

**THEORETICAL AND PRACTICAL STUDIES
OF EFFECTIVE PHYSICAL AND MATHEMATICAL MODELS
OF DESIGNING EDUCATIONAL RESOURCES
IN AGRARIAN INSTITUTIONS OF HIGHER EDUCATION**

Viktor Dubchak¹

Elvira Manzhos²

DOI: <https://doi.org/10.30525/978-9934-26-298-2-5>

Abstract. *The purpose* of this paper to investigate the issue of optimal, extreme (minimum) coverage of the area of one flat figure, namely, a square (rectangle) of the area of another flat geometric figure, such as a circle (ellipse). The value of a certain function that determines the difference in the areas of the mismatch of the given geometric figures at the intersection has been obtained, in the first case the function is dependent on one variable argument, in the second case – on two arguments, in both cases the study of the extreme of this function has been carried out, it is shown that in the found extremum point, the function that determines the mismatch of the areas of the figures acquires a minimum value. *Methodology.* The conditions of such extremely minimal covering of one area with another area are established, drawings are provided for a better understanding of the formulation and solution of the given problem. *Results.* The conclusions have been made, in which the values of the sought arguments are given, when the corresponding function will acquire minimum values. Also, as an alternative criterion for the optimality of such a mutual placement of one figure relative to another, this paper proposes such a criterion as an effective assessment of the length of the set of lines along which the specified figures diverge. The value of a certain function that determines the length of such a sum of the lines of divergence of the figures in both cases has been obtained, a study has been carried out on the extremity of this function, it has been shown that at

¹ Candidate of Technical Sciences, Associate Professor
at the Department of Physics, Mathematics and Computer Sciences,
Vinnytsya National Agrarian University, Ukraine

² Candidate of Pedagogical Sciences, Associate Professor
at the Department of Foreign Languages,
National Pirogov Memorial Medical University, Ukraine

the found extremum point the function that determines the total length of the line of divergence of the figures acquires a minimum value. The issue of body movement under the influence of gravitational field forces and environmental resistance has been considered in these studies, the obtained solutions and their practical use in the field of agro-industrial complex of the economic sphere have been analyzed. *Practical implications.* As a result of the conducting research using a fundamental analytical and mathematical apparatus, an algorithmic model has been developed. It describes the movement of seeds or mineral fertilizers on the path of movement from sowing machines to the grooves that are formed in the soil after the passage of coulters of seeders; the movement of grain, which moves from top to bottom in the cleaning systems of grain harvesters or during unloading from the hopper, etc. Such processes and phenomena occur during the production operation of planters and planters, machines for post-harvest processing of grain, harvesting of root vegetables, fruits, berries, etc. *Value/originality.* Drawings are provided for a better understanding of the formulation and solution of the set problems, conclusions are drawn, as the meaning of the sought arguments, when the corresponding function acquires minimum values, the corresponding differential equations of body motion are drawn up and the directions of using their solutions are indicated, particularly in the agro-industrial sector of the economy.

1. Introduction

The results of the work obtained in the above studies are based on the use of the basic mathematical apparatus for establishing the corresponding extreme values using the basics of differential calculus. Also, on the basis of Newton's 2nd law regarding the movement of a body under the action of relevant external active forces, certain differential equations have been formulated, their solutions have been solved and analyzed. The analysis of the results of solving this problem has a rather significant practical significance, including in solving the issues of increasing productivity in the agrarian sector of the economy.

The search for extreme values in various mathematical, technical, economic applied problems is always an interesting and relevant problem. In geometric problems, in our case, the size of the area of non-coincidence or the length of the line of non-coincidence serves as an extreme value

in the case of the corresponding relative location of the reduced and correspondingly located flat geometric figures.

Also, significant practical scientific interest is manifested in the issue of the study of body movement under the influence of gravitational field forces and environmental resistance, in particular, liquid particles in various irrigation systems during watering and soil and plant treatment. Therefore, the results of these studies are original and have important practical significance.

The problem of effective extreme (minimum) covering of one flat geometric figure with another [1–12] is relevant both from the point of view of theory and from a number of specific practical applications, in particular, covering the area of a different shape with the area of a circle, such as a square (Figure 1). Also, only with certain ratios of the radius of the circle in relation to the length of the side of a square or triangle, the length of the arc of the total line, along which the divergence of the specified geometric objects is observed, acquires an extreme (minimum) value. The results of these studies are devoted to the search for such mutual relations of the mentioned geometric characteristics of these figures.

Also, the main factors that determine the yield of agricultural crops, the quality of the obtained products, the competitiveness of production include technological operations of field irrigation, sowing or planting, applying fertilizers, chemical protection of plants, harvesting and processing of grain crops, root vegetables, fruits, berries, etc. The execution of most operations is provided by modern technical means of mechanization, which are designed and developed on the basis of a powerful modern mathematical apparatus, as the fundamental basis of theoretical research. During the implementation of technological processes of moving materials under the influence of the gravitational field, it should be taken into account that the impact force of rain particles, other liquid for processing or feeding the crop at the moment of fall often leads to negative consequences. So, for example, falling drops as a result of the impact break the lumps and put the soil in a state that is easily susceptible to erosion processes; the fall of tubers, root crops, fruits, berries can lead to their injury and further spoilage and decrease in market value.

As the main criterion for such an effective covering of one figure by another, it is proposed to logically choose such an indicator as the minimum area [1; 2; 6; 11], on which the specified geometric figures do not coincide. A case of a more general statement of a similar problem is the effective

(minimum) coverage of the area of the ellipse of the corresponding area of the rectangle (Figure 2). Also, as one of the criteria for such an effective covering of one figure with another with a common center, it is proposed to choose such an indicator as the minimum length of the line [1; 2; 6; 12] along which the indicated geometric figures do not coincide. On the basis of this geometric indicator, a corresponding function dependent on a certain argument is constructed, this function is studied by the methods of differential calculus, it is shown that at extreme values of this argument, this function acquires minimum values, which, accordingly, are also set.

The innovative development of technological processes, which are characterized by the movement of materials under the influence of the force of the gravitational field, has practical application in the latest designs (agricultural machines for sowing and planting, applying fertilizers, grain harvesters, machines for post-harvest processing of grain, etc.) and reclamation machines (first of all in sprinkler systems) [13–20].

The authors: M.S. Kravchenko, Yu.A. Zlobin, O.M. Tsarenko in the textbook “Agriculture” partially analyzed the destructive effect of raindrops on the surface structure of the soil and the water erosion processes that occur at the same time [16]. Scientists have indicated that the greatest risk of losing fertile soil occurs when raindrops fall on the surface with little surface water flow, because in this case the breaking of structured soil clods occurs the strongest: structured soil turns into unstructured, and erosion processes increase. If the thickness of the water layer exceeds the diameter of the drops, the force of the rain on the lumps is significantly reduced.

In addition to the movement of particles in air environments, sometimes some processes of body movement occur in liquid environments. The simplest models of liquid media are considered (using the laws of conservation of mass, momentum, conservation of energy, moments of momentum) by A.G. Kutsenko, S.M. Bondar, V.M. Prishlyak in the monograph “Biomechanics of continuous environments” [17]. It is known that in theoretical mechanics for a mechanical system the law of moments of momentum has the following meaning: the time derivative of the total moment of momentum of a certain system of material points is equal to the principal moment of external forces. The authors in [17] have separately focused on this law, have analyzed it and some development have been given. The theoretical features of such concepts and environments as

ideal, viscous (Newtonian), non-heat-conducting, incompressible and compressible fluids have also been disclosed [17].

Therefore, an important element of research is the optimization of the parameters of technological operations and machines of agro-industrial production for the processes of movement of agricultural materials in liquid and air environments under the influence of the gravitational field [14; 18–20].

The purpose of these studies is to establish analytical conditions regarding the extremity (minimum) of the area and the length of the line of non-coincidence when covering some flat geometric structures with other geometric shapes. Also, the analysis of the movement of liquid particles under the influence of the forces of the gravitational field and the variable value of the resistance force of the environment and the application of the obtained results in the agro-industrial sector of the economy.

Special attention should be paid to falling raindrops or drops created by irrigation systems. The impact of the impact of drops on the field surface was analyzed, which ensures the development of practical recommendations for preventing soil erosion processes, which is especially relevant in fields with complex topography. Some technical systems for creating “artificial rain” are considered, their technological schemes are substantiated, and constructive and technological parameters should be optimized.

