of the resonance of voltages and currents in the simple electric circuits (simple circuits). Note that the resonance of voltage and currents in the simple electric circuits and the resonance in the electric circuits with a mixed connection of elements is described by the same mathematical apparatus, and it is based on the same physical laws.

In the simple alternating current circuits, there can be two types of resonance: voltage (series); currents (parallel).

Voltage resonance. Voltage resonance [11; 13; 16] is observed in an electric circuit with a series connection of inductors and capacitors (series oscillating circuit), Figure 1.

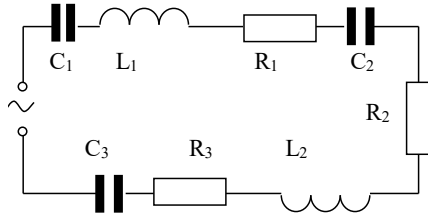


Figure 1. Voltage resonance

The resonance frequency of the circuit during voltage resonance is recognized only by the value of the inductive and capacitive resistance of the circuit [11; 13; 16]. At resonance, the resistances of the reactive elements are the same:

$$X_L = X_C,$$

$$\omega_p L = \frac{1}{\omega_p C},$$

$$\omega_p = \omega_0 = \frac{1}{\sqrt{LC}}, \quad (2)$$

$$v_p = \frac{\omega_p}{2\pi} = \frac{1}{2\pi\sqrt{LC}}, \quad (3)$$

where L – is the total inductance, C – is the total capacity, v_p – is the resonant frequency ω_p – is the resonant cyclic frequency.

At resonance, the resistances of the reactive elements are the same:

$$\omega_p L = \frac{1}{\omega_p C} = \sqrt{\frac{L}{C}} = \rho, \quad (4)$$

where ρ – is the wave resistance.

The voltage values across the inductor and the capacitor will be the same:

$$U_L = U_C$$

$$U_L = I_p \omega_p L = I_p \rho, \quad (5)$$

$$U_c = \frac{I_p}{\omega_p C} = I_p \rho, \quad (6)$$

where I_p – is the current at resonance.

At resonance, the value of the voltage across the active resistance is equal to

$$U_R = I_p R. \quad (7)$$

A physical quantity that is numerically equal to the ratio of the reactive voltage component (capacitance or inductance voltage) to the active one at resonance is called the Q factor of the circuit.

$$Q = \frac{U_L}{U_R} = \frac{U_c}{U_R} = \frac{\omega_p L}{R} = \frac{1}{\omega_p C} = \frac{1}{R} \sqrt{\frac{L}{C}} = \frac{\rho}{L}, \quad (8)$$

and the inverse of the factor is called the extinction coefficient:

$$d = \frac{U_R}{U_L} = \frac{U_R}{U_c} = \frac{R}{\omega_p L} = R \omega_p L = R \sqrt{\frac{C}{L}} = \frac{L}{\rho}. \quad (9)$$

For the electrical circuit shown in Figure 1, we have:

$$R = R_1 + R_2 + R_3 = \sum_{i=1}^3 R_i$$

$$L = L_1 + L_2,$$

$$C = \frac{C_1 C_2 C_3}{C_1 C_2 + C_1 C_3 + C_2 C_3}.$$

The total resistance of such an electric circuit in the state of voltage resonance is purely active in nature and the smallest in magnitude.

$$Z = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C} \right)^2}, \quad (10)$$

Accordingly, the current and power consumed reach the highest values.

Characteristic signs of the voltage resonance:

- the total resistance of the electric circuit is minimal and equal to the active resistance of the circuit;

- the effective value of the voltage on the active resistor is equal to the effective value of the applied external voltage;

- voltage and current are in the same phases;

- the supplied energy is transmitted only to the active resistance;
- useful power of the external source is maximum.

Resonance of currents. The phenomenon of setting the minimum value of the current in the unbranched part of the circuit with the parallel connection of circuit elements (inductance coils, capacitors) is called current resonance.

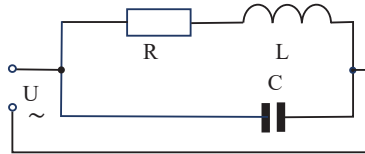


Figure 2. Resonance of currents

Resonance of currents is observed in an electric circuit with a parallel connection (parallel oscillating circuit), Figure 2.

If the oscillating circuit is ideal, then the active resistance $R = 0$ and the resonant frequency

$$\omega_p = \omega_0 = \frac{1}{\sqrt{LC}}. \quad (11)$$

In the real electric circuits, the active resistance of the inductance is $R \neq 0$, so the resonant frequency is determined from the ratio [11; 13; 16]:

$$\frac{\omega L}{R^2 + (\omega L)^2} - \omega C = 0,$$

$$\omega_p = \sqrt{\frac{1}{LC} - \left(\frac{R}{L}\right)^2}. \quad (12)$$

If the active resistance is included in the capacitor branch (Figure 3),

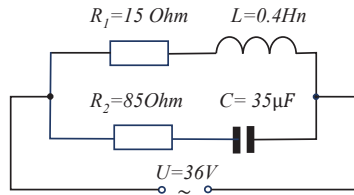


Figure 3. Resonance of currents (general case)

then the resonant frequency is determined from the ratio:

$$\frac{\omega_p L}{R_1^2 + (\omega_p L)^2} = \frac{1/\omega_p C}{R_1^2 + (1/\omega_p C)^2},$$

$$\omega_p = \frac{1}{\sqrt{LC}} \sqrt{\frac{L/C - R_1^2}{L/C - R_2^2}} = \frac{1}{\sqrt{LC}} \sqrt{\frac{\rho^2 - R_1^2}{\rho^2 - R_2^2}},$$

$$\nu_p = \frac{\omega_p}{2\pi} = \frac{1}{2\pi\sqrt{LC}} \sqrt{\frac{L/C - R_1^2}{L/C - R_2^2}} = \frac{1}{2\pi\sqrt{LC}} \sqrt{\frac{\rho^2 - R_1^2}{\rho^2 - R_2^2}}. \quad (13)$$

Let's analyze the ratio (13). Resonance of currents can flow at

$$\frac{L/C - R_1^2}{L/C - R_2^2} \geq 0. \quad (14)$$

If
$$\frac{L/C - R_1^2}{L/C - R_2^2} > 0,$$

then the systems of inequalities must be fulfilled:

$$\begin{cases} L/C > R_1^2 \\ L/C > R_2^2 \end{cases} \text{ or } \begin{cases} L/C < R_1^2 \\ L/C < R_2^2 \end{cases}. \quad (15)$$

The systems of inequalities (15) are equivalent to the system of inequalities

$$\begin{cases} \rho > R_1 \\ \rho > R_2 \end{cases} \text{ or } \begin{cases} \rho < R_1 \\ \rho < R_2 \end{cases}. \quad (16)$$

If $R_1 = R_2 \neq \rho$, then the resonance is described by dependence (11) for an ideal circuit.

An interesting case is when $R_1 = R_2 = \rho$. The contour resonates at all frequencies and we observe the phenomenon of so-called "indifferent" resonance.

When $\frac{L/C - R_1^2}{L/C - R_2^2} < 0$, resonance in the electric circuit does not occur.

When the currents resonate in the branches of the inductor and the capacitor, a small current may flow, the value of which is determined by the active components of the resistance. If the inductor and capacitor are ideal, then there is no current in the branched part of the circuit, because the currents in the

inductor and capacitor are opposite in phase and compensate each other in the non-branched part of the circuit. However, the current in the inductance and capacity branches can reach much higher values than in the unbranched part.

The dielectric of the capacitor may also be imperfect. Then the capacitor is shunted by an active resistance and losses in the capacitor must be taken into account.

Example 1. Construct a resonance curve and determine the resonance frequency for an electric circuit (Figure 3).

We can find the resonant frequency in two ways: analytical (equation 13) and numerical methods of the Mathcad system. The numerical method is reduced to the solution of the optimization problem [3].

For solving optimization problems, two functions **minimize** and **maximize** are built into Mathcad. The first function defines the arguments of the objective function for the minimum value, the second – the maximum value. The functions have the format:

$$\text{minimize (f, x, y), maximize (f, x, y),}$$

where f – is the name of the function, x, y – are the arguments of the function.

The minimize and maximize functions are used in the Given block.

The Given block has the following structure:

- we define the target function;
- we set the initial approximate values of the resonance frequency (initial values are found from the resonance curve);
- we set the accuracy of calculations;
- open the Given block;
- we write down the restrictions imposed on the objective function through the signs of Boolean algebra;
- we write an expression with the minimize or maximize functions to find the optimal value of the arguments of the optimization tasks (in the Given computing block, the minimize or maximize functions can only be used once);
- we determine the optimal value of the function argument.

We will make calculations using the Mathcad system (Listing 1).

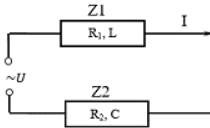
Therefore, the results of calculations by two methods agree within the limits of error.

