CHAPTER «ENGINEERING SCIENCES»

SYNTHESIS OF INFORMATION TECHNOLOGIES FOR DECISION SUPPORT UNDER UNCERTAINTY: PROBABILISTIC ASPECTS

Nadiya Honcharova[1]
Alyona Shved[2]

DOI: https://doi.org/10.30525/978-9934-26-355-2-10

Abstract. The scenario approach for decision-making under uncertainty and risk based on probabilistic inference methods is currently being intensively developed. *The purpose* of the paper is to analyze a number of modern methods of probabilistic inference, to develop a set of principles for construction of integrated information technology for their application as a part of mathematical models for decision support systems. *Methodology* of the study is based on general probabilistic inference methods: probabilistic inference in probability trees, the method of condensation of probability distributions, probabilistic inference in Bayesian networks, abductive inference in Bayesian belief networks, probabilistic inference in algebraic Bayesian networks. *Results*. A set of probabilistic inference methods has been analyzed, which represent a powerful means of modeling initial uncertain situations in many applications of artificial intelligence and decision theory. Their advantages and disadvantages, limitations and capabilities are studied. Based on a systematic approach that takes into account the features and conditions of application of probabilistic inference methods an integrated information technology for decision support has been proposed. The approach of aggregation of group expert assessments of the probability of realization of random events for solving the problems of probabilistic inference on probability trees has been proposed. Such an approach allows to synthesize of generalized estimates of

[1] PhD Student,
Petro Mohyla Black Sea National University, Ukraine
[2] Doctor of Technical Sciences, Associate Professor of the Department of Software Engineering,
Petro Mohyla Black Sea National University, Ukraine

the probability of realization of random events formed by a group of experts based on interviews, surveys, focus groups, methods of expert evaluations. To identify and analyze expert information, it is proposed to use the mathematical apparatus of the theory of evidence and the theory of plausible and paradoxical reasoning, which are an effective tool for analyzing and modeling specific types of uncertainty, in the form of incompleteness, inaccuracy, vagueness, and their possible combinations. The synthesis of a group decision is carried out on the basis of a mechanism of expert evidence combination. *Practical implications.* The proposed methodology and information technology provide a theoretical basis for the design of decision support systems under uncertainty and risk in various spheres of human activity. *Value/originality.* The feature of the proposed information technology for the analysis of group expert assessments by using methods of probabilistic inference is that it allows to correctly operate with expert assessments formed under uncertainty (for example, an expert cannot assess the possibility of the occurrence of the analyzed event), incompleteness, inconsistencies (contradictions, conflicts) due to the application of the rules of redistribution of conflicting information in the process of group decision synthesis.

1. Introduction

Currently, probabilistic inference methods are widely used to generate recommendations to the decision maker (DM) and occupy an important place in the mathematical support of various decision support systems (DSS).

Initially, probabilistic inference was presented by methods of testing statistical hypotheses, where, as a rule, one random event is considered, for which the probabilities of its implementation or non-realization are determined. At the same time, real problems in various practical applications can be characterized by systems of random events and diverse connections between these systems.

To solve such problems, instrumental methods of probabilistic analysis were created: probability trees, decision trees, goal trees, belief networks, abductive inference, etc. A fairly large number of publications are devoted to the listed methods, but there is no information about their systematic (complex) application based on the analysis of the initial information characterizing the conditions of their application, advantages and disadvantages of such methods.

The purpose of the paper is to analyze a number of modern methods of probabilistic inference, and to develop a set of principles for construction of integrated information technology for their application as part of mathematical models for decision support systems.

2. Decision support using probabilistic inference techniques

In general case, by the problem of probabilistic inference will mean the task of determining the probability of random events or their combinations, as well as the probability of other events stochastically related to them, based on all the initial information. For construction of integrated information technologies of their application, consider the following group of methods that have become widely used recently in various practical tasks: probabilistic inference in probability trees, the method of condensation of probability distributions, probabilistic inference in Bayesian networks, abductive inference in Bayesian belief networks [13; 14].

To present the listed methods within the framework of integrated technology, it is necessary to analyze the following aspects:

1. To consider methods of obtaining estimates of the probability of occurrence of random events.

2. Determine the influence of dependence or independence of systems of random events on the process of construction of probability trees and networks.

3. To investigate the possibility of applying methods of probabilistic inference depending on the increase in the number of simulated systems of random events.