2. Research on the non-coincidence of the areas of flat figures

There is a square with the length of its equal side $OA = 2a$ (Figure 1). This square covers the area of the radius circle $OM = ON = R$. The coordinates of the point N are dependent on the angle β , as it is shown in the figure, and the point itself N forms within the first 45° two areas S_1 and S_2 , where the given geometric shapes do not match. The total area, on which they do not coincide within the limits of the first 45° figure, is defined as $S = S_1 + S_2$. Each of the two given terms is the value of the corresponding function depending on the angle β .

$$\text{There are the conditions: } x = a \Rightarrow r \cos \phi = a, \cos \phi = \frac{a}{r}, \quad (1)$$

where r, ϕ – polar coordinates of arbitrary points of the corresponding areas, along which the figures do not coincide, at the same time $\cos \beta = \frac{a}{R}$. (2)

The values of areas of both terms of the required area S are set. It is now:

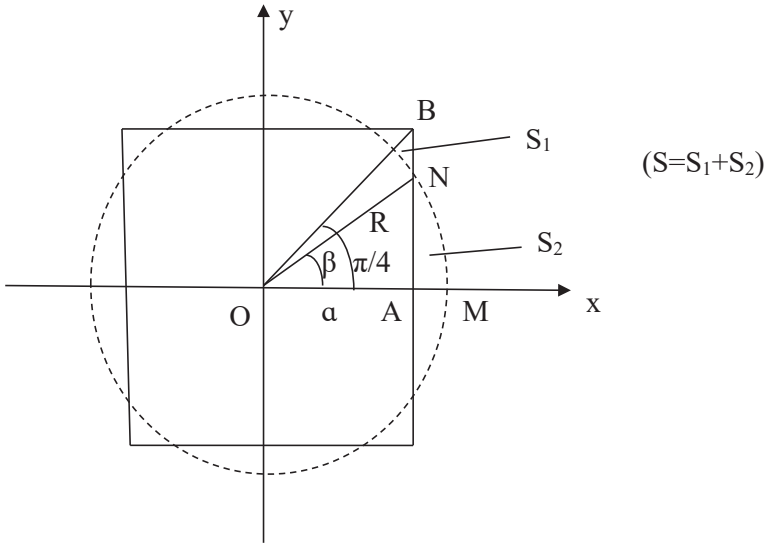


Figure 1. Covering the area of a circle with a square, determining the unknown argument β , on which the size of the area depends S , according to which the areas of these figures do not match

Source: developed by the authors according to [11]

$$\begin{aligned}
 S_1 &= \iint_{D_1} r dr d\phi = \int_{\beta}^{\pi/4} d\phi \int_R^{\frac{a}{\cos\phi}} r dr = \int_{\alpha}^{\pi/4} \frac{r^2}{2} \Big|_R^{\frac{a}{\cos\phi}} d\phi = \frac{1}{2} \int_{\beta}^{\pi/4} \left(\frac{a^2}{\cos^2\phi} - R^2 \right) d\phi = \frac{1}{2} (a^2 \operatorname{tg}\phi - R^2 \phi) \Big|_{\beta}^{\pi/4} = \\
 &= \frac{1}{2} (a^2 - R^2 \frac{\pi}{4} - (a^2 \operatorname{tg}\beta - R^2 \beta)) = \frac{1}{2} (a^2 (1 - \operatorname{tg}\beta) - R^2 (\frac{\pi}{4} - \beta)) = \frac{1}{2} (R^2 \cos^2 \beta (1 - \operatorname{tg}\beta) - \\
 &R^2 (\frac{\pi}{4} - \beta)) = \frac{R^2}{2} (\cos^2 \beta - \sin \beta \cos \beta - \frac{\pi}{4} + \beta). \tag{3}
 \end{aligned}$$

$$\begin{aligned}
 S_2 &= \iint_{D_2} r dr d\phi = \int_0^{\beta} d\phi \int_{\frac{a}{\cos\phi}}^R r dr = \frac{1}{2} \int_0^{\beta} r^2 \Big|_{\frac{a}{\cos\phi}}^R d\phi = \frac{1}{2} \int_0^{\beta} \left(R^2 - \frac{a^2}{\cos^2\phi} \right) d\phi = \frac{1}{2} (R^2 \phi - \\
 &a^2 \operatorname{tg}\phi) \Big|_0^{\beta} = \frac{1}{2} R^2 (\beta - \cos^2 \beta \operatorname{tg}\beta) = \frac{1}{2} R^2 (\beta - \cos \beta \sin \beta). \tag{4}
 \end{aligned}$$

$$S = S_1 + S_2 = \frac{R^2}{2} (\cos^2 \beta - 2 \sin \beta \cos \beta + 2\beta - \frac{\pi}{4}). \quad (5)$$

When searching for the extreme (minimum) value of function S we find the first derivative of this function with respect to its argument β :

$$S'_\beta = -R^2 \sin \beta \cos \beta - \frac{1}{2} R^2 \cdot 2 \cos 2\beta + R^2 = 0. \quad (6)$$

Hence, it is the trigonometric equation for the desired angle β :

$$2 \sin^2 \beta - \sin \beta \cos \beta = 0 \Rightarrow \text{Whereas } \sin \beta \neq 0 \Rightarrow \operatorname{tg} \beta = \frac{1}{2} \Rightarrow$$

$$\beta = \operatorname{arctg} \frac{1}{2}, \quad (7)$$

$$\sin \beta = \sqrt{1 - \frac{4}{5}} = \frac{1}{\sqrt{5}}, \quad (8)$$

$$\cos \beta = \cos(\operatorname{arctg} \frac{1}{2}) = \frac{1}{\frac{1}{\cos(\operatorname{arctg} \frac{1}{2})}} = \frac{1}{\sqrt{1 + \operatorname{tg}^2(\operatorname{arctg} \frac{1}{2})}} = \frac{1}{\sqrt{1 + \frac{1}{4}}} = \frac{2}{\sqrt{5}}.$$

$$\text{Accordingly } \cos \beta = \frac{a}{R} = \frac{2}{\sqrt{5}}. \quad (9)$$

Let us find the derivative of the second order and set its sign:

$$\begin{aligned} S''_{\beta\beta} &= R^2 (-\cos 2\beta + 2 \sin 2\beta) = R^2 (\sin^2 \beta - \cos^2 \beta + 4 \sin \beta \cos \beta) = \\ &= R^2 (\frac{1}{5} - \frac{4}{5} + 4 \cdot \frac{1}{\sqrt{5}} \cdot \frac{2}{\sqrt{5}}) = R^2 \frac{5}{\sqrt{5}} > 0. \end{aligned} \quad (10)$$

This inequality indicates that the value of the angle has been found β within the first 45° provides the minimum value of the function S . This value of this function is equal to:

$$\begin{aligned} S \Big|_{\beta=\operatorname{arctg} \frac{1}{2}} &= \frac{R^2}{2} (\cos^2 \beta - 2 \sin \beta \cos \beta + 2\beta - \frac{\pi}{4}) = \frac{R^2}{2} (\frac{4}{5} - 2 \cdot \frac{1}{\sqrt{5}} \cdot \frac{2}{\sqrt{5}} + 2 \cdot \operatorname{arctg} \frac{1}{2} - \frac{\pi}{4}) = \\ &= \frac{R^2}{2} (2 \operatorname{arctg} \frac{1}{2} - \frac{\pi}{4}) = \frac{5}{8} a^2 (2 \operatorname{arctg} \frac{1}{2} - \frac{\pi}{4}). \end{aligned} \quad (11)$$

Since within 360° , the area on which the given geometric figures do not coincide minimally is 8 times larger, the total total area is equal to:

$$4R^2 \left(2 \operatorname{arctg} \frac{1}{2} - \frac{\pi}{4} \right) = 5a^2 \left(2 \operatorname{arctg} \frac{1}{2} - \frac{\pi}{4} \right). \quad (12)$$

A slightly more general case of the essence of the previous problem, namely the principles of effective (minimum) coverage of the area bounded by the arc of the ellipse and the corresponding rectangle covering the previous flat area (Figure 2) is considered.

The equation of an ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, we can specify this equation by

curvilinear coordinates:
$$\begin{cases} x = ar \cos \phi \\ y = br \sin \phi \\ 0 \leq r \leq 1 \end{cases} \quad (13)$$

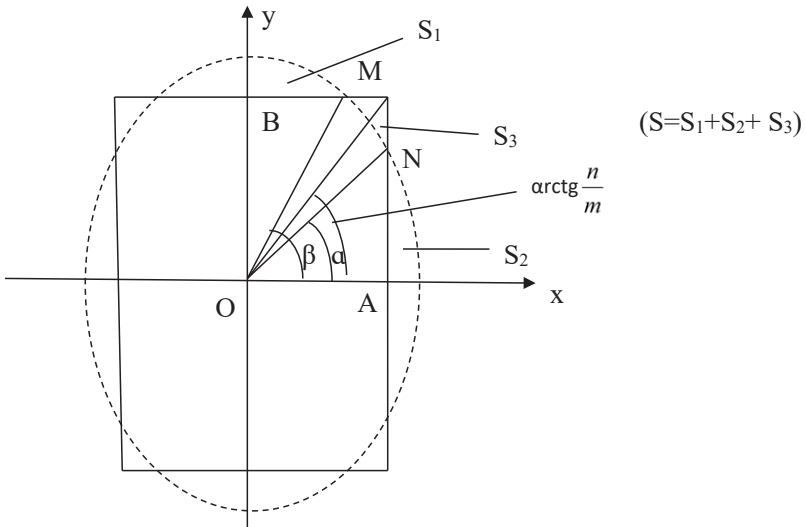


Figure 2. Covering the area bounded by the arc of the ellipse with a rectangle, determining the unknown arguments α and β , from which the size of the area S depends, on which the areas of the given figures do not match

Source: developed by the authors according to [11]

On the other hand, there is a rectangle, as shown in Figure 2, with sides, respectively, $2m \times 2n$. The problem within the first quarter ($x \geq 0, y \geq 0$) is analyzed and finally the results, using the obvious symmetry in the other quarters are generalized.