ORIGIN := 1

Active resistance, Ohm $R := \begin{pmatrix} 15 \\ 5 \end{pmatrix}$ Inductivities, Hn $L := 0.4$
 Electrical capacity capacitors, F $C := 35 \cdot 10^{-6}$ Voltage, V $U := 36$

Numerical method

$i := \sqrt{-1}$ <== An imaginary unit



Equivalent electrical circuit

Functions for calculating the resistance of sections and a complete circle

$$Zk1(\nu) := R_1 + 2\pi\nu \cdot L \cdot i \quad Zk2(\nu) := R_2 - \frac{1}{2\pi \cdot \nu \cdot C} \cdot i \quad Zk(\nu) := \frac{Zk1(\nu) \cdot Zk2(\nu)}{Zk1(\nu) + Zk2(\nu)}$$

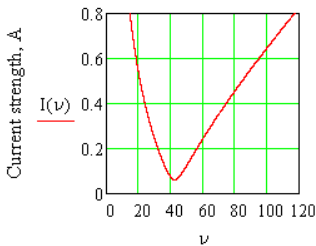
Construction of resonance curve

$\nu := 1, 1.01..120$

$$Ik(\nu) := \frac{U}{Zk(\nu)} \quad \text{<== Function for calculating the complex value of the current}$$

$$I(\nu) := |Ik(\nu)| \quad \text{<== Function for calculating the value of the modulus of the current}$$

Resonance curve



Resonant frequencies ν_p , Hz

$$\nu_1 := 45 \quad \text{Given} \quad 1 \leq \nu_1 \leq 400$$

$$\nu_{pz} := \text{Minimize}(I, \nu_1) = 42.5296$$

Analytical method

$$\frac{\frac{L}{C} - (R_1)^2}{\frac{L}{C} - (R_2)^2} = 0.982 \quad \nu_{pa} := \frac{1}{2 \cdot \pi \cdot \sqrt{L \cdot C}} \cdot \sqrt{\frac{\frac{L}{C} - (R_1)^2}{\frac{L}{C} - (R_2)^2}} = 42.1613$$

3. Frequency characteristics of an alternating current circuit with a mixed connection of elements

Resonance in alternating current electric circuits with a mixed connection of elements has its own specific features [2; 3; 10]. Resonance phenomena can occur in individual branches or circuits of alternating current circuits with a mixed connection of elements (complex circuits), and the general frequency characteristic is the result of superposition of individual resonances on branches and circuits. The elementary processes occurring in the elements of such circuits have been studied and described in detail in the scientific and scientific-methodical literature, but their analysis presents a number of difficulties caused by computational procedures and process analysis.

Consider an example (Figure 4).

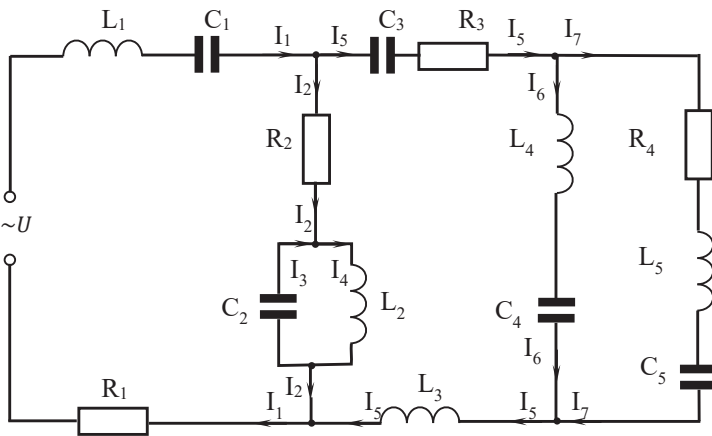


Figure 4. An electric circuit with a mixed connection of elements

For an electric circuit, investigate resonance characteristics. Determine the voltages and currents on the circuit elements at a frequency of 50 Hz and resonance frequencies. Estimate current errors in the unbranched section of the circuit. Electric circuit parameters:

$$R_1 = 4 \pm 0,1 \text{ Ohm}, R_2 = 12 \pm 0,3 \text{ Ohm}, R_3 = 8 \pm 0,2 \text{ Ohm}, R_4 = 7 \pm 0,1 \text{ Ohm},$$

$$L_1 = 0,4 \pm 0,002 \text{ Hn}, L_2 = 0,1 \pm 0,001 \text{ Hn}, L_3 = 0,35 \pm 0,002 \text{ Hn}, L_4 = 0,4 \pm 0,002 \text{ Hn},$$

$$L_5 = 0,1 \pm 0,0061 \text{ Hn}, C_1 = 50 \pm 0,2 \mu\text{f}, C_2 = 12 \pm 0,1 \mu\text{F}, C_3 = 1 \pm 0,01 \mu\text{F}, \\ C_4 = 5 \pm 0,1 \mu\text{F}, C_5 = 15 \pm 0,1 \mu\text{F}, U = 220 \pm 5 \text{ V}, \nu = 50 \pm 0,002 \text{ Hz}.$$

Let's develop a mathematical model of an electric circuit. We will record mathematical functions and expressions in a form close to the format of the Mathcad system. The model is based on the symbolic method of calculating electric circuits. Let's replace the elements of the branches of the electric circuit with the complex resistances equivalent to them (Figure 5).

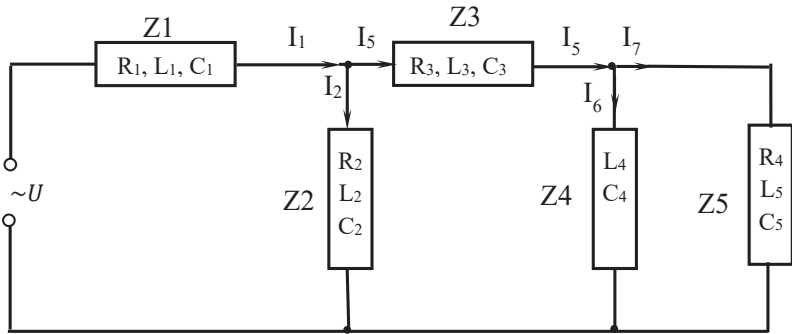


Figure 5. Equivalent electrical circuit

Let's determine the complex resistances of the branches of an electric circuit as a function of frequency.

$$\bar{Z}1(\nu) = R_1 + \left(2\pi\nu L_1 - \frac{1}{2\pi\nu C_1} \right) i; \quad (17)$$

$$\bar{Z}2(\nu) = R_2 + \frac{2\pi\nu L_2 i \cdot \left(\frac{-i}{2\pi\nu C_2} \right)}{2\pi\nu L_2 i + \left(\frac{-i}{2\pi\nu C_2} \right)} = R_2 - \frac{2\pi\nu L_2 i}{4\pi^2 \nu^2 L_2 C_2 - 1}; \quad (18)$$

$$\bar{Z}3(\nu) = R_3 + \left(2\pi\nu L_3 - \frac{1}{2\pi\nu C_3} \right) i; \quad (19)$$

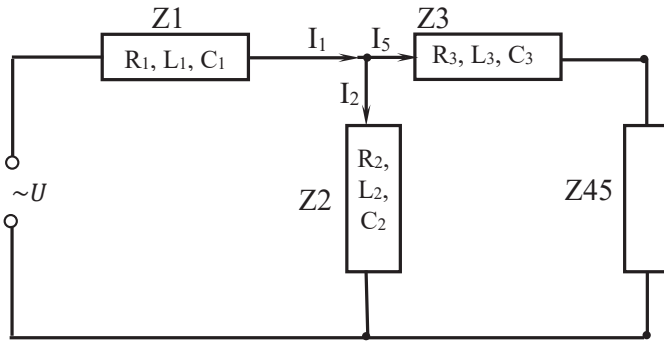
$$\bar{Z}4(\nu) = \left(2\pi\nu L_4 - \frac{1}{2\pi\nu C_4} \right) i; \quad (20)$$

$$\bar{Z}_5(v) = R_4 + \left(2\pi\nu L_5 - \frac{1}{2\pi\nu C_5} \right) i; \quad (21)$$

where $i = \sqrt{-1}$ – is an imaginary unit.

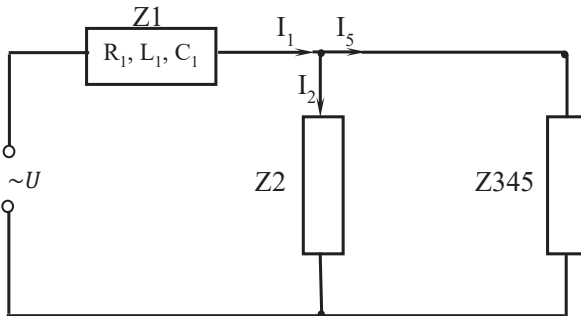
By means of the equivalent substitutions, we will reduce the electric circuit to a simple form.

Let's replace the complex supports \bar{Z}_4, \bar{Z}_5 with their equivalent \bar{Z}_45 :



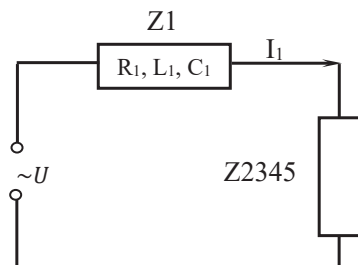
$$\bar{Z}_{45}(v) = \frac{\bar{Z}_4(v) \cdot \bar{Z}_5(v)}{\bar{Z}_4(v) + \bar{Z}_5(v)}. \quad (22)$$

Let's replace the complex supports \bar{Z}_3, \bar{Z}_45 with their equivalent \bar{Z}_{345} :



$$\bar{Z}_{345}(v) = \bar{Z}_3(v) + \bar{Z}_{45}(v). \quad (23)$$

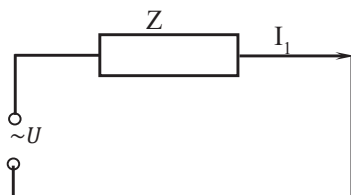
Let's replace the complex supports \bar{Z}_2, \bar{Z}_{345} with their equivalent \bar{Z}_{2345} :



$$\bar{Z}_{2345}(\nu) = \frac{\bar{Z}_2(\nu) \cdot \bar{Z}_{345}(\nu)}{\bar{Z}_2(\nu) + \bar{Z}_{345}(\nu)}. \quad (24)$$

Let's replace the complex supports $\bar{Z}_1, \bar{Z}_{2345}$ with their equivalent \bar{Z} :

$$\bar{Z}(\nu) = \bar{Z}_1(\nu) + \bar{Z}_{2345}(\nu). \quad (25)$$



So, the dependence of the total complex resistance of the electric circuit on the frequency is described by the function

$$\bar{Z}(\nu) = \bar{Z}_1(\nu) + \bar{Z}_{2345}(\nu),$$

and, accordingly, the modulus of total resistance:

$$Z(\nu) = |\bar{Z}(\nu)|. \quad (26)$$

The dependence of the complex value of the current on the frequency in an electric circuit is described by the dependence

$$\bar{F}(\nu) = \frac{U}{\bar{Z}(\nu)}, \quad (27)$$

and the current module

$$F(\nu) = \left| \frac{U}{\bar{Z}(\nu)} \right|. \quad (28)$$

We implement a mathematical model of an electric circuit in Mathcad (Listing 2).