Let us analyze the considered aspects in more detail. The need to consider methods of obtaining estimates of probabilistic events is dictated by the fact that, as a rule, their preliminary determination is necessary for the implementation of probabilistic inference techniques.

There are two main types of such estimates:

1. Objective (empirical) probabilities.

2. Subjective (expert) probabilities.

The first of them is obtained on the basis of the frequency approach, which consists in obtaining a share from the division of the number of equivalent manifestations (n), which contribute to the realization of events, by the total number of equally possible events (N). At the same time, information about the past implementation of events over a long period of time is used.

Subjective estimates of probabilities, the source of which is an expert or a group of experts, are formed in unique situations, when there is no background history of the realization of random events. This approach entails solving the problems of expert assessments processing.

The existence of the fact of dependence (independence) of systems of random events determines the order of construction of probability trees:

1. The presence of independence allows combining different systems of random events into a tree, in an arbitrary order.

2. The dependence of such systems requires compliance with a certain order when combining them.

Let us give a definition of the dependence (independence) of random events. Two events are independent (an event e_i and event e_j) when the probability of the first event does not depend on whether or not the second event has occurred. In this case, the following expression is used: $p(e_i, e_j) = p(e_i) \cdot p(e_j)$.

Two events (an event e_i and event e_j) are dependent when the probability of event e_i depends on whether the event e_j occurred or not. The probability of an event e_i determined under the condition that the event e_j occurred is called the conditional probability of the event e_j : $p(e_i / e_j)$. Accordingly: if the events e_i and e_j are independent, then $p(e_i / e_j) = p(e_i)$. If the events e_i and e_j are dependent, then the probability of their joint realization (intersection) is equal to $p(e_i \cap e_j) = p(e_j) \cdot p(e_i / e_j)$.

The number of systems of random events is important, since their growth entails an exponential growth in the size of the probability trees. This is especially evident when the number of systems of random events is $m > (3 \div 4)$.

The size of the probability tree can be determined based on the following approach. Each path in the tree from the root node to the final position represents one of all possible combinations of events, called a scenario. Since each scenario forms one possible combination of events, one from each complete system of events, the total number of scenarios can be determined before constructing the probability tree, according to the expression:

$$N = \prod_{i=1}^{m} n_i , \qquad (1)$$

where n_i is the number of events in the i-th system; m is the total number of systems of random events [21, p. 100].

The stated judgments form the basis of the algorithm for the implementation of decision support technology using probabilistic inference methods expressed in IF (antecedent) THEN (consequent) form:

1. (there is a connection between the events) $\wedge$ (interval probability estimates) $\rightarrow$ probabilistic inference in algebraic Bayesian networks;

2. (there is a connection between the events) $\wedge$ (crisp probability estimates) $\wedge$ (the number of random event systems > 4) $\wedge$ (the acyclicity condition is imposed) $\rightarrow$ probabilistic inference in Bayesian belief networks;

3. (there is a connection between the events) $\wedge$ (crisp probability estimates) $\wedge$ (the number of random event systems > 4) $\wedge$ (the acyclicity condition is imposed) $\wedge$ (condensation of probability distributions) $\rightarrow$ probabilistic inference in probability trees with a certain sequence of composite systems of random events;

4. (there is a connection between the events) $\wedge$ (crisp probability estimates) $\wedge$ (the number of random event systems > 4) $\wedge$ (the acyclicity condition is not imposed) $\rightarrow$ markov networks;

5. (there is a connection between the events) $\wedge$ (crisp probability estimates) $\wedge$ (the number of random event systems > 4) $\wedge$ (the acyclicity condition is imposed) $\wedge$ (there is a need to determine the most plausible probabilities) $\rightarrow$ abductive inference in belief networks;

6. (there is a connection between the events) $\wedge$ (crisp probability estimates) $\wedge$ (the number of random event systems ≤ 4) $\rightarrow$ probabilistic inference in probability trees with a certain order of random event systems;

7. (there is not a connection between the events) $\wedge$ (crisp probability estimates) $\wedge$ (the number of random event systems > 4) $\wedge$ (condensation of probability distributions) $\rightarrow$ probabilistic inference in probability trees with arbitrary alternation of composite systems of random events;

8. (there is not a connection between the events) $\wedge$ (crisp probability estimates) $\wedge$ (the number of random event systems ≤ 4) $\rightarrow$ probabilistic inference in probability trees with arbitrary alternation of systems of random events.