Total area S , according to which the given geometric shapes do not coincide within the first quarter, the sum of three terms, as shown in Figure 2 is written, i.e. $S = S_1 + S_2 + S_3$. The angle between the diagonal of the rectangle and the positive direction of the axis OX is equal to $\arctg \frac{n}{m}$. Points N and M are the points of intersection of the arc of the ellipse with the corresponding sides of the rectangle, the radius vectors of which are inclined at angles α and β to the positive direction of the axis OX , that is, the coordinates of these points are dependent on the values of the entered angles.

It is the ratio:

$$OA = x = m = ar \cos \phi, \quad OB = y = n = ar \sin \phi. \quad (14)$$

$$S_1 = \int_{\beta}^{\pi/2} d\phi \int_0^1 abrd r - \frac{1}{2} n^2 \operatorname{tg}(\frac{\pi}{2} - \beta) = \frac{1}{2} ab(\frac{\pi}{2} - \beta) - \frac{1}{2} n^2 \operatorname{ctg}\beta. \quad (15)$$

$$S_2 = \frac{1}{2} ab\alpha - \frac{1}{2} n^2 \operatorname{tg}\alpha. \quad (16)$$

$$\begin{aligned} S_3 &= \int_{\beta}^{\arctg \frac{n}{m}} d\phi \int_1^{\frac{m}{a \cos \phi}} abrd r + \int_{\arctg \frac{n}{m}}^{\beta} d\phi \int_1^{\frac{n}{b \sin \phi}} abrd r = \frac{1}{2} ab \int_{\alpha}^{\arctg \frac{n}{m}} (\frac{m^2}{a^2 \cos^2 \phi} - 1) d\phi + \\ &+ \frac{1}{2} ab \int_{\arctg \frac{n}{m}}^{\beta} (\frac{n^2}{b^2 \sin^2 \phi} - 1) d\phi = \frac{1}{2} ab \left(\frac{m^2}{a^2} \operatorname{tg}\phi - \phi \right) \Big|_{\alpha}^{\arctg \frac{n}{m}} + \frac{1}{2} ab \left(-\frac{n^2}{b^2} \operatorname{ctg}\phi - \phi \right) \Big|_{\arctg \frac{n}{m}}^{\beta} = (17) \\ &= \frac{1}{2} ab \left(\frac{mn}{a^2} + \frac{mn}{b^2} + \alpha - \beta - \frac{m^2}{a^2} \operatorname{tg}\alpha - \frac{n^2}{b^2} \operatorname{ctg}\beta \right). \end{aligned}$$

Thus, the total area S over which the given geometric figures do not coincide, and the value of which is dependent on the two entered angles α and β , is as follows:

$$S = S_1 + S_2 + S_3 = \frac{1}{2}ab\left(\frac{\pi}{2} - \beta\right) - \frac{1}{2}n^2 \operatorname{ctg}\beta + ab\alpha - \frac{1}{2}m^2 \operatorname{tg}\alpha + \frac{1}{2}ab\left(\frac{mn}{a^2} + \frac{mn}{b^2} + \alpha - \beta - \frac{m^2}{a^2} \operatorname{tg}\alpha - \frac{n^2}{b^2} \operatorname{ctg}\beta\right). \quad (18)$$

So, it is a certain function S dependent on two arguments α and β which we have to examine for extremum (minimum):

$$\begin{cases} S'_\alpha = \frac{1}{2}ab - \frac{1}{2} \cdot \frac{m^2}{\cos^2 \alpha} + \frac{1}{2}ab - \frac{1}{2} \cdot \frac{m^2 b}{a \cos^2 \alpha} = 0 \\ S'_\beta = -\frac{1}{2}ab + \frac{1}{2} \cdot \frac{n^2}{\sin^2 \beta} - \frac{1}{2}ab + \frac{1}{2} \cdot \frac{n^2 a}{b \sin^2 \beta} = 0 \end{cases}. \quad (19)$$

Regarding the values of the trigonometric functions of the sought angles, the following relations are:

$$\begin{cases} \cos^2 \alpha = \frac{m^2(a+b)}{2a^2b} \\ \sin^2 \beta = \frac{n^2(a+b)}{2ab^2} \end{cases},$$

and finally:

$$\begin{cases} \cos \alpha = \frac{m}{a} \sqrt{\frac{a+b}{2b}} \\ \sin \beta = \frac{n}{b} \sqrt{\frac{a+b}{2a}} \end{cases}. \quad (20)$$

The effective value of the size of the area, on which the difference between the areas of the geometric objects specified in clause 2 is observed. It is determined as follows:

$$S^* = 4 \cdot \left[\frac{1}{2}ab \cdot \arccos\left(\frac{n}{b} \sqrt{\frac{a+b}{2a}}\right) - \frac{1}{2}n^2 \sqrt{\frac{1 - \frac{n^2(a+b)}{2ab^2}}{\frac{n}{b} \sqrt{\frac{a+b}{2a}}}} + ab \cdot \arccos\left(\frac{m}{a} \sqrt{\frac{a+b}{2b}}\right) - \right.$$

$$\begin{aligned}
 &-\frac{1}{2}m^2 \frac{\sqrt{1-\frac{m^2(a+b)}{2a^2b}}}{\frac{m}{a}\sqrt{\frac{a+b}{2b}}} + \frac{1}{2}ab\left(\frac{mn}{a^2} + \frac{mn}{b^2} + \arccos\left(\frac{m}{a}\sqrt{\frac{a+b}{2b}}\right) - \right. \\
 &\left. -\arccos\left(\frac{n}{b}\sqrt{\frac{a+b}{2a}}\right) - \frac{m^2}{a^2} \frac{\sqrt{1-\frac{m^2(a+b)}{2a^2b}}}{\frac{m}{a}\sqrt{\frac{a+b}{2b}}} - \frac{n^2}{b^2} \frac{\sqrt{1-\frac{n^2(a+b)}{2ab^2}}}{\frac{n}{b}\sqrt{\frac{a+b}{2a}}}\right) \square
 \end{aligned} \quad (21)$$

(A well-known trigonometric identity is used here: $\arccos \xi = \frac{\pi}{2} - \arcsin \xi$ and the fact of symmetry is taken into account, that is, the area on which the specified figures do not match is located in all quarters, so the multiplier appears).

Consequence 1. If the rectangle as a partial case turns into a square, i.e. $m = n = a, a = b = R \Rightarrow$ then $\cos \alpha = \frac{a}{R}$, and it is a partial result of item 1.

Consequence 2. The case of effective extreme (minimum) mutual locations of the area of the circle of radius R with the rectangle $2m \times 2n$ is achieved from the condition: $a = b = R$, then the following relations are:

$$\left\{ \begin{array}{l} \cos \alpha = \frac{m}{R} \\ \sin \beta = \frac{n}{R} \end{array} \right., \text{ which makes it possible to set the values of unknown}$$

angles α and β .

3. Research on the non-coincidence of arc lengths of plane figures

Now let us have a square with the length of its side equal to $2a$ (Figure 3). That is, $OA = a, OM = ON = r$ is the radius of the circle centered at point O . This point is also the center of the entered square. φ is the value of the desired optimal angle between the leg OA and the hypotenuse ON , which depends on the ratio between the length of the side of the square and the radius of the circle to obtain the optimal (minimum) value of the function that characterizes the value of the length of the total line along which the divergence of the mentioned geometric figures is observed.

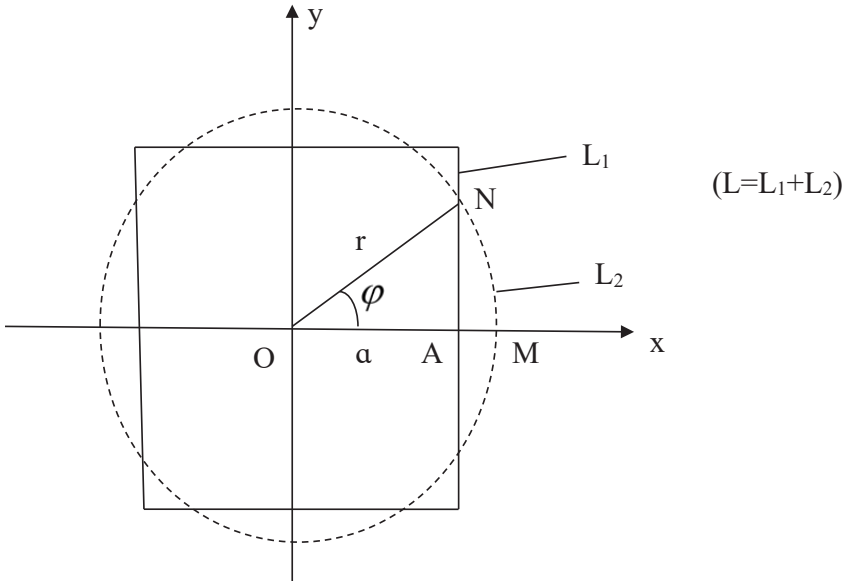


Figure 3. Covering the circle with a square with a common center, determining the unknown argument ϕ , which depends on the length of the line L along which the figures do not coincide

Source: developed by the authors according to [12]

Let the length L be the total value of the lengths L_1 and L_2 within 45° of the first quarter, where the figures indicated at the beginning of the work do not match (have a discrepancy). Obviously, due to the symmetry of the drawing, the total length of the divergence line L_{full} is equal to $8L$.