Research of resonance in an electric circuit with mixed connection of elements

ORIGIN := 1

Active resistance, Om $R := \begin{pmatrix} 10 \\ 12 \\ 8 \\ 7 \end{pmatrix}$ Absolute errors, Om $\Delta R := \begin{pmatrix} 0.1 \\ 0.3 \\ 0.2 \\ 0.1 \end{pmatrix}$

Inductivities, Hn $L := \begin{pmatrix} 0.4 \\ 0.10 \\ 0.35 \\ 0.4 \\ 0.1 \end{pmatrix}$ Absolute errors, Hn $\Delta L := \begin{pmatrix} 0.002 \\ 0.001 \\ 0.002 \\ 0.002 \\ 0.001 \end{pmatrix}$

Electrical capacity capacitors, F $C := \begin{pmatrix} 50 \\ 12 \\ 15 \\ 5 \\ 15 \end{pmatrix} \cdot 10^{-6}$ Absolute errors, F $\Delta C := \begin{pmatrix} 0.2 \\ 0.1 \\ 0.1 \\ 0.1 \\ 0.1 \end{pmatrix} \cdot 10^{-6}$

Voltage, V $U := 220$ $\Delta U := 5$

Frequency, Hz $\Delta \nu := 0.002$

$i := \sqrt{-1}$ \Leftarrow An imaginary unit

Functions for calculating complex resistances of electric circuit branches (equations 17-21):

$$Z1_k(\nu) := R_1 + \left(2 \cdot \pi \cdot \nu \cdot L_1 - \frac{1}{2 \cdot \pi \cdot \nu \cdot C_1} \right) i$$

$$Z2_k(\nu) := R_2 - \frac{2 \pi \cdot \nu \cdot L_2 \cdot i}{4 \pi^2 \cdot \nu^2 \cdot L_2 \cdot C_2 - 1}$$

$$Z3_k(\nu) := R_3 + \left(2 \cdot \pi \cdot \nu \cdot L_3 - \frac{1}{2 \cdot \pi \cdot \nu \cdot C_3} \right) i$$

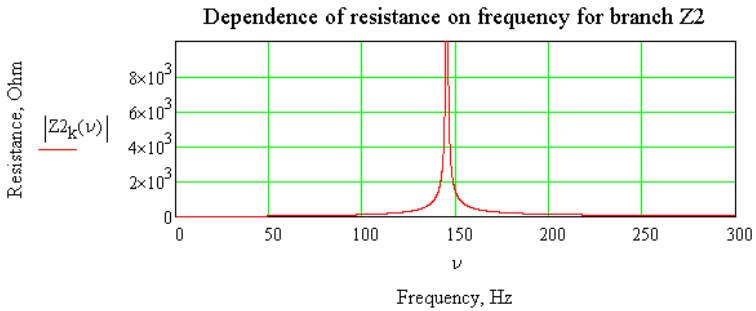
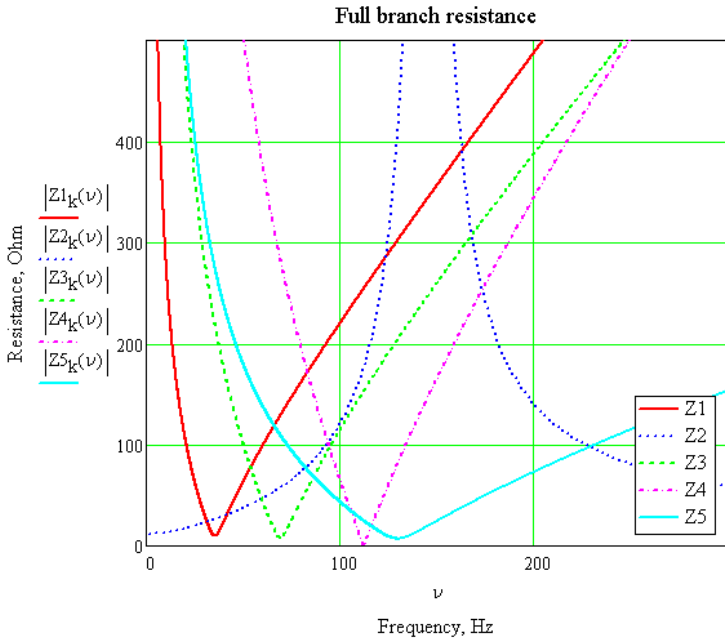
$$Z4_k(\nu) := \left(2 \cdot \pi \cdot \nu \cdot L_4 - \frac{1}{2 \cdot \pi \cdot \nu \cdot C_4} \right) i$$

(Continuation of listing 2)

$$Z5_k(\nu) := R_4 + \left(2 \cdot \pi \cdot \nu \cdot L_5 - \frac{1}{2 \cdot \pi \cdot \nu \cdot C_5} \right) i$$

Graphs of the dependence of the resistance of the branches of the electric circuit on the frequency

$$\nu := 1, 1.05 \dots 300$$



Functions for calculating the resistance of sections and a complete circle

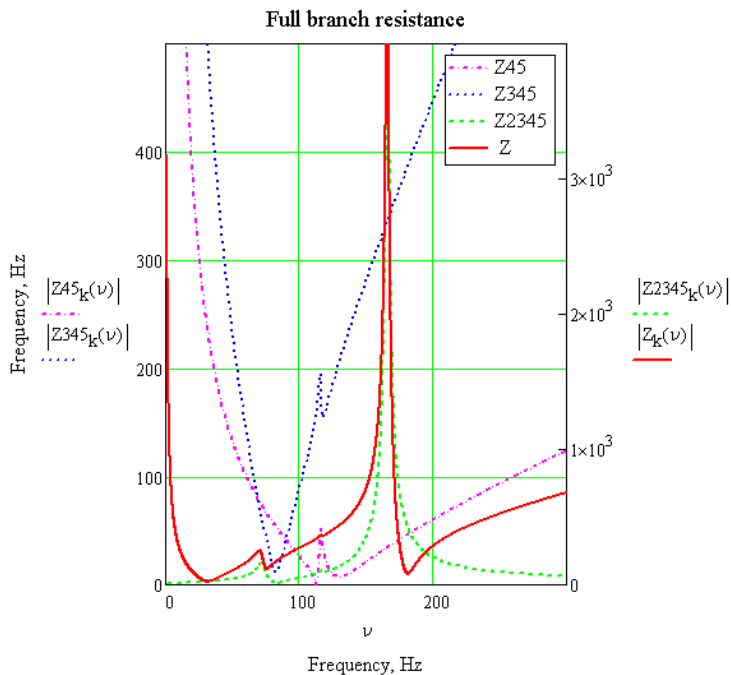
$$Z_{45k}(\nu) := \frac{Z_{4k}(\nu) \cdot Z_{5k}(\nu)}{Z_{4k}(\nu) + Z_{5k}(\nu)}$$

$$Z_{345k}(\nu) := Z_{3k}(\nu) + Z_{45k}(\nu)$$

$$Z_{2345k}(\nu) := \frac{Z_{2k}(\nu) \cdot Z_{345k}(\nu)}{Z_{2k}(\nu) + Z_{345k}(\nu)}$$

$$Z_k(\nu) := Z_{1k}(\nu) + Z_{2345k}(\nu)$$

The resistance of sections of the electric circuit varies in a wide range of 0-4000 Ohms. For clarity, we build graphs on different scales: along the main Y axis (interval 0-500 Ohms) and auxiliary (interval 1-4000 Ohms).