As an antecedent in the above proposed production rules, a set of criteria and conditions for the applicability of probabilistic inference methods were used; methods of probabilistic inference are considered as a consequent. If the antecedent is true, then the corresponding method of probabilistic inference has been chosen. The antecedent can be constructed on the basis of operations $\vee$ and $\wedge$, and their combinations.

The problem of increasing the number of systems of random events can be solved by using the method of condensation of probability distributions [12, p. 305], which allows to reduce their dimension.

Condensation of distributions can be performed both by combining or removing some values of random variables, and by a combination of such procedures. The main disadvantage of this method is that some important relevant information may be loss.

At the same time, as a fundamental solution to this problem, the authors of [14] proposed an approach based on a belief network (alternative name: Bayesian network (BN), causal network, probabilistic network). Such a network is a directed acyclic graph, and is a graphical probabilistic model for representing probabilistic dependencies, or their absence.

The Bayesian network can be represented as a pair <G, B>, in which the first component is a directed acyclic graph G = (*X, E*), where *X* is a set of vertices; *E* is a set of arcs. Vertices $X_j \in X$, called nodes, represent all complete systems of random events; each element of the set $X_j \in X$ is a random variable (event), which can be both discrete and continuous in nature. The vertices of the graph $X_j \in X$ are connected in pairs by oriented edges and describe the relation of conditional independence.

The relationship $A \to C$ is causal when event *A* is the cause of event *C*; the vertex *A* is called the parent of *C*, and affects its value. The second component B is a set of parameters defining the network. It contains parameters $\Theta_{x_j|pa(X_j)} = P(x_j \mid pa(X_j))$ for each possible value x_j in X_j, and $pa(X_j)$ in $Pa(X_j)$, where $Pa(X_j)$ is a set of all parents of the variable X_j in G. With a given network structure, the complete compatible probability is determined by the formula:

$$P(X_1, X_2, ..., X_N) = \prod\nolimits_{j=1}^{N} P(X_j \mid Pa(X_j)) . \qquad (2)$$

To describe the BN, it is necessary to define:

1. The network structure (optimal topology of graph G).

2. The parameters of each node $X_j \in X$, that are tensors of conditional probabilities $P(X_j \mid Pa(X_j))$ in the nodes. In nodes that do not have parents, tensors of conditional probabilities degenerate into tensors of marginal probabilities $P(X_j)$.

The process of construction of acyclic graph corresponding to the variables is called network training. Currently, there is a wide class of BN training methods. The choice of such methods depends on two features:

1. Whether the topology (structure) of the network is known and the presence of hidden variables (nodes) in the network is observed.

2. A situation in which part of the data is incorrect or completely missing. Let us consider 4 cases given in [2, p. 18], Table 1.

Table 1

Conditions of BN teaching methods application

Structure	Observation	Method
Known	Complete	Maximum likelihood estimation
	Partial	Maximization of mathematical expectation or the greedy extremum search algorithm
Unknown	Complete	Search in the space of models
	Partial	Structural algorithm for maximization of mathematical expectation or compression of boundaries

In general, the task of BN learning can be formulated as follows [23, p. 82]. For a set of related events X_j, $j=\overline{1,N}$, a set of training data $D=\{d_i \mid i=\overline{1,n}\}$, $d_i=\{x_i^{(j)}\} \mid j=1,N\}$, $N \geq 2$ is given (the lower index is the observation number, the upper one is the variable number), n is the number of observations, each observation consists of N variables, each variable has $A_j=\{0,1,\ldots,\alpha_j-1\}, \alpha_j \geq 2$ states. Based on the training sample, it is necessary to construct an acyclic graph corresponding to the variables X_j, $j=\overline{1,N}$. The task of training the network is NP-hard, because with a complete search, it is necessary to perform an analysis of the $3^{\frac{n(n-1)}{2}}-k$ models, where n is the number of vertices; k is the number of models with cycles [3, p. 396].

To evaluate the quality of training, it can be used the number of redundant, missing and reverse arcs in the training BN compared to the original BN. As a measure of the learning error, it can be used the structured difference, as well as the cross entropy measure, between the training and original BN [8].