From the right triangle OAN , obvious trigonometric relations are:

$$\cos\phi = \frac{a}{r} \Rightarrow a = r\cos\phi. \quad (22)$$

For the first half of the first quarter, a corner limit ϕ is:

$$\phi \in [0; 45^\circ].$$

$$\text{There is also: } L_1 = a - r\sin\phi = r(\cos\phi - \sin\phi), \quad (23)$$

$$L_2 = r\phi, \quad (24)$$

$$L = L_1 + L_2 = r\phi + r(\cos\phi - \sin\phi) = r(\phi + \cos\phi - \sin\phi). \quad (25)$$

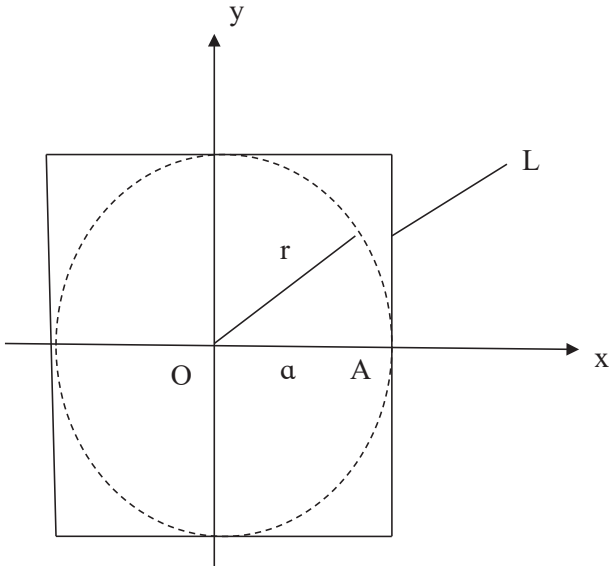


Figure 4. A circle inscribed in a square as the limit maximum value of the outer line of divergence of geometric figures at $\varphi = 0$

Source: developed by the authors according to [12]

Here L – the function that sets the total length of the arc of divergence of geometric figures. Having found the first derivative of this function by its argument φ and equating this value of the found derivative to 0, the equation is obtained:

$$L'_{\varphi} = r(1 - \sin\varphi - \cos\varphi) = 0, \quad (26.1)$$

$$\text{whence: } \sqrt{2}\cos(\varphi - 45^{\circ}) = 1. \quad (26.2)$$

Taking into account the condition (23), the critical value of the angle is $\varphi = 0$. The second derivative of the function L is found and its sign at $\varphi = 0$ is set:

$$L''_{\varphi\varphi} = r(-\cos\varphi + \sin\varphi),$$

$$L''(\varphi = 0) = r(-\cos 0 + \sin 0) = -r < 0 \Rightarrow L \rightarrow \max \quad (27)$$

For $\varphi = 0$, the ratio between the geometric characteristics of the radius of the circle and half the length of the side of the square are obtained:

$$a = r\cos 0 \Rightarrow a = r \quad (28)$$

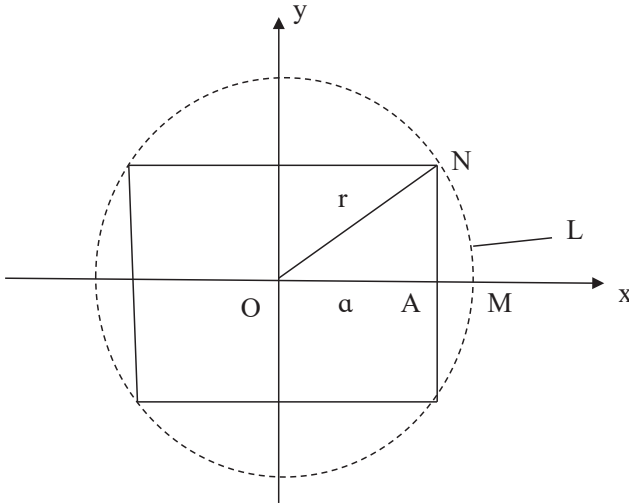


Figure 5. A square inscribed in a circle as the marginal minimum value of the outer line of divergence of geometric figures at $\varphi = 45^\circ$

Source: developed by the authors according to [12]

It means that in the case of a circle inscribed in a square (Figure 4), the value of L , and therefore the total length of the outer line along which the geometric shapes diverge, acquires a maximum number that is equal within all quarters:

$$L_{full\ max}(\varphi = 0) = 8L = 8a = 8r. \quad (29)$$

Another extreme situation is if a square is inscribed in a circle (Figure 5). In this case

$$L_{full\ min}(\varphi = 45^\circ) = 2\sqrt{2}a = 2\pi r. \quad (30)$$

In this section, a problem similar to the one considered in the previous point 1 is solved and investigated, with the difference that the condition of the effective location of the arc of a circle relative to an equilateral triangle with the common center of these figures is established (Figure 6). Let ABC be a triangle with side a whose center coincides with the plane of a circle of radius r . The following geometric characteristics of the lengths of the

corresponding segments are: $OH = r$, $OR = \frac{a\sqrt{3}}{6}$, $RH = \sqrt{r^2 - \frac{a^2}{12}}$,

$$HC = \frac{a}{2} - \sqrt{r^2 - \frac{a^2}{12}}, \quad (31)$$

$$\cos\beta = \frac{a\sqrt{3}}{6r} \Rightarrow r = \frac{a\sqrt{3}}{6\cos\beta}, \quad \beta \in [0; 60^\circ]. \quad (32)$$

We can introduce the function L , which sets the total length of the line of divergence of the reduced geometric figures with respect to one of the vertices of the triangle, namely the vertex C , and this function is defined as follows:

$$L = 2TC + \cup TP = 2\left(\frac{a}{2} - \sqrt{r^2 - \frac{a^2}{12}}\right) + 2r\beta, \quad a \leq 6r \quad (33.1)$$

$$L = \left(a - 2\sqrt{r^2 - \frac{a^2}{12}}\right) + 2rarccos \frac{a\sqrt{3}}{6r}. \quad (33.2)$$

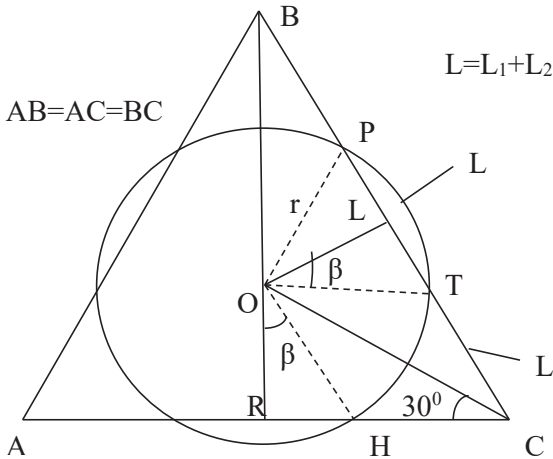


Figure 6. Covering the circle with an equilateral triangle ABC with a common center O, determining the unknown argument β , which depends on the length of the line L along which the figures do not coincide

Source: developed by the authors according to [12]

In this problem, it is useful to introduce the function l reduced to L , namely:

$$l = \frac{L}{r} = \frac{a}{r} - 2\sqrt{1 - \frac{a^2}{12r^2}} + 2\arccos \frac{a\sqrt{3}}{6r}. \quad (34.1)$$

Let $t = \frac{a}{r}$ be the only argument of the function l that depends on the share of the ratio a to r , then with regard to l , it is:

$$l = t - 2\sqrt{1 - \frac{t^2}{12}} + 2\arccos \frac{t\sqrt{3}}{6}. \quad (34.2)$$

Find and equate to 0 the first derivative l' : $l' = 1 + \frac{t - 2\sqrt{3}}{6\sqrt{1 - \frac{t^2}{12}}} = 0. \quad (35.1)$

The obtained equation (35.1) with regard to the unknown t in the form is rewritten:

$$6^2 \left(1 - \frac{t^2}{12}\right) = (2\sqrt{3} - t)^2. \quad (35.2)$$

Wherefrom $t^2 - \sqrt{3}t - 6 = 0 \Rightarrow D = 27, \sqrt{D} = 3\sqrt{3} \Rightarrow t = 2\sqrt{3}$, (is the only logical solution of equation (35.2), since $t > 0$). At this value of t , according to relation (32), the equation is $\cos\beta = 1$ or $\beta = 0$. The found answer of the desired angle β means, due to the necessary condition of the existence of extreme values, that on the interval $\beta \in [0; 60^\circ]$ the function l (and therefore the entire total line L) is monotonic depending on the value of its argument, and therefore reaches its smallest and largest possible values at the ends of the corresponding segment from the specified area of definition of these functions.

When $\beta = 0$, it is the situation of reaching the minimum of the function l , which is geometrically interpreted by placing the initial figures as a circle inscribed in an equilateral triangle (Figure 7). In this case ($\beta = 0$), the full length of the line that determines the divergence of the given geometric figures is equal to:

$$L_{full\ min} = L_{\Delta ABC} = 3a = \frac{18}{\sqrt{3}}r. \quad (36)$$

Another limit, but already the maximum value of the total length of the divergence line, is reached at the value of the β parameter equal to 60° . In this case, geometrically, we have the situation when the arc of a circle describes a given equilateral triangle (Figure 8).