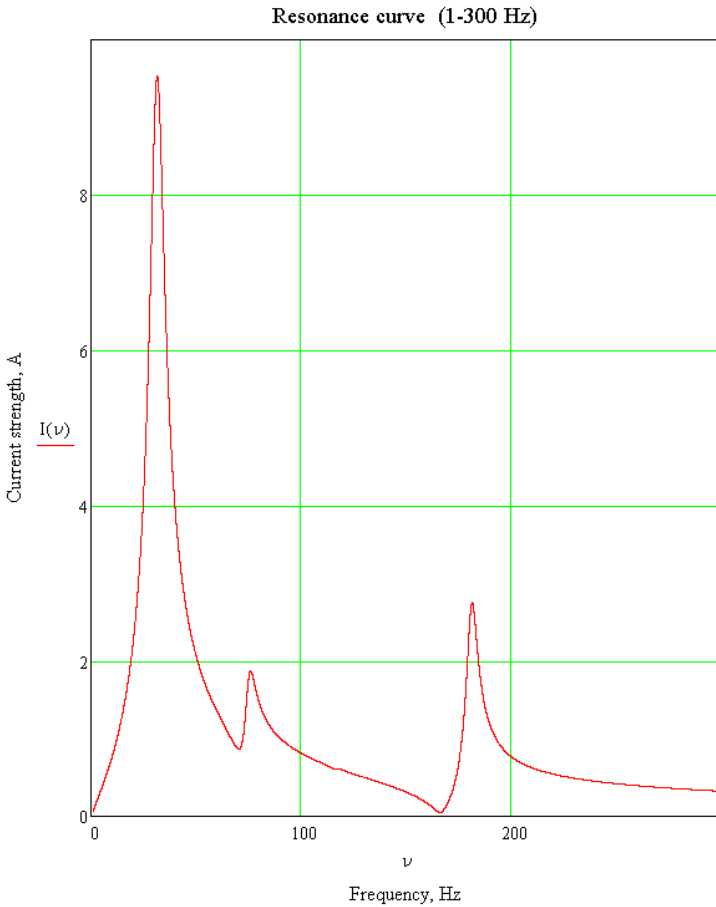


Construction of resonance curve

$$K(\nu) := \frac{U}{Z_k(\nu)} \quad \leftarrow \text{Function for calculating the complex value of the current}$$

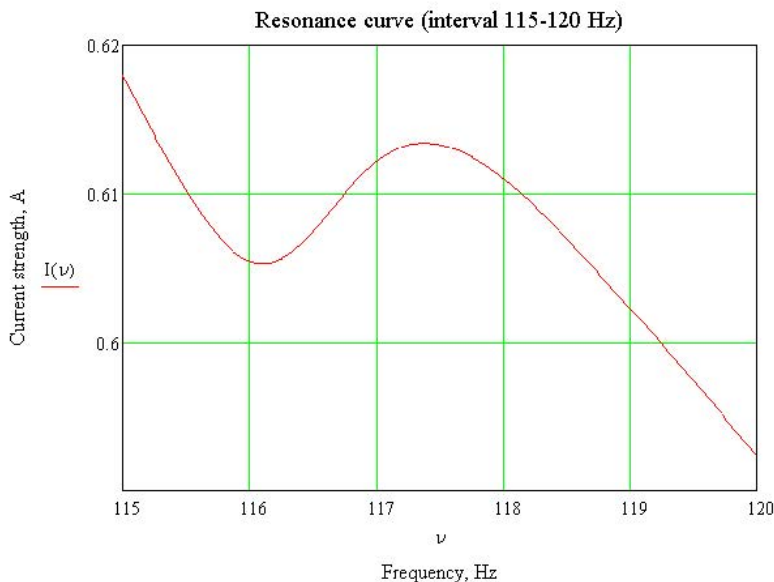
$$I(\nu) := \left| \frac{U}{Z_k(\nu)} \right| \quad \leftarrow \text{Function for calculating the value of the modulus of the current}$$

$$\nu := 1, 1.01 \dots 300$$



Checking the 115-120 Hz interval for the existence of critical points

$$\nu := 115, 115.01 \dots 120$$



Let's clarify the found approximations of resonant frequencies . To do this, we will use the built-in functions of the Mathcad system: **minimize(f,x)** and **maximize(f,x)**

$TOL := 1 \cdot 10^{-9}$	Resonant frequencies ν_p, Hz	
$\nu1 := 30$	Given $10 \leq \nu1 \leq 50$	
	$\nu1_p := \text{Maximize}(I, \nu1) = 31.54$	\Leftarrow Voltage resonance
$\nu2 := 65$	Given $50 \leq \nu2 \leq 73$	
	$\nu2_p := \text{Minimize}(I, \nu2) = 70.392$	\Leftarrow Resonance of currents
$\nu3 := 77$	Given $71 \leq \nu3 \leq 90$	
	$\nu3_p := \text{Maximize}(I, \nu3) = 75.989$	\Leftarrow Voltage resonance
$\nu4 := 110$	Given $110 \leq \nu4 \leq 117$	
	$\nu4_p := \text{Minimize}(I, \nu4) = 116.104$	\Leftarrow Resonance of currents

(Continuation of listing 2)

$$\begin{aligned}
 v5 &:= 116.5 \quad \text{Given} \quad 116 \leq v5 \leq 120 \\
 v5_p &:= \text{Maximize}(I, v5) = 117.379 \quad \Leftarrow \text{Voltage resonance} \\
 v6 &:= 150 \quad \text{Given} \quad 125 \leq v6 \leq 170 \\
 v6_p &:= \text{Minimize}(I, v6) = 166.141 \quad \Leftarrow \text{Resonance of currents} \\
 v7 &:= 175 \quad \text{Given} \quad 170 \leq v7 \leq 200 \\
 v7_p &:= \text{Maximize}(I, v7) = 181.442 \quad \Leftarrow \text{Voltage resonance}
 \end{aligned}$$

Currents in the branches of an electric circuit and currents on its elements for an industrial frequency of 50 Hz and resonant frequencies: 31.54; 70.392; 75.989; 116.104; 117.379; 166.141; 181.442 Hz.

Calculation of an electric circuit for an industrial frequency 50 Hz

$$\begin{aligned}
 v &:= 50 & U &= 220 \\
 Xl(L0) &:= 2 \cdot \pi \cdot v \cdot L0 \cdot i & & \Leftarrow \text{Functions for calculating the value of} \\
 Xc(C0) &:= \frac{-1}{2 \cdot \pi \cdot v \cdot C0} \cdot i & & \text{complex reactive resistance}
 \end{aligned}$$

Current strength in the unbranched section of the circle (element Z1), A

Complex current value	Current module
$I_{k1} := I_k(v) = 0.525 - 2.005i$	$I1 := I(v) = 2.073$

Phase shift between current and voltage, deg

$$\cos \left(\frac{\text{Re}(Z_k(v))}{|Z_k(v)|} \right) = 0.968 \quad \varphi := \frac{180}{\pi} \cdot \arccos \left(\cos \left(\frac{\text{Re}(Z_k(v))}{|Z_k(v)|} \right) \right) = 14.506$$

Stresses on the components of the element Z1, V

Complex voltage value	Voltage module
$U_{kr1} := I_{k1} \cdot R_1 = 5.248 - 20.052i$	$U_{r1} := U_{kr1} = 20.727$
$U_{kL1} := I_{k1} \cdot Xl(L_1) = 251.979 + 65.944i$	$U_{L1} := U_{kL1} = 260.465$
$U_{kC1} := I_{k1} \cdot Xc(C_1) = -127.654 - 33.407i$	$U_{C1} := U_{kC1} = 131.953$
$U_{Z1k} := I_{k1} \cdot Z_{1k}(v) = 129.573 + 12.484i$	$U_1 := U_{Z1k} = 130.173$

Voltage and current on Z2

$U_{Z2k} := I_{k1} \cdot Z_{2345k}(v) = 90.427 - 12.484i$	$U_2 := U_{Z2k} = 91.285$
$I_{k2} := \frac{U_{Z2k}}{Z_{2k}(v)} = 0.453 - 2.385i$	$I_2 := I_{k2} = 2.428$
$U_{kr2} := I_{k2} \cdot R_2 = 5.433 - 28.62i$	$U_{r2} := U_{kr2} = 29.131$

(Continuation of listing 2)

Voltage and current on the circuit $L_2 C_2$

$$ULk_2 := Ik_2 \left[\frac{Xl(L_2) \cdot Xc(C_2)}{(Xl(L_2) + Xc(C_2))} \right] = 84.994 + 16.136i \quad ULC2 := |ULk_2| = 86.512$$

$$ILk_2 := \frac{ULk_2}{Xl(L_2)} = 0.514 - 2.705i \quad IL_2 := |ILk_2| = 2.754$$

$$ICk_2 := \frac{ULk_2}{Xc(C_2)} = -0.061 + 0.32i \quad IC_2 := |ICk_2| = 0.326$$

Voltage and current on Z3

$$Ik_3 := \frac{UZ2k}{Z345_k(v)} = 0.072 + 0.38i \quad I3 := |Ik_3| = 0.387$$

$$Ukr_3 := Ik_3 \cdot R_3 = 0.576 + 3.039i \quad Ur_3 := |Ukr_3| = 3.093$$

$$UkL_3 := Ik_3 \cdot Xl(L_3) = -41.765 + 7.914i \quad UL_3 := |UkL_3| = 42.508$$

$$UkC_3 := Ik_3 \cdot Xc(C_3) = 80.604 - 15.273i \quad UC_3 := |UkC_3| = 82.038$$

$$UZ3k := Ik_3 \cdot Z3_k(v) = 39.414 - 4.321i \quad U_3 := |UZ3k| = 39.651$$

Voltage and current on Z45, Z4, Z5

$$U45k := Ik_3 \cdot Z45_k(v) = 51.013 - 8.164i \quad U45 := |U45k| = 51.662$$

$$Ik_4 := \frac{U45k}{Z4_k(v)} = 0.016 + 0.1i \quad U4 := U45 \quad U5 := U45$$

$$I4 := |Ik_4| = 0.101$$

$$UkL_4 := Ik_4 \cdot Xl(L_4) = -12.546 + 2.008i \quad UL_4 := |UkL_4| = 12.706$$

$$UkC_4 := Ik_4 \cdot Xc(C_4) = 63.559 - 10.171i \quad UC_4 := |UkC_4| = 64.368$$

$$Ik_5 := \frac{U45k}{Z5_k(v)} = 0.056 + 0.28i \quad I5 := |Ik_5| = 0.286$$

$$UkL_5 := Ik_5 \cdot Xl(L_5) = -8.796 + 1.759i \quad UL_5 := |UkL_5| = 8.971$$

$$UkC_5 := Ik_5 \cdot Xc(C_5) = 59.417 - 11.883i \quad UC_5 := |UkC_5| = 60.594$$

$$Ukr_4 := Ik_5 \cdot R_4 = 0.392 + 1.96i \quad Ur_4 := |Ukr_4| = 1.999$$

(Continuation of listing 2)