All probabilistic inference problems solved using BN can be divided into two classes [14; 21, p. 118]: calculation of a priori conditional probabilities of events on all nodes of the network and calculation of posterior probabilities of events in all separate nodes of the network, provided that in a separate node an event is occurred. The choice of one or another method is determined by the type of problem to be solved, the structure of the network and the availability of initial information.

However, in many practical situations, the interest may not be the probabilities of all events related to the problem being solved, but the determination of such a set of events that most plausibly explain the existing facts. To solve such a problem, the author of the work [13] proposed abductive deduction, which is understood as such a reasoning process that gives the best explanation (or a number of explanations) for the available analyzed factors under the conditions of a specific problem. In this case, the events that have occurred, we mean such a set of events in the nodes of the network in which no events have occurred, provided that an event (events) has occurred in one or more nodes of the network. The set of groups of events containing such an explanation is called an explanatory set.

The measure of the quality of the explanation is the total probability of the set of values of the variables (events) forming this explanation.

All problems of abductive inference can be divided into two classes [21, p. 142]:

1. Problem of determination of the most probable explanation.

2. Problem of finding the most probable events in the nodes forming the explanatory set.

Despite all their attractiveness, BNs are not without a number of disadvantages, among which the following can be highlighted:

1. A strict restriction on the presence of directed cycles (according to the definition the BN is an acyclic graph). However, when solving real problems, the structure of the model is often set by an expert, which can lead to the emergence of directed cycles in the real model. Attempts to overcome this shortcoming and to develop the BN apparatus to take into account directional cycles are being carried out in [20, p. 210].

2. Methods of probabilistic derivation on confidence networks cannot be applied for interval estimates of probabilities.

3. Probabilistic inference cannot be carried out directly in a BN with a multi-link structure – such a network must first be transformed into an articulations tree.

All the above-mentioned limitations can be overcome by using the mathematical apparatus of algebraic BN (ABN) – a logical-probabilistic graphic model of a knowledge system with uncertainty [7, p. 233; 20, p. 210]. ABN is an undirected graph (adjacency graph), in the nodes of which there are fragments of knowledge – sets of variables with adjacent values. For any nodes containing common variables, there is a path between the nodes, each vertex of which also contains these variables.

ABN allows the presence of cycles in the basic graph of the network; allows to operate with crisp (scalar), binary and interval estimates of probabilities. ABN has a well-developed apparatus for carrying out a priori and a posteriori derivation.

3. Synthesis of information technology for the analysis of expert assessments under uncertainty using methods of probabilistic inference

Let us consider a set of random events $X = \{X_j \mid j = \overline{1, m}\}$. The set X can represent both a set of connected ($\forall X_i, X_j \in X : (X_i \to X_j) \vee (X_i \to X_j \to null)$) and a set of unconnected ($\forall X_j \in X : (null \to X_j \to null)$) events $X_j \in X$.

A logical-probabilistic graphic model of systems of random events (knowledge) in the form of a pair <G, B> is associated with the set *X*. The component G is a directed graph G = (*X, E*), where *X* is the set of vertices; *E* is a set of arcs; component B is a set of parameters that determine the event $X_j \in X$.

The methodology for synthesis of information technology (IT) for decision support using probabilistic inference techniques can be formally presented in the form of the following successive stages, Figure 1 [10, p. 52]:

1. Formation of a set of random events $X = \{X_j \mid j = \overline{1, m}\}$.

2. The choice of the method of obtaining probabilistic events, which, in turn, depends on the method of obtaining initial information – expert, analytical, for example, based on the analysis of static information that can accumulate in the database, etc.

3. Construction of a priori distribution of probabilities of random events, determination of unconditional probabilities of variables. All random events (variables) can be conditionally divided into two categories: evidence variables, target variables. For example, a symptom can be considered as an evidence variable, and the diagnosis, in turn, is a target variable. Evidence variables are related to target variables forming causal relationships between all variables.