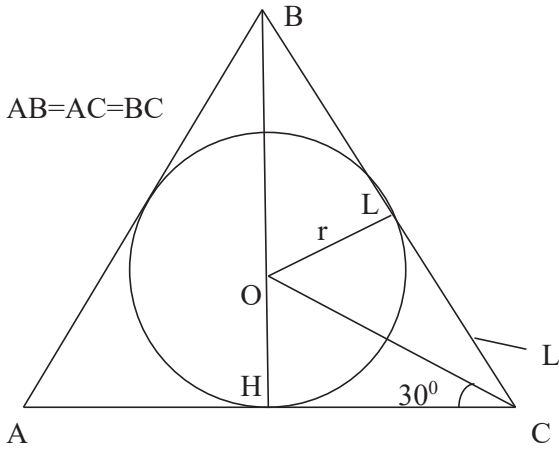
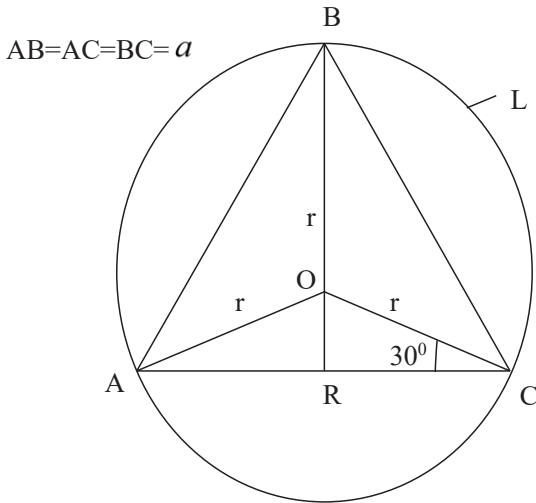


Figure 7. The limit position of geometric shapes as a circle inscribed in an equilateral triangle at the value of the parameter $\beta = 0$, for which the function L reaches its minimum value

Source: developed by the authors according to [12]



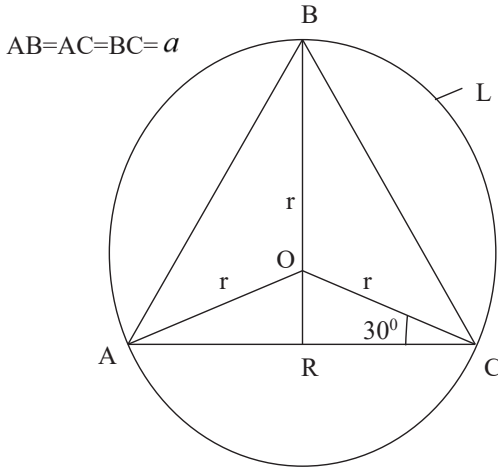


Figure 8. The limit arrangement of geometric figures as described around an equilateral triangle by a circle with center at point O at the value of the parameter $\beta = 60^\circ$, for which the function L reaches its maximum value.

Source: developed by the authors according to [12]

At the same time, the full length of the outer line, along which the specified geometric curves diverge, obviously coincides with the length of the arc of the entire circle:

$$L_{full\ max} = 2\pi r = \frac{2\pi}{\sqrt{3}} a . \quad (37)$$

4. Investigation of the movement of fluid particles in the gravitational field and its application

In real agro-industrial production, the processes and phenomena of the movement of various particles in an air or liquid environment take place under the condition of the action of the gravitational field. For example: in the liquid state, water in the form of drops falls on the soil during precipitation or irrigation [16]; seeds or mineral fertilizers are moved from seeding machines to grooves that are formed in the soil after the coulters of seeders pass through; grain moves from top to bottom in the

cleaning systems of grain harvesters or during unloading from the hopper, etc. The same phenomena occur in machines for post-harvest processing of grain, harvesting root vegetables, fruits, etc. Falling particles have both positive and negative results. It has been established that during rain, the speed of falling drops reaches about 10 m/s [16], which leads to the separation of soil particles and contributes to the development of water erosion. Experimental separation of soil particles under the action of raindrops is studied with the help of special white boards with divisions in centimeters applied to them. Installation of these boards on a clean cultivated field makes it possible to measure the height and number of soil particles separated by drops during rain of different intensities [16]. Due to the erosion and pushing of small soil particles into the pores of the field surface, raindrops falling with high kinetic energy lead to soil compaction. The compacted surface of the field reduces the absorption of rainwater by the soil, which contributes to the growth of water erosion, especially on sloping lands with significant steepness.

Some applied problems that theoretically describe the processes and phenomena of real agro-industrial production should be considered.

The body of mass m falls vertically downward under the force of its own weight. At the same time, this body is acted upon by the resistance force of the medium (air) in which the body moves. At the same time, this resistance force is proportional to the speed of the body at the given moment of time with the coefficient of proportionality k . The dependence of the path traveled by the body during its vertical flight is established, if we know at the initial moment of time at $t = 0$ its coordinates S , its location and initial speed, which is equal to 0, i.e. $S(0) = 0$, $V(0) = 0$.

On the basis of Newton's 2nd law, which describes the movement of a body, $ma = \sum F_i$, where $a = a(t)$ is the function of the acceleration of the body at the moment of time t , the algebraic sum of all active forces is written on the right side of the equation, which acts on this body. Since $a = V'$, where $V = V(t)$ is a function of the speed of the body, which changes with the passage of time t . Then, using the condition of the problem, it is:

$$mV' = mg - kV, \quad (38)$$

whence g – has become the acceleration of free fall.

Using simple algebraic transformations, we write the last equality (38) in the form:

$$V' + \frac{k}{m}V = g, \quad V(0) = 0. \quad (39)$$

This differential equation of the 1st order is subject to an analytical solution:

$$\begin{aligned} V(t) &= \alpha(t)\beta(t), \\ V' &= \alpha'\beta + \beta'\alpha, \\ \alpha'\beta + \frac{k}{m}\alpha\beta + \beta'\alpha &= g, \\ \beta\left(\alpha' + \frac{k}{m}\alpha\right) + \beta'\alpha &= g. \end{aligned}$$

Regarding the first unknown function $\alpha(t)$, the following condition is:

$$\frac{d\alpha}{dt} = -\frac{k}{m}\alpha \quad \text{or} \quad \frac{d\alpha}{\alpha} = -\frac{k}{m}dt.$$

Integrating the last equality, we get: $\ln|\alpha| = -\frac{k}{m}t$. Herefrom

$$\alpha(t) = e^{-\frac{k}{m}t}. \quad (40)$$

Having found $\alpha(t)$, we get the following differential equation with respect to the unknown function $\beta = \beta(t)$:

$$e^{-\frac{k}{m}t} \frac{d\beta}{dt} = g, \quad \text{або} \quad d\beta = ge^{\frac{k}{m}t} dt.$$

The last equality and equality is integrated: $\beta(t) = \frac{mg}{k}e^{\frac{k}{m}t} + C_1$. (41)

Regarding to the function $V(t)$, the following value is:

$$V(t) = e^{-\frac{k}{m}t} \left(\frac{mg}{k}e^{\frac{k}{m}t} + C_1 \right) = \frac{mg}{k} + C_1e^{-\frac{k}{m}t}.$$

Whereas $V(0) = 0$ – due to one of the initial conditions. Then:

$$V(0) = 0 = \frac{mg}{k} + C_1e^0.$$

So

$$C_1 = -\frac{mg}{k}.$$

Thereby:

$$V(t) = \frac{mg}{k} \left(1 - e^{-\frac{k}{m}t} \right). \quad (42)$$

It is known that the coordinates $S = S(t)$, which establish the position of the body during its vertical fall, are defined as $S'(t) = V(t)$, therefore

$$\frac{ds}{dt} = \frac{mg}{k} \left(1 - e^{-\frac{k}{m}t} \right),$$

or

$$ds = \frac{mg}{k} \left(1 - e^{-\frac{k}{m}t} \right) dt. \quad (43)$$

The right and left sides of the last equality are integrated and then we get:

$$S(t) = \frac{mg}{k} \left(t + \frac{m}{k} e^{-\frac{k}{m}t} \right) + C_2, \quad S(0) = 0.$$

Therefore, with regard to constants C_2 the equality is: $0 = \frac{m^2g}{k^2} e^0 + C_2$.

herefrom $C_2 = -\frac{m^2g}{k^2}$.

Thus, coordinate S changes along the vertical axis according to the following law:

$$S(t) = \frac{mg}{k} \left(t + \frac{m}{k} \left(e^{-\frac{k}{m}t} - 1 \right) \right). \quad (44)$$

Asymptotically at infinity, this function converges to a linear dependence of the proportional value

$$\frac{mg}{k} \left(t - \frac{m}{k} \right).$$

As an intermediate conclusion, we can note that this mathematical model takes place at relatively small velocities. For example, the vertical immersion of a body (a diver, a hydrobathyscaphe, a submarine, etc.) agrees well with the obtained result (44) of the above mathematical model studies.

The above mathematical model becomes more complicated to some extent when the vertical fall of the same body at a relatively high speed is investigated, for example, a body falling from a great height in the atmosphere. Then it is assumed that the drag force of the medium is proportional to the square or even the cube of the velocity of the body at the given moment of time.