Formation of the final table

$$D := \begin{pmatrix} \text{"Element"} & \text{"Denomination: Ohm, Hn, F"} & \text{"Current, A"} & \text{"Voltage, V"} \\ \text{"R1"} & R_1 & I1 & U_{r1} \\ \text{"R2"} & R_2 & I2 & U_{r2} \\ \text{"R3"} & R_3 & I3 & U_{r3} \\ \text{"R4"} & R_4 & I5 & U_{r4} \\ \text{"L1"} & L_1 & I1 & U_{L1} \\ \text{"L2"} & L_2 & I_{L2} & U_{LC2} \\ \text{"L3"} & L_3 & I3 & U_{L3} \\ \text{"L4"} & L_4 & I4 & U_{L4} \\ \text{"L5"} & L_5 & I5 & U_{L5} \\ \text{"C1"} & C_1 & I1 & U_{C1} \\ \text{"C2"} & C_2 & I_{C2} & U_{LC2} \\ \text{"C3"} & C_3 & I3 & U_{C3} \\ \text{"C4"} & C_4 & I4 & U_{C4} \\ \text{"C5"} & C_5 & I5 & U_{C5} \end{pmatrix}$$

Summary table (frequency 50 Hz)

$$\nu = 50 \quad \text{Phase shift between current and voltage, deg} \quad \varphi = 14.51 \quad \cos\left(\frac{\pi}{180} \cdot \varphi\right) = 0.968$$

$$D = \begin{pmatrix} \text{"Element"} & \text{"Denomination: Ohm, Hn, F"} & \text{"Current, A"} & \text{"Voltage, V"} \\ \text{"R1"} & 10 & 2.073 & 20.727 \\ \text{"R2"} & 12 & 2.428 & 29.131 \\ \text{"R3"} & 8 & 0.387 & 3.093 \\ \text{"R4"} & 7 & 0.286 & 1.999 \\ \text{"L1"} & 0.4 & 2.073 & 260.465 \\ \text{"L2"} & 0.1 & 2.754 & 86.512 \\ \text{"L3"} & 0.35 & 0.387 & 42.508 \\ \text{"L4"} & 0.4 & 0.101 & 12.706 \\ \text{"L5"} & 0.1 & 0.286 & 8.971 \\ \text{"C1"} & 5×10^{-5} & 2.073 & 131.953 \\ \text{"C2"} & 1.2×10^{-5} & 0.326 & 86.512 \\ \text{"C3"} & 1.5×10^{-5} & 0.387 & 82.038 \\ \text{"C4"} & 5×10^{-6} & 0.101 & 64.368 \\ \text{"C5"} & 1.5×10^{-5} & 0.286 & 60.594 \end{pmatrix}$$

(Continuation of listing 2)

Summary table. The resonance frequency 31.54 Hz

Phase shift between current and voltage, deg $\varphi = 57.29 \cos\left(\frac{\pi}{180} \cdot \varphi\right) = 0.54$

"Element"	"Denomination: Ohm, Hn, F"	"Current, A"	"Voltage, V"
"R1"	10	9.539	95.388
"R2"	12	9.941	119.287
"R3"	8	0.474	3.793
"R4"	7	0.354	2.476
"L1"	0.4	9.539	756.129
"L2"	0.1	10.432	206.737
"L3"	0.35	0.474	32.887
"L4"	0.4	0.12	9.548
"L5"	0.1	0.354	7.01
"C1"	5×10^{-5}	9.539	962.681
"C2"	1.2×10^{-5}	0.492	206.737
"C3"	1.5×10^{-5}	0.474	159.507
"C4"	5×10^{-6}	0.12	121.56
"C5"	1.5×10^{-5}	0.354	118.994

$\nu 1_p = 31.54$
 $U = 220$

Summary table. The resonance frequency 70.392 Hz

Phase shift between current and voltage, deg $\varphi = 32.67 \cos\left(\frac{\pi}{180} \cdot \varphi\right) = 0.842$

"Element"	"Denomination: Ohm, Hn, F"	"Current, A"	"Voltage, V"
"R1"	10	0.865	8.655
"R2"	12	2.279	27.349
"R3"	8	1.825	14.6
"R4"	7	1.316	9.209
"L1"	0.4	0.865	153.113
"L2"	0.1	2.978	131.718
"L3"	0.35	1.825	282.501
"L4"	0.4	0.51	90.244
"L5"	0.1	1.316	58.189
"C1"	5×10^{-5}	0.865	39.136
"C2"	1.2×10^{-5}	0.699	131.718
"C3"	1.5×10^{-5}	1.825	275.077
"C4"	5×10^{-6}	0.51	230.666
"C5"	1.5×10^{-5}	1.316	198.308

$\nu 2_p = 70.392$
 $U = 220$

(Continuation of listing 2)

Summary table. The resonance frequency 75.989 Hz

Phase shift between current and voltage, deg $\varphi = 30.91 \cos\left(\frac{\pi}{180} \cdot \varphi\right) = 0.858$

D =

"Element"	"Denomination: Ohm, Hn, F"	"Current, A"	"Voltage, V"
"R1"	10	1.869	18.687
"R2"	12	2.048	24.574
"R3"	8	3.438	27.503
"R4"	7	2.45	17.147
"L1"	0.4	1.869	356.886
"L2"	0.1	2.819	134.592
"L3"	0.35	3.438	574.5
"L4"	0.4	0.99	189.151
"L5"	0.1	2.45	116.952
"C1"	5×10^{-5}	1.869	78.278
"C2"	1.2×10^{-5}	0.771	134.592
"C3"	1.5×10^{-5}	3.438	480.031
"C4"	5×10^{-6}	0.99	414.875
"C5"	1.5×10^{-5}	2.45	342.024

$\nu_3^p = 75.989$
 $U = 220$

Summary table. The resonance frequency 116.104 Hz

Phase shift between current and voltage, deg $\varphi = 4.19 \cos\left(\frac{\pi}{180} \cdot \varphi\right) = 0.997$

D =

"Element"	"Denomination: Ohm, Hn, F"	"Current, A"	"Voltage, V"
"R1"	10	0.605	6.053
"R2"	12	0.298	3.573
"R3"	8	0.311	2.488
"R4"	7	0.779	5.451
"L1"	0.4	0.605	176.634
"L2"	0.1	0.824	60.11
"L3"	0.35	0.311	79.396
"L4"	0.4	0.87	253.996
"L5"	0.1	0.779	56.807
"C1"	5×10^{-5}	0.605	16.595
"C2"	1.2×10^{-5}	0.526	60.11
"C3"	1.5×10^{-5}	0.311	28.417
"C4"	5×10^{-6}	0.87	238.64
"C5"	1.5×10^{-5}	0.779	71.163

$\nu_4^p = 116.104$
 $U = 220$

(Continuation of listing 2)

Summary table. The resonance frequency 166.141 Hz

Phase shift between current and voltage, deg $\varphi = 55.99$ $\cos\left(\frac{\pi}{180} \cdot \varphi\right) = 0.559$

"Element"	"Denomination: Ohm, Hn, F"	"Current, A"	"Voltage, V"
"R1"	10	0.048	0.478
"R2"	12	0.637	7.645
"R3"	8	0.643	5.146
"R4"	7	0.545	3.817
"L1"	0.4	0.048	19.973
"L2"	0.1	2.071	216.151
"L3"	0.35	0.643	235.014
"L4"	0.4	0.099	41.435
"L5"	0.1	0.545	56.916
"C1"	5×10^{-5}	0.048	0.916
"C2"	1.2×10^{-5}	2.708	216.151
"C3"	1.5×10^{-5}	0.643	41.079
"C4"	5×10^{-6}	0.099	19.012
"C5"	1.5×10^{-5}	0.545	34.82

$v_6^p = 166.141$
 $U = 220$

Summary table. The resonance frequency 181.442 Hz

Phase shift between current and voltage, deg $\varphi = 56.03$ $\cos\left(\frac{\pi}{180} \cdot \varphi\right) = 0.559$

"Element"	"Denomination: Ohm, Hn, F"	"Current, A"	"Voltage, V"
"R1"	10	2.749	27.487
"R2"	12	5.754	69.054
"R3"	8	3.033	24.263
"R4"	7	2.531	17.719
"L1"	0.4	2.749	1.253×10^3
"L2"	0.1	10.283	1.172×10^3
"L3"	0.35	3.033	1.21×10^3
"L4"	0.4	0.505	230.237
"L5"	0.1	2.531	288.572
"C1"	5×10^{-5}	2.749	48.221
"C2"	1.2×10^{-5}	16.037	1.172×10^3
"C3"	1.5×10^{-5}	3.033	177.354
"C4"	5×10^{-6}	0.505	88.575
"C5"	1.5×10^{-5}	2.531	148.023

$v_7^p = 181.442$
 $U = 220$

Evaluation of errors

To find the error of the result of the experimental studies for a nonlinear functional dependence $Z = f(a_1, a_2, a_3, \dots, a_i \dots a_m)$, it is necessary to construct a composition of the distribution of the random and systematic errors. But its construction is cumbersome, so an empirical formula is used to estimate the confidence error:

$$\Delta(P) = t_{\Sigma} \cdot S_{\Sigma}, \quad (29)$$

where,
$$t_{\Sigma} = \frac{\Theta(P) + \varepsilon(P)}{S(\bar{Z}) + \frac{\Theta(P)}{\sqrt{3}}}, \quad S_{\Sigma} = \sqrt{S^2(\bar{Z}) + \frac{\Theta^2(P)}{3}}.$$

The confidence limits of the non-excluded systematic error at the confidence probability are determined P by the formula:

$$\Theta(P) = k \sqrt{\sum_{i=1}^m \left(\frac{\partial f}{\partial a_i} \right)^2} \Theta_i^2, \quad (30)$$

$k = 1$ at $P = 0,9$

where, $k = 1,1$ at $P = 0,95$, m – is the number of dependence

$k = 1,4$ at $P = 0,99$

parameters $Z = f(a_1, a_2, a_3, \dots, a_i \dots a_m)$,

Θ_i – component of the non-excluded systematic error of the i th parameter.