Essentially the same problem and its solution in the case when the resistance force of the medium when a body falls is proportional to the square of the speed of this body is considered. The equation of the form is:

$$mV' = mg - kV^2. \quad (45)$$

Under the initial condition $S(0) = 0, V(0) = 0$.

The equation is rewritten in the form:

$$\frac{m}{k} \frac{dV}{dt} = \frac{dV}{k} - V^2, \text{ а } \text{б} \text{о } \frac{dV}{\frac{mg}{k} - V^2} = \frac{k}{m} dt.$$

Integrating the last equality, it is:

$$\frac{1}{2\sqrt{\frac{mg}{k}}} \ln \left(\frac{\sqrt{\frac{mg}{k}} + V}{\sqrt{\frac{mg}{k}} - V} \right) = \frac{k}{m} t + C_1. \quad (46)$$

Due to the condition $V(0) = 0$, the value of the constant is $C_1 = 0$, then

$$\frac{1}{2} \sqrt{\frac{k}{mg}} \ln \left| \frac{\sqrt{\frac{mg}{k}} + V}{\sqrt{\frac{mg}{k}} - V} \right| = \frac{k}{m} t, \quad (46^*)$$

or

$$\ln \left| \frac{\sqrt{\frac{mg}{k}} + V}{\sqrt{\frac{mg}{k}} - V} \right| = 2\sqrt{\frac{kg}{m}} t, \text{ or } \frac{\sqrt{\frac{mg}{k}} + V}{\sqrt{\frac{mg}{k}} - V} = e^{2\sqrt{\frac{kg}{m}} t}.$$

Let us find $V = V(t)$ from the last equality:

$$V(t) = \sqrt{\frac{mg}{k}} \frac{e^{2\sqrt{\frac{kg}{m}} t} - 1}{e^{2\sqrt{\frac{kg}{m}} t} + 1} = \sqrt{\frac{mg}{k}} \frac{e^{\sqrt{\frac{kg}{m}} t} - e^{-\sqrt{\frac{kg}{m}} t}}{e^{\sqrt{\frac{kg}{m}} t} + e^{-\sqrt{\frac{kg}{m}} t}} = \sqrt{\frac{mg}{k}} \frac{Sh\left(\sqrt{\frac{kg}{m}} t\right)}{Ch\left(\sqrt{\frac{kg}{m}} t\right)} = \sqrt{\frac{mg}{k}} th\left(\sqrt{\frac{kg}{m}} t\right). \quad (47)$$

As in the previous model, $\frac{dS}{dt} = V(t)$, therefore

$$dS = \sqrt{\frac{mg}{k}} th\left(\sqrt{\frac{kg}{m}} t\right) dt. \quad (48)$$

Integrating the last equality, we get:

$$S(t) = \sqrt{\frac{mg}{k}} \int th \left(\sqrt{\frac{kg}{m}} t \right) dt = \sqrt{\frac{mg}{k}} \cdot \sqrt{\frac{m}{kg}} \int \frac{dch \left(\sqrt{\frac{kg}{m}} t \right)}{ch \left(\sqrt{\frac{kg}{m}} t \right)} = \frac{m}{k} \ln ch \left(\sqrt{\frac{kg}{m}} t \right) + C_2. \quad (49)$$

Taking into account the condition $S(0) = 0$, $ch0 = 1$, we obtain:
 $S(0) = 0 = \frac{m}{k} \ln 1 + C_2$. Wherefrom $C_2 = 0$.

Thus, the final value of the sought function $S(t)$ takes the form:

$$S(t) = \frac{m}{k} \ln ch \left(\sqrt{\frac{kg}{m}} t \right). \quad (50)$$

Now the speed of the body is proportional to the cube of the speed of the given body, that is, the equation of motion of such a body is as follows:

$$mV' = mg - kV^3. \quad (51)$$

Analytically, this equation can be solved halfway by integrating the first time and setting the value of the velocity function that satisfies condition (51):

$$\frac{m}{k} \frac{dV}{dt} = \frac{mg}{k} - V^3, \text{ wherefrom}$$

$$\frac{dV}{h^3 - V^3} = \frac{m}{k} dt, \text{ here } h = \sqrt[3]{\frac{mg}{k}} \text{ or } \frac{k}{m} = \frac{g}{h^3}.$$

When integrating the left part of the resulting differential equation in differentials, the following transformations are used:

$$\frac{1}{h^3 - V^3} = \frac{A}{h - V} + \frac{BV + C}{h^2 - hV + V^2}. \quad (52)$$

Skipping the search for undefined coefficients, their found values are presented:

$$A = B = \frac{1}{h^2}, C = 0.$$

$$\int \frac{dV}{h^3 - V^3} = -\frac{1}{h^2} \ln|h - V| + \frac{1}{h^2} \ln|V^2 - hV + h^2| +$$

$$+ \frac{1}{\sqrt{3}h^2} \operatorname{arctg} \left(\frac{h - 2V}{\sqrt{3}h} \right). \quad (53)$$

Therefore, the following complex expression for the function $V=V(t)$ is:

$$\frac{1}{h^2} \left(\ln \left| \frac{\sqrt{V^2 - hV + h^2}}{h - V} \right| + \frac{1}{\sqrt{3}} \operatorname{arctg} \left(\frac{h - 2V}{\sqrt{3}h} \right) \right) = \frac{g}{h^3} t + C_1. \quad (54)$$

Since the first initial condition, under which $V(0) = 0$, is preserved, this makes it possible to set the value of the integration constant C_1 :

$$\frac{1}{h^2} \cdot \frac{1}{\sqrt{3}} \operatorname{arctg} \frac{1}{\sqrt{3}} = C_1. \quad (55)$$

So

$$\ln \left| \frac{\sqrt{V^2 - hV + h^2}}{h - V} \right| + \frac{1}{\sqrt{3}} \operatorname{arctg} \left(\frac{h - 2V}{\sqrt{3}h} \right) - \frac{gt}{h} = \frac{\pi}{6\sqrt{3}}, \quad (56)$$

where $h = \sqrt[3]{\frac{mg}{k}}$.

The condition obtained in relation to $V(t)$ is an implicit rather complex mathematical function, from which it is quite difficult to analytically establish the function V itself, and therefore to find another function $S(t)$, such that $S'(t) = V(t)$.

The development of theoretical prerequisites for some technological processes of agro-industrial production and the functioning of technical systems contributed to the optimization of the parameters of special agricultural machines. In addition, the results of the conducted scientific research showed that the high quality of the educational process is achieved when future agricultural engineers conduct in-depth theoretical calculations of machines, participate in experimental laboratory and field studies of developments.

An example of the use of elements of the algorithm of theory and calculation of nozzles and devices of sprinkler machines and installations in scientific

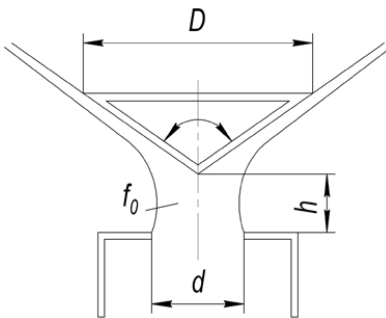


Figure 9. Scheme of the deflector nozzle

Source: [18]

and technical activities and technologies for training agricultural engineers for innovative design activities is given.

In Figure 9 it is shown a scheme of a reflector nozzle, which is used in sprinkler machines and installations.

The following ratios $h = d$; $D = 2d$; $2\theta = 1200 h = d$ are usually accepted (Figure 9).

The irrigation radius R can be determined by the following formula:

$$R = \frac{H}{0,43 + 0,0014 \frac{H}{d}}, \quad (57)$$

where H – pressure in front of the nozzle opening.

One of the important advantages of slot nozzles (Figure 10) is their simplicity.

Such a nozzle can be obtained by sawing on any pipe. The angle of the spray torch φ_p is determined from the following ratio: $\varphi_p = (0,7 \div 0,9) \varphi$.

Smaller values of the digital coefficient correspond to smaller values of the angle φ . The radius of the irrigated sector is determined by the formula:

$$R = \frac{H}{1,15 + 0,0003 \frac{H}{h}}. \quad (58)$$

The recommended ratio of the length of the gap to its width is 1:5, 1:10. The irrigated area has the shape of a sector with an angle φ_p . At a distance of approximately $1/5R$ from the nozzle, irrigation does not occur.

Medium jet sprinklers are used on most modern sprinkler machines and installations. Their designs are mostly of the same type, although they have some significant differences.

The principle of operation is approximately as follows: the stream flowing out of the barrel meets a rotary deflector and a deflector blade on its way. At the same time, the front end of the rocker

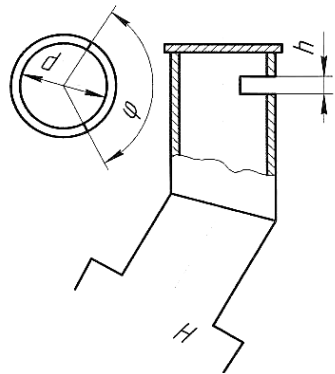


Figure 10. Scheme of a slot nozzle

Source: [18]

arm is pushed to the side, and the rocker arm itself turns. When exiting the jet, the deflector turns relative to the rocker arm. The rocker arm, twisting the spring, turns to an angle of about 90° , then under the action of the spring it returns to its previous position.