The confidence limits of the random error at the confidence probability are determined P by the formula:

$$\varepsilon(P) = t \cdot S^2(\bar{Z}), \quad (31)$$

where, $S(\bar{Z}) = \sqrt{\sum_{i=1}^m \left(\frac{\partial f}{\partial a_i} \right)^2} S^2(\bar{a}_i)$, t – is the Student coefficient. It

depends on the number of effective degrees of freedom f_e and the selected confidence probability P [10].

$$f_e = \frac{\left[\sum_{i=1}^m \left(\frac{\partial f}{\partial a_i} \right)^2 S^2(\bar{a}_i) \right]^2}{\sum_{i=1}^m \left[\left(\frac{\partial f}{\partial a_i} \right)^4 S^4(\bar{a}_i) / (n_i - 1) \right]} - 2, \quad (32)$$

where m – is the number of arguments, n_i – is the number of dimensions a_i of the argument.

The considered method is mainly used to estimate the errors of scientific research. When estimating the errors of a physical experiment or engineering calculations, the logarithm method is most often used [2; 3; 7; 10].

The logarithmic method gives an over-estimate of the error, so it is called the margin of error. Such an assessment is in many cases sufficient for technical purposes. Note that we will not be able to calculate its exact value, but can only estimate its limiting (maximum) value. For a more accurate assessment of the error, it is necessary to apply the apparatus of mathematical statistics and probability theory, to find the distribution of the composition of the distribution of the random and systematic errors.

The method of estimating errors for functions of a real variable is developed in detail and described in the literature [3; 10], however, the theory of errors of functions of a complex variable, which includes calculations of alternating current circuits, is not sufficiently covered in the literature. Let's consider a simplified method of estimating the marginal value of the error using the Mathcad system, while applying the well-known method of logarithm, modernizing it for the function of a complex variable:

– we write down the calculation formula as a complex function of several variables:

$$g = g(x_i), i = 1 \dots n, \quad (33)$$

– we logarithmize the function:

$$g(x_i) = \ln[g(x_i)], \quad (34)$$

– we find the partial derivatives of the function (34):

$$g'(x_i) = \frac{\partial}{\partial x_i} \ln[g(x_i)], \quad (35)$$

– we find the components of the relative non-excluded systematic error through the modules of the products of the partial derivatives on the corresponding absolute systematic errors of the arguments Δx_i :

$$\varepsilon_{se} = |g'(x_i) \cdot \Delta x_i|, \quad (36)$$

– the limit value of the relative non-excluded systematic error is estimated by the ratio:

$$\varepsilon_s = \sum_{i=1}^n \varepsilon_{se}; \quad (37)$$

– we find the components of the relative random error through the modules of the products of the partial derivatives on the corresponding absolute random errors of the arguments δx_i :

$$\varepsilon_{re} = |g'(x_i) \cdot \delta x_i|, \quad (38)$$

– the limit value of the relative random error is estimated by the ratio:

$$\varepsilon_r = \sum_{i=1}^n \varepsilon_{re}, \quad (39)$$

– the limit value of the total relative random error is estimated by the ratio:

$$\varepsilon = \sqrt{\varepsilon_s^2 + \varepsilon_r^2}. \quad (40)$$

The errors estimated by ratios (37, 39) are significantly overestimated. In these ratios, we found the usual sum of the component mean values of the errors. If a systematic or random error includes four or more components and the largest of them have the same order, then the theory of errors recommends estimating their values by ratios:

$$\varepsilon_s = \begin{cases} \sum_{i=1}^n \varepsilon_{se}, & n < 4 \\ \sqrt{\sum_{i=1}^n \varepsilon_{se}^2}, & n \geq 4 \end{cases}, \quad (41)$$

$$\varepsilon_r = \begin{cases} \sum_{i=1}^n \varepsilon_{re}, & n < 4 \\ \sqrt{\sum_{i=1}^n \varepsilon_{re}^2}, & n \geq 4 \end{cases}, \quad (42)$$

The Mathcad system [3; 6; 7; 12; 18] has a complete set of tools for performing calculations with ratios (29-42), so we will evaluate the errors using the Mathcad system (*Continuation of listing 2*).

Functions for estimating current error components. Complex branch supports:

$$Z1(R1, L1, C1, \nu) = R1 + \left(2 \pi \cdot \nu \cdot L1 - \frac{1}{2 \pi \cdot \nu \cdot C1} \right) \cdot i$$

$$Z2(R2, L2, C2, \nu) = R2 - \frac{2 \pi \cdot \nu \cdot L2 \cdot i}{4 \pi^2 \cdot \nu^2 \cdot L2 \cdot C2 - 1}$$

$$Z3(R3, L3, C3, \nu) = R3 + \left(2 \pi \cdot \nu \cdot L3 - \frac{1}{2 \pi \cdot \nu \cdot C3} \right) \cdot i$$

$$Z4(L4, C4, \nu) = \left(2 \pi \cdot \nu \cdot L4 - \frac{1}{2 \pi \cdot \nu \cdot C4} \right) \cdot i$$

$$Z5(R4, L5, C5, \nu) = R4 + \left(2 \pi \cdot \nu \cdot L5 - \frac{1}{2 \pi \cdot \nu \cdot C5} \right) \cdot i$$

$$Z4(L4, C4, \nu) \cdot Z5(R4, L5, C5, \nu) = \frac{Z4(L4, C4, \nu) \cdot Z5(R4, L5, C5, \nu)}{Z4(L4, C4, \nu) + Z5(R4, L5, C5, \nu)}$$

$$Z345(R3, R4, L3, L4, L5, C3, C4, C5, \nu) = Z3(R3, L3, C3, \nu) + Z45(R4, L4, L5, C4, C5, \nu)$$

$$Z2345(R2, R3, R4, L2, L3, L4, L5, C2, C3, C4, C5, \nu) = \frac{Z2(R2, L2, C2, \nu) \cdot Z345(R3, R4, L3, L4, L5, C3, C4, C5, \nu)}{Z2(R2, L2, C2, \nu) + Z345(R3, R4, L3, L4, L5, C3, C4, C5, \nu)}$$

$$Z(R1, R2, R3, R4, L1, L2, L3, L4, L5, C1, C2, C3, C4, C5, \nu) = Z1(R1, L1, C1, \nu) + Z2345(R2, R3, R4, L2, L3, L4, L5, C2, C3, C4, C5, \nu)$$

Function for calculating current strength

$$F1(R1, R2, R3, R4, L1, L2, L3, L4, L5, C1, C2, C3, C4, C5, \nu, U) = \frac{U}{Z(R1, R2, R3, R4, L1, L2, L3, L4, L5, C1, C2, C3, C4, C5, \nu)}$$

A function for calculating the value of the modulus of the current

$$I(\nu) = |F1(R1, R2, R3, R4, L1, L2, L3, L4, L5, C1, C2, C3, C4, C5, \nu, U)|$$

(Continuation of listing 2)

$$qL4'(R1, R2, R3, R4, L1, L2, L3, L4, L5, C1, C2, C3, C4, C5, \nu, U) := \frac{\partial}{\partial L4} LF(R1, R2, R3, R4, L1, L2, L3, L4, L5, C1, C2, C3, C4, C5, \nu, U)$$

$$qL5'(R1, R2, R3, R4, L1, L2, L3, L4, L5, C1, C2, C3, C4, C5, \nu, U) := \frac{\partial}{\partial L5} LF(R1, R2, R3, R4, L1, L2, L3, L4, L5, C1, C2, C3, C4, C5, \nu, U)$$

$$qC1'(R1, R2, R3, R4, L1, L2, L3, L4, L5, C1, C2, C3, C4, C5, \nu, U) := \frac{\partial}{\partial C1} LF(R1, R2, R3, R4, L1, L2, L3, L4, L5, C1, C2, C3, C4, C5, \nu, U)$$

$$qC2'(R1, R2, R3, R4, L1, L2, L3, L4, L5, C1, C2, C3, C4, C5, \nu, U) := \frac{\partial}{\partial C2} LF(R1, R2, R3, R4, L1, L2, L3, L4, L5, C1, C2, C3, C4, C5, \nu, U)$$