After reaching the initial position, the rocker arm strikes the trunk with the tide that is on it and turns it to an angle of $2-5^\circ$ in the direction of movement. At this moment, the jet hits the back of the deflector and returns it to its original position. Then the front end of the rocker arm is pushed out of the jet again and the process repeats.

The barrel rotates intermittently. The stream irrigates a circular area. The design can work without a chopper blade. In this case, a more accurate adjustment of the position of the deflector relative to the jet is required.

Under the conditional replacement of the action of the jet on the curved rotary deflector and the vane, the action is only on one vane, located on the middle radius, according to Figure 11, *a* and *b*, the angular velocity of the rocker arm after the deflector exits the jet is

$$\omega = 2 \sqrt{\frac{\rho f r \phi \sin \alpha}{J_k}}, \quad (59)$$

where J_k – moment of inertia of the rocker arm; ϕ is the rocker rotation angle at which the jet affects the deflector, $\phi = (2 \div 3)\alpha_1$; p is the pressure in front of the nozzle.

Due to the fact that the flow is not completely deflected by the deflector

$$\alpha = (0,7 - 0,8)\alpha_1, \quad (60)$$

The maximum angle of rotation of the rocker arm

$$\phi_{\max} = \frac{2\rho f r \sin \alpha}{M_n}, \quad (61)$$

where M_n – the average value of the moment created by the spring during the rotation of the rocker arm.

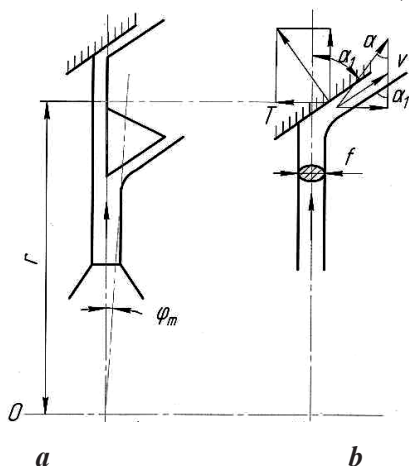


Figure 11. Scheme of the deflector of the medium stream sprinkler

Source: [18]

The angle of rotation of the device in one stroke of the rocker arm

$$\psi_a = \frac{8\rho fr\phi \sin \alpha}{M_T} \left[\frac{me^2}{J_k} + \cos^2 \theta \right] \cos^2 \theta, \quad (62)$$

where M_T – the moment of friction, which inhibits the rotation of the device and is equal to the sum of the moments of friction in the bearing and the rubber cuff.

The full time of the rocker arm in both directions

$$t = 4 \frac{\sqrt{J_k \rho fr \sin \alpha}}{M_n}. \quad (63)$$

Long-range sprinklers are used in those cases when it is possible and expedient to create a pressure of 4...6 atm in the pipes.

Worm or gear rotating machines are common, powered by a small turbine that rotates under the power of the main jet generated by the turbine-driven machine.

Mechanisms are often introduced into the design to provide the apparatus with reverse-rotational movement inside the sector of the circle for watering the sector in a given direction. The sector angle is usually adjustable.

The stream of water flowing out of the nozzle causes the turbine to rotate. On the axis of the turbine there is a worm, which rotates a worm gear. The gear through the shaft transmits rotation to the worm connected to the worm gear. There is an eccentric finger on the axis of the gear, which forces the frame to swing. A ratchet mechanism consisting of two hinged rods, a spring and a dog is installed on the frame. Under the action of a compressed spring, the dog is pushed away from the fixed gear on one side, due to which the sprinkler rotates slowly and intermittently around the vertical axis. The fixed gear has a row of extendable fingers. When the dog shank rests on one of them, its position changes, its other side starts working and the direction of rotation of the device changes. In this way, irrigation can be carried out by sector. The number of revolutions of the turbine per minute is determined by the formula:

$$n_r = \frac{60u}{\pi D}, \quad (64)$$

where u – circular velocity of blade ends in m/s; D – diameter of the turbine at the ends of the small blades, m.

The circular speed of the ends of the blades can be determined by the formula

$$u = k_0 v, \quad (65)$$

where k_0 – coefficient of circular speed, $k_0 = 0,7 \dots 0,8$; v – velocity of liquid in the jet, m/s.

The speed of the liquid in the jet is determined by the formula

$$v = \phi \sqrt{2gH}, \quad (66)$$

where ϕ – speed coefficient, $\phi = 0,97$; g – free fall acceleration, $g = 9,8 \text{ m/c}^2$; H – pressure in front of the nozzle, meters of water column.

The number of revolutions of the far jet sprinkler is equal to:

$$n_a = \frac{n_T}{i}, \quad (67)$$

where i – the total gear ratio of the gearbox between the turbine and the body of the device.

The flight range of the jet for medium and long jet nozzles can be determined by one of the following empirical formulas:

$$R = 0,42H + 1000d \quad (68)$$

or

$$R = \frac{H}{0,5 + 0,25 \frac{H}{d}}. \quad (69)$$

The formulas are valid for an angle of inclination of the jet to the horizon of 30° , that is, for the angle corresponding to the greatest range, and the ratio

$$\frac{H}{d} \geq 800, \quad (70)$$

де H – pressure in front of the nozzle, meters of water column,
 d – jet diameter, meters.

The range of the jet can also be determined using the experimental graph shown in Figure 12. A series of curves defining the relationship between H and R has been constructed for different diameters of the jets. The decay of the jet on the droplet is determined by the H/d ratio (Table 1).

The range of the jet, determined on the graph (Figure 12) can be obtained only with a correctly designed barrel and a non-rotating sprinkler.

The flight range of the jet during the rotation of the device around the vertical axis with a number of revolutions of $0.3 \dots 1$ per minute decreases by $10 \dots 15\%$ compared to the flight range of the jet without rotation of the device. The rational shape of the barrel is shown in Figure 13. A larger angle

Table 1

Disintegration of the jet on a droplet

H/d	Characteristics of the jet
до 900	Solid, which does not fall apart in drops
900–1500	Weak disintegration into drops, not suitable for irrigation
1500–1600	Disintegration into drops of medium size, which are suitable for watering grass in meadows and pastures
1700–1800	Disintegration into drops of medium fineness, suitable for irrigation of closed rural areas of agricultural cultures
2000–2200	Disintegration into small drops, suitable for irrigation of all crops
2500–2600	Disintegration into very small drops, suitable for watering the seedlings of the most delicate plants and flowers

Source: [19]

of inclination to the horizon is selected at pressure of 1.5...3 atm, a smaller one at pressure of more than 6...8 atm. The stabilizer 1, which consists of a number of partitions parallel to the axis of the trunk, dividing its live section into a number of narrow channels, is of significant importance for the formation of the initial section of the jet.

The length of the cells of the tranquilizer should be 12-15 times greater than their width. The diameters of the barrel and tranquilizer should be 3-4 times larger than the diameter of the nozzle. In the elbow, it is necessary to install

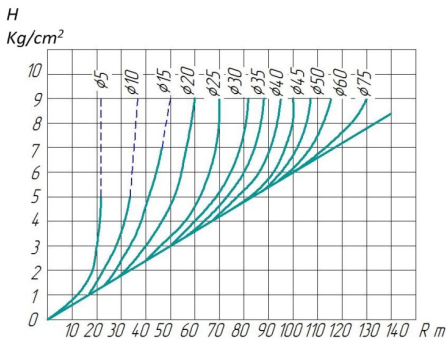


Figure 12. Graph for determining the flight range of the jet

Source: [19]

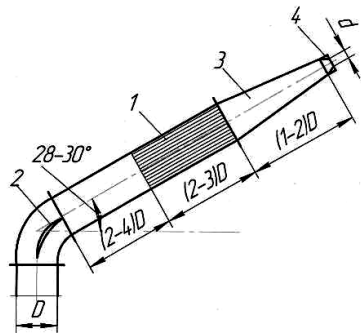


Figure 13. Scheme of a long-range barrel: 1 – sedative; 2 – rib; 3 – cone; 4 – nozzle

Source: [19]

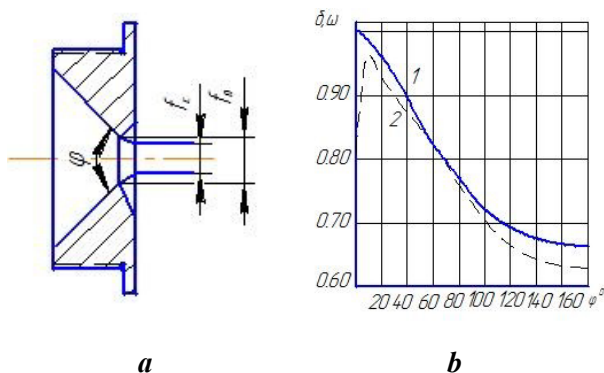


Figure 14. Rational shape of the nozzle

Source: [19]

ribs 2, which prevent the occurrence of transverse velocity circulation in the flow. In medium-flow and some long-flow barrels, the cone 3 is not placed, and the nozzle 4 is located directly behind the outlet end of the tranquilizer. The rational shape of the nozzle is shown in Figure 14, a.

Water consumption through such a nozzle is determined by the formula:

$$Q = \delta f_0 \sqrt{2gH}, \quad (71)$$

where δ – flow coefficient, which is determined depending on the taper angle of the nozzle φ along curve 2 (Figure 14, b).

Curve 1 gives the value of the compression coefficient ε of the jet:

$$\varepsilon = \frac{f_e}{f_0}. \quad (72)$$

This coefficient determines the diameter of the nozzle needed to create the desired jet.

5. Conclusions

After studying the mechanical and technological properties of the soil and other agricultural materials, it is recommended to carry out engineering, technological and energy calculations of machines and equipment for agricultural reclamation. As an example, the laboratory-practical work “Compilation of complete and incomplete schedules of water supply for crop rotation” is partially considered, which is performed for students

of specialty 208 “Agroengineering” during the study of machines and equipment in agricultural reclamation with the aim of studying the features and mastering the method of calculating internal economic network and the method of drawing up an incomplete schedule of water supply in technological processes of irrigation.

First, it is recommended to analyze the main theoretical provisions, as well as general information about irrigation systems.

It is known that the operation of irrigation systems is a set of organizational and technical measures that are necessary to maintain all structures of the system in working condition, to ensure more efficient use of irrigation water.

On the main distribution channels, up to the watersheds on farm land, operation is carried out by the management of the irrigation system by operational sections. Distribution of water on farm lands between individual irrigation plots, as well as irrigation of agricultural crops, is carried out through the intra-farm irrigation network.

The operation of the intra-farm irrigation network carried out by the farm consists of the following technological processes: irrigation of agricultural crops in accordance with the water use plan, in the appropriate agrotechnical terms according to the necessary norms, which ensure the cultivation of high and sustainable crops; maintenance of domestic irrigation and drainage canals and structures on them in working order; diversion of excess water into the drainage network to prevent waterlogging and salinization of land; planting of canals with tree fruit plantations and organization of care for plantations.

Farm workers – irrigators – should take an active part in the specified works related to the operation of the intra-farm irrigation network. They should not only irrigate agricultural crops, but also repair and prepare the irrigation network and structures for irrigation, mow and spray canal slopes with herbicides; to ensure that channels and structures are in good working order; participate in the preparation of intra-household water use plans. Irrigation in farms should be carried out on the basis of a prepared water use plan. The domestic water use plan of the farm is drawn up simultaneously with the production plan.

The results of the research on the search for the conditions of the minimum difference in the coverage of the area of one flat geometric figure by another area can be reflected and applied in various systems of technical vision, navigation and search systems, means of aiming and impression of military objects, tasks of extremely effective use of agricultural areas land, etc.

In this work, the question of the optimal effective mutual placement of two geometric figures with a common center, namely the arc of a circle first relative to a square, and then relative to an equilateral triangle, was investigated. As the main criterion for such mutual locations, it is proposed to take the length of the total line along which such figures diverge (outer line of divergence). The conducted study of the function, which sets the entire length of the line along which the divergence of figures is observed, to the extremum indicates that this function acquires minimum values in the case of such an arrangement of figures, when a circle is inscribed in a square or an equilateral triangle, respectively (Figures 4, 7), and with a circle described around these figures (Figures 5, 8), this function is maximal. The results of these studies, on the one hand, are purely theoretical and classical, on the other hand, these results can be used in some transport problems for efficiency, in solving agronomic issues of optimal land management, etc.

As a result of conducting scientific research with the involvement of fundamental mathematical apparatus, the theoretical foundations of the movement of agricultural materials in liquid and air environments under the influence of a gravitational field, as well as the method of calculating a sprinkler installation, which are expedient to use in design practice and agricultural engineering training of future specialists in modern agro-industrial production, have been developed.

References:

1. Klepko V.Yu., Golets V.L. (2009). *Vyshcha matematyka v prykladakh i zadachakh* [Higher mathematics in examples and Tasks]. 2nd edition. Kyiv: Center of educational literature. (in Ukrainian)
2. Moklyachuk M.P. (2014). *Zbirnyk zadach iz variatsiynoho chyslennya ta metodiv optymizatsiyi* [A collection of problems from calculus of variations and optimization methods]. Education help. Kyiv: Kyiv. University Publishing and Printing Center. (in Ukrainian)
3. Korolsky V.V., Kramarenko T.G., Semerikov S.O., Shokalyuk S.V. (2009). *Inovatsiyini informatsiyno-komunikatsiyini tekhnolohiyi navchannya matematyky* [Innovative information and communication technologies for teaching mathematics]: Tutorial. Kryvyi Rih: Kireevsky book publishing house. (in Ukrainian)
4. Pak V.V., Nosenko Yu.L. (1996). *Vyshcha matematyka* [Higher mathematics]. Kyiv: "Libid" publication. (in Ukrainian)
5. Vysshaya matematika. Sbornik zadach (1991) [Higher Mathematics. Collection of problems] / H.I. Gavrilenko, A.F. Krivoy, P.S. Kropivnyansky and others. Kyiv: Vishcha school. (in Ukrainian)

6. Berezansky Yu.M., Us G.F., Sheftel Z.G. (1990). *Funktsional'nyy analiz* [Functional analysis]. Kyiv: Vishcha schoola. (in Ukrainian)
7. Dorogovtsev A.Ya. (1993). *Matematychnyy analiz* [Mathematical analysis]. Kyiv: Publishing house “Lybid”, part 1. (in Ukrainian)
8. Trish B.M. (2008). *Osnovy vyshchoyi matematyky. Teoremy, pryklady i zadachi* [Basics of higher mathematics. Theorems, examples and problems]: Tutorial. Lviv: Ivan Franko LNU Publishing Center. (in Ukrainian)
9. Valeev K.G., Jalladova I.A. (2001). *Vyshcha matematyka* [Higher mathematics]. Study guide. Part 1. Kyiv. (in Ukrainian)
10. Vasiliev F.P. (2002). *Chysel'ni metody rozv'yazannya ekstremal'nykh zadach* [Numerical methods for solving extreme problems]. M.: Science. (in Ukrainian)
11. Dubchak V.M. (2018). Vstanovlennya umov efektyvnoho pokryttya ploscheyu kruha ploschi kvadrata ta deyaki vypadky uzahal'nennya [Establishing the conditions for effective coverage of the area of a circle by the area of a square and some cases of generalization]. *Vibrations in engineering and technology*, no. 4 (91), pp. 15–20. (in Ukrainian)
12. Dubchak V.M. (2019). Vstanovlennya umov efektyvnoho rozmishchennya kruha vidnosno kvadrata ta rivnostoronn'oho trykutnyka zi spil'nym tsentrom za otsinkoyu dovezhyny liniyi nespivpadinnya [Establishing the conditions for the effective placement of a circle relative to a square and an equilateral triangle with a common center by estimating the length of the line of non-coincidence]. *Vibrations in engineering and technology*, no. (94), pp. 52–58. (in Ukrainian)
13. Liu Zhong (2011). *Hydropower generation unit and its selection / Training course on renewable energy for developing countries*. Changsha: Ministry of Commerce of the People's Republic of China, pp. 178–274.
14. Pryshlyak V.M., Dubchak V.M. (2021). Teoriya i praktyka rukhu sil's'kohospodars'kykh materialiv u ridynnomu ta povitryanomu seredovyshchakh za umovy diyi hravitatsiynoho polya [Theory and practice of the movement of agricultural materials in liquid and air environments under the influence of a gravitational field]. *Technology, energy, transport of agricultural industry*, no. 2 (113), pp. 121–133. (in Ukrainian)
15. Dubchak, V.M., Pryshlyak V.M., Novitskaya, L.I. (2018). *Vyshcha matematika v prykladakh ta zadachakh* [Higher Mathematics in Examples and Tasks]. Vinnytsia: VNAU. (in Ukrainian)
16. Kravchenko, M.S., Zlobin, Yu.A., Tsarenko, O.M. (2002). *Zemlerobstvo* [Agriculture]. Kyiv: Lybid. (in Ukrainian)
17. Kutsenko, A.G., Bondar, M.M., Pryshliak, V.M. (2014). *Biomekhanika sut-sil'nykh seredovyshch: monohrafiya* [Biomechanics of continuous media]. Nizhin: Aspect Polygraph Publishing House LLC. (in Ukrainian)
18. Kaletnik, G.M., Chausov, M.G., Bondar, M.M., Pryshliak, V.M. and others (2011). *Mashyny ta obladnannya v sil's'kohospodars'kyy melioratsiyi* [Machines and equipment in agricultural land reclamation]. Kyiv: High-tech Press. (in Ukrainian)
19. Chovnyk, Yu.V., Pryshliak, V.M., Shymko, L.S., Prykhodko, S.P. (2016). *Optimizatsiya konstruktivnykh tekhnichnykh system* [Optimization of designs of technical systems]. Nizhin: Aspect Polygraph Publishing House LLC. (in Ukrainian)
20. Kutsenko, A.M., Bondar, M.M., Pryshliak, V.M., Shymko L.S. (2015). *Prykladna mekhanika v prykladakh ta zadachakh* [Applied mechanics in examples and problems]. Nizhyn: LLC Publishing House “Aspect Polygraph”. (in Ukrainian)