$$qC3'(R1, R2, R3, R4, L1, L2, L3, L4, L5, C1, C2, C3, C4, C5, \nu, U) := \frac{\partial}{\partial C3} LF(R1, R2, R3, R4, L1, L2, L3, L4, L5, C1, C2, C3, C4, C5, \nu, U)$$

$$qC4'(R1, R2, R3, R4, L1, L2, L3, L4, L5, C1, C2, C3, C4, C5, \nu, U) := \frac{\partial}{\partial C4} LF(R1, R2, R3, R4, L1, L2, L3, L4, L5, C1, C2, C3, C4, C5, \nu, U)$$

$$qC5'(R1, R2, R3, R4, L1, L2, L3, L4, L5, C1, C2, C3, C4, C5, \nu, U) := \frac{\partial}{\partial C5} LF(R1, R2, R3, R4, L1, L2, L3, L4, L5, C1, C2, C3, C4, C5, \nu, U)$$

$$q\nu'(R1, R2, R3, R4, L1, L2, L3, L4, L5, C1, C2, C3, C4, C5, \nu, U) := \frac{\partial}{\partial \nu} LF(R1, R2, R3, R4, L1, L2, L3, L4, L5, C1, C2, C3, C4, C5, \nu, U)$$

$$qU'(R1, R2, R3, R4, L1, L2, L3, L4, L5, C1, C2, C3, C4, C5, \nu, U) := \frac{\partial}{\partial U} LF(R1, R2, R3, R4, L1, L2, L3, L4, L5, C1, C2, C3, C4, C5, \nu, U)$$

Evaluation of the values of the component relative systematic errors, %

The function for evaluation the limit value of the relative error of the current forse

$$\xi(\nu) := \begin{pmatrix} |qR1'(R_1, R_2, R_3, R_4, L_1, L_2, L_3, L_4, L_5, C_1, C_2, C_3, C_4, C_5, \nu, U)| \cdot \Delta R_1 \\ |qR2'(R_1, R_2, R_3, R_4, L_1, L_2, L_3, L_4, L_5, C_1, C_2, C_3, C_4, C_5, \nu, U)| \cdot \Delta R_2 \\ |qR3'(R_1, R_2, R_3, R_4, L_1, L_2, L_3, L_4, L_5, C_1, C_2, C_3, C_4, C_5, \nu, U)| \cdot \Delta R_3 \\ |qR4'(R_1, R_2, R_3, R_4, L_1, L_2, L_3, L_4, L_5, C_1, C_2, C_3, C_4, C_5, \nu, U)| \cdot \Delta R_4 \\ |qC1'(R_1, R_2, R_3, R_4, L_1, L_2, L_3, L_4, L_5, C_1, C_2, C_3, C_4, C_5, \nu, U)| \cdot \Delta C_1 \\ |qC2'(R_1, R_2, R_3, R_4, L_1, L_2, L_3, L_4, L_5, C_1, C_2, C_3, C_4, C_5, \nu, U)| \cdot \Delta C_2 \\ |qC3'(R_1, R_2, R_3, R_4, L_1, L_2, L_3, L_4, L_5, C_1, C_2, C_3, C_4, C_5, \nu, U)| \cdot \Delta C_3 \\ |qC4'(R_1, R_2, R_3, R_4, L_1, L_2, L_3, L_4, L_5, C_1, C_2, C_3, C_4, C_5, \nu, U)| \cdot \Delta C_4 \\ |qC5'(R_1, R_2, R_3, R_4, L_1, L_2, L_3, L_4, L_5, C_1, C_2, C_3, C_4, C_5, \nu, U)| \cdot \Delta C_5 \\ |qL1'(R_1, R_2, R_3, R_4, L_1, L_2, L_3, L_4, L_5, C_1, C_2, C_3, C_4, C_5, \nu, U)| \cdot \Delta L_1 \\ |qL2'(R_1, R_2, R_3, R_4, L_1, L_2, L_3, L_4, L_5, C_1, C_2, C_3, C_4, C_5, \nu, U)| \cdot \Delta L_2 \\ |qL3'(R_1, R_2, R_3, R_4, L_1, L_2, L_3, L_4, L_5, C_1, C_2, C_3, C_4, C_5, \nu, U)| \cdot \Delta L_3 \\ |qL4'(R_1, R_2, R_3, R_4, L_1, L_2, L_3, L_4, L_5, C_1, C_2, C_3, C_4, C_5, \nu, U)| \cdot \Delta L_4 \\ |qL5'(R_1, R_2, R_3, R_4, L_1, L_2, L_3, L_4, L_5, C_1, C_2, C_3, C_4, C_5, \nu, U)| \cdot \Delta L_5 \\ |q\nu'(R_1, R_2, R_3, R_4, L_1, L_2, L_3, L_4, L_5, C_1, C_2, C_3, C_4, C_5, \nu, U)| \cdot \Delta \nu \\ |qU'(R_1, R_2, R_3, R_4, L_1, L_2, L_3, L_4, L_5, C_1, C_2, C_3, C_4, C_5, \nu, U)| \cdot \Delta U \end{pmatrix} \cdot 100$$

Evaluation of the limit value of the relative error of the current force for

frequencies: 31.54; 50; 70.392; 75.989; 166.141; 181.442 Hz, %

$$M := \begin{pmatrix} \text{"Frequency, Hz"} \\ \text{"Current error, \%"} \end{pmatrix}$$

$$D0 := \left(\sum \xi(31.54) \quad \sum \xi(50) \quad \sum \xi(70.392) \quad \sum \xi(75.989) \quad \sum \xi(166.141) \quad \sum \xi(181.442) \right)$$

$$PO := \text{augment}(M, D0)$$

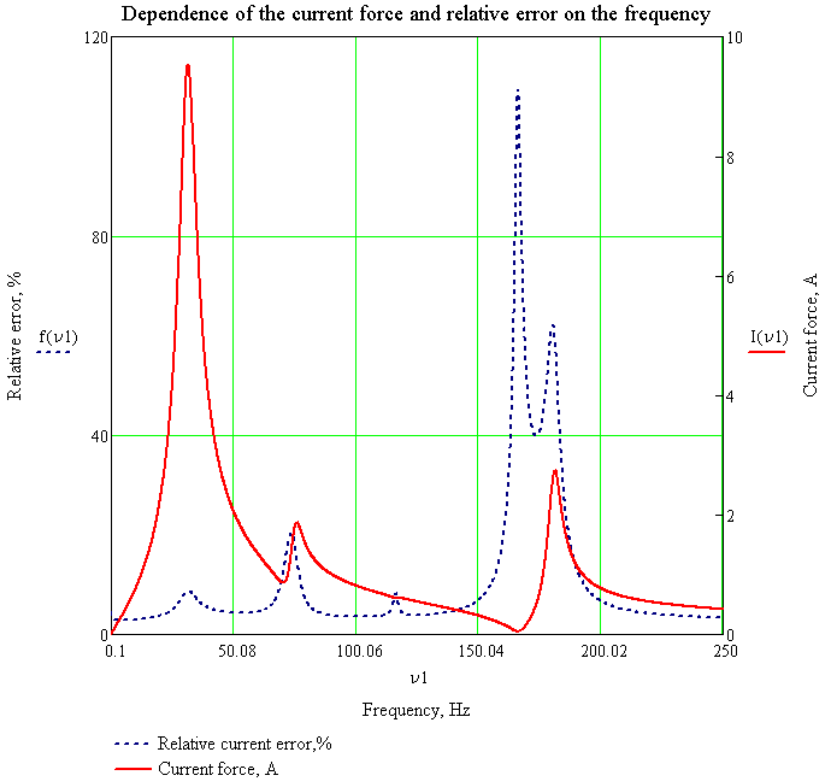
$$PO = \begin{pmatrix} \text{"Frequency, Hz"} & 31.54 & 50 & 70.392 & 75.989 & 166.141 & 181.442 \\ \text{"Current error, \%"} & 8.772 & 4.307 & 12.458 & 15.214 & 108.16 & 59.229 \end{pmatrix}$$

The function for calculating the value of the modulus of the curren, A

$$I(\nu) := |F1(R_1, R_2, R_3, R_4, L_1, L_2, L_3, L_4, L_5, C_1, C_2, C_3, C_4, C_5, \nu, U)|$$

(Continuation of listing 2)

$f(\nu) := \sum \xi(\nu) \Leftarrow$ The function for estimating the limit value of the relative error of the current force, %
 $\nu_1 := 0.1, 0.2 \dots 250$



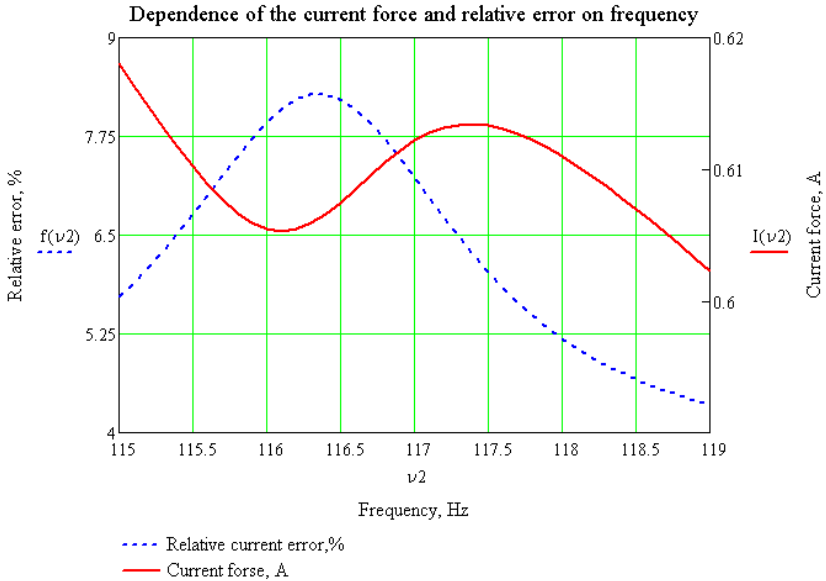
A series of peaks is observed on the graph of the dependence of the relative error of the current force in the electric circuit on the frequency at the resonance frequencies of the circuit.

A comparison of the graph of the dependence of the current force in the electric circuit on the frequency and the graph of the dependence of the relative error of the current on the frequency shows that at a frequency of $\nu \approx 116 \text{ Hz}$, a slight local maximum is observed on the graph of the relative

error. Therefore, a frequency of 116 Hz for this circuit may also be resonant, but this requires a detailed additional study of the resonance curve of the circuit (Listing 2).

Continuation of listing 2

$$\nu 2 := 115, 115.1 .. 119$$



Therefore, a detailed study of the frequency interval 115-119 Hz shows that the frequencies 116,1 Hz and 117,38 Hz are also resonant.

4. Conclusions

To study the frequency characteristics of an alternating current circuit (of an oscillating circuit) a method is proposed that makes it possible to determine the main characteristics of the circuit quite effectively and to evaluate their errors.

In the proposed method, there is no need to explicitly find the complex roots of complex equations. The resonant frequencies of the oscillating circuit are determined by the method of optimizing the mathematical

model. Initial approximations of resonance frequencies are determined from the resonance curve and refined to the required accuracy by means of the Mathcad system.

The limit values of the oscillating contour parameter errors are estimated by the classical method of logarithmization, which has been modernized for the function of a complex variable.

It was found that at resonant frequencies of the circuit, the value of the relative error of the current increases sharply.

The proposed method can be useful in solving a number of applied problems in physics, radio engineering, and electrical engineering.

It is important that the use of the Mathcad system in the educational process when studying physics stimulates the student's work. To develop a working document of the Mathcad system, a student must know mathematics, clearly understand the essence of physical phenomena and the laws of their course. At the same time, the task can be conditionally divided into two components: mathematical and physical. Calculations are performed by the system according to the algorithm developed by the student. The algorithm is based on the laws of physical processes that are considered in the given problem.

Therefore, the use of the Mathcad system during the study of the academic discipline "Physics" makes it possible to effectively solve problems of various levels of complexity, not only physical, but also professional, which require a complex mathematical apparatus. Graphs built using the Mathcad system contribute to the reliable learning of educational material and a deeper understanding of physical phenomena and laws.

References:

1. Dzis V. G. (2019). Avtomatyzovana informatsiina systema dlia aproksymatsii ekonomichnykh pokaznykiv [Automated information system for approximating economic indicators]. *Ekonomika, finansy, menedzhment: aktualni pyttannia nauky i praktyky – Economics, finance, management: topical issues of science and practice*, vol. 8 (48), pp. 31–39. DOI: 10.37128/2411-4413-2019-8-4 (in Ukrainian)
2. Dzis V. G. (2019). *Otsinka pokhybok zasobamy systemy Mathcad* [Evaluation of errors by means of the Mathcad system]. Proceedings of the *All-Ukrainian conference "MATLAB and computer calculations in education, science and engineering"* (Ukraine, Kyiv, May 16–18, 2019). Kyiv: NAU, pp. 23–24.
3. Dzis V. G., Levchuk O. V., Diachynska O. M. (2020). *Prykladna matematyka na osnovi Mathcad: navch. posib.* [Applied mathematics based on Mathcad: teach-

ing. Study guide]. Vinnytsia: VNAU, 378 p. Available at: <http://socrates.vsau.org/repository/getfile.php/23375.pdf> (accessed January 2, 2023).

4. Dzis V., Diachynska O. (2020). Pobudova matematychnoi modeli rostu roslyn [Construction of the mathematical plant growth model]. *Naukova spadshchyna – The scientific heritage*, vol. 3, no. 56 (56), pp. 48–54. DOI: 10.24412/9215-0365-2020-56-3-48-54

5. Dzis V., Dyachynska O. (2022). Imitatsiine modeliuвання investytsiinykh proektiv u sferi posluhova spadshchyna [Simulation modeling of investment projects in the service sector]. *Ekonomichnyi zhurnal trokh moriv – Three Seas Economic Journal*, vol. 3, no. 1, pp. 68–77. DOI: 10.30525/2661-5150/2022-1-9

6. Herhagel M., Partol H. (2000). *Mathcad-2000. Polnoe rukovodstvo* [Mathcad-2000. Complete guide]. Kyiv, 412 p. (in Ukrainian)

7. Isai V. M., Korol A. M. (2011). *Fizyka: metodychni vkazivky do elektronnoho posibnyka “Rozrakhunok pokhybok rezultativ vykonannya laboratornykh robit z fizyky z vykorystanniam paketa MATHCAD”* [Physics: methodological instructions for the electronic manual “Calculation of errors of the results of laboratory works in physics using the MATHCAD package”]. Kyiv: NUHT, 114 p. (in Ukrainian)

8. Karsyibaev M. Sh. (2008). *Fizika. Reshenie zdach v kompyuternoy srede Mathsad. Uchebnoe posobie* [Physics. Solving problems in the computer environment Mathcad. Study guide]. Almaty: AIES, 102 p. Available at: https://libr.aues.kz/facultet/tef/kaf_fiziki/fiz/umm/fiz_9.htm (accessed January 5, 2023).

9. Kravchenko I. V., Mykytenko V. I. (2018). *Informatsiini tekhnolohii. Systemy komp'uternoi matematyky [Elektronnyi resurs]: navchalnyi posibnyk dlia studentiv spetsialnosti “Avtomatyzatsiia ta kompiuterno-intehrovani tekhnolohii”* [Information Technology. Systems of computer mathematics [Electronic resource]: study guide for students majoring in “Automation and computer-integrated technologies”]. Kyiv: KPI im. Ihoria Sikorskoho, 243 p. Available at: <https://ela.kpi.ua/handle/123456789/22913> (accessed January 6, 2023).

10. Kucheruk I. M., Andrianov V. M. (1981). *Obrobka rezultativ fizychnykh doslidzhen* [Processing of the physical research results]. Kyiv: Higher School, 216 p. (in Ukrainian)

11. Kucheruk I. M., Gorbachuk I. T., Lutsyk P. P. (2006). *Zahalnyi kurs fizyky: navchalnyi posibnyk dlia stud. tekhn. i ped. spets. vyshch. navch. zakladiv: U 3 t. T. 2. Elektryka i mahnetyzm* [General course of physics: study guide for students. technical and ped. special higher education institutions: In 3 vols. Vol. 2. Electricity and magnetism]. Kyiv: Technika, 452 p. (in Ukrainian)

12. Lysenko V. P., Bolbot I. M. (2010). *Kompiutery ta kompiuterni tekhnolohii: navch. posib. Ch. 1. Prohramuvannya v matematychnomu paketi Mathcad* [Computers and computer technologies: study guide Part 1. Programming in Mathcad mathematical package]. Kyiv: Agrarian Education, 229 p. (in Ukrainian)

13. Menyailov M. E. (1974). *Zahalna fizyka. Elektryka ta mahnetyzm* [General Physics. Electricity and magnetism]. Kyiv: Higher School, 392 p. (in Ukrainian)

14. Miliukova I. R. (2019). Zastosuvannya matematychnoho paketu MathCAD pry rozviazanni zadach z fizyky [Application of the MathCAD mathematical package when solving physics problems]. *Fizyko-matematychna osvita – Physical and*

mathematical education, vol. 2, pp. 99–106. Available at: http://nbuv.gov.ua/UJRN/fmo_2019_2_18 (accessed January 4, 2023).

15. Odnovol D. H. (2009). Praktyka vykorystannia prykladnykh matematychnykh paketiv prohram pid chas laboratornykh robit z fizyky [The practice of using applied mathematical software packages during physics laboratory works]. *Zbirnyk naukovykh prats Kamianets-Podilskoho natsionalnoho universytetu im. I. Ohienka. Ser. pedahohichna – Collection of scientific works of the Kamianets-Podilskiy National University named after I. Ohienko. Pedagogical series*, vol. 15, pp. 155–157. (in Ukrainian)

16. Rozvodiuk M. P., Blinkin E. Y., Tkach V. C. (2006). *Elektrotehnika. Chastyna I. Doslidzhennia elektrychnykh kil: navchalnyi posibnyk* [Electrical engineering. Part I. Study of the electric circuits: study guide]. Vinnytsia: VNTU, 206 p. Available at: <http://ir.lib.vntu.edu.ua/handle/123456789/5015> (accessed January 6, 2023).

17. Yefymenko S. M. (2018). Zasoby MathCAD u navchalnomu fizychnomu eksperymenti [Means of Mathcad in the educational physical experiment]. *Fizyko-matematychna osvita – Physical and mathematical education*, vol. 1 (15), pp. 195–199. (in Ukrainian)

18. Zachek I. R., Lopatynskiy I. Ie., Yuriev S. O., Rybak O. V., Dubelt S. P. (2019). *Fizyka i kompiuterni tekhnolohii. Navchalnyi posibnyk*. [Physics and computer technologies. Study guide]. Lviv: Vydavnytstvo Lvivskoi politekhniky, 360 p. (in Ukrainian)