4. Establishing of dependencies between variables (random events). A causal event (cause) is such an event that, under certain conditions, generates, affects or changes another event that is its consequence. Such event has called a consequential event. For example, in a relationship $A \to B$, event A is a cause event, and event B is a consequence event, provided that A is the cause of event B and affects its meaning. A cause-

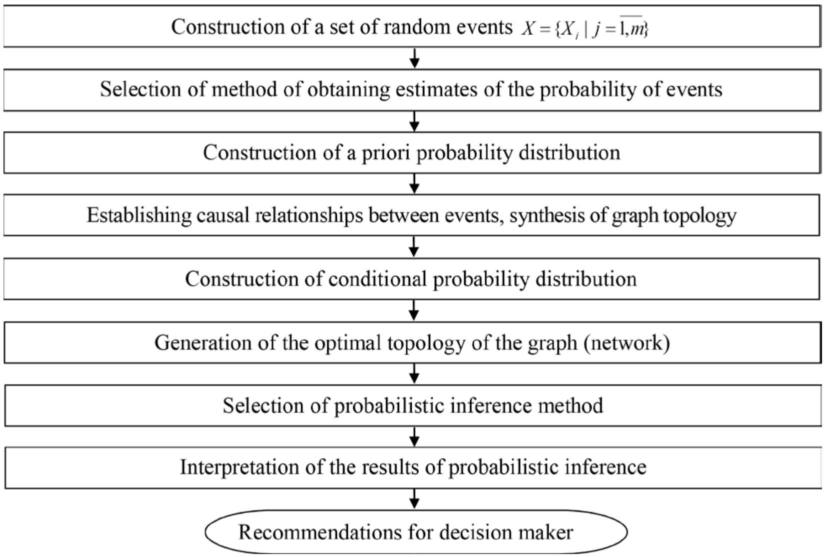

Figure 1. Structure of IT for decision support based on graphic probabilistic models

and-effect relationship can involve a pair of events in which one event causes the other.

5. After establishing causal relationships between all variables (random events), or establishing the fact of their absence, it is possible to construct a conditional distribution of probabilities, which can be specified in the form of tables of conditional probabilities, for consequent events. In the graphical interpretation the events-consequences are the graph nodes (networks) that have output edges, and their probability estimates depend on the combination of values of ancestor variables on the graph.

6. Synthesis of graph (network) topology. If the number of systems of random events does not exceed four and on the basis of a priori information a single-connected graph with a tree-like structure can be synthesized, then a probability tree can be chosen as a probabilistic model of knowledge representation. In this case, there may or may not be a probabilistic dependence between systems of random events. The algorithm for probability trees construction is shown in Figure 2.

In the case of existence of a rigid dependence between all variables (graph nodes, networks), the graph topology can be synthesized on the basis of a priori information – expert judgments, but obtaining an optimal structure requires its training. The choice of learning method depends on the presence of hidden nodes, the completeness of a priori information, or is the network structure known. The algorithm for the BN construction is shown in Figure 3.

7. Selection of the method of probabilistic inference.

8. Interpretation of the results of probabilistic inference and development the reccomendations for decision maker.

Consider the situation when the probabilities of an event $X_j \in X$ can be obtained simultaneously from several independent sources, for example, assessed by a group of experts.

Let $\mathrm{E} = \{E_i \mid i = \overline{1,n}\}$ be a group of experts, that evaluating some initial set of random events $X = \{X_j \mid j = \overline{1,m}\}$ have formed expert profiles (EPs) $\mathrm{B} = \{B_i \mid i = \overline{1,n}\}$. Each expert was asked to estimate the possibility (probability) of the occurrence of the event $X_j \in X$. The profile $B_i = \{b_j^i \mid j = \overline{1,m}\}$ formed by the expert E_i reflects his preferences regarding the realization (possibility of occurrence) of all the analyzed elements of the set *X*.

The assessment $b_j^i \to [0; N]$ represents the possibility (probability) of the occurrence of a random event $X_j \in X$, determined by an expert E_i.

The assessment b_j^i can be expressed, for example, using a scale from 0 to 1: insignificant probability of implementation (0.1); low (0.3); average (0.5); high (0.7); critical (0.9); absolute (1). Values 0.2; 0.4; 0.6; 0.8 are correspond to intermediate judgments between each gradation. If $\mathrm{N} \neq 1$, then the obtained value b_j^i should be reduced to a unit interval, i.e. $b_j^i \in [0; 1]$.

Thus, *m*-systems of random events can be constructed by each expert, which can be graphically represented in the form of distribution trees, each branch of which reflects the probability of the analyzed events.

1. The events of the set X are independent [17, p. 179].

The procedure for the synthesis of a priori expert evaluations of the implementation of events $X_j \in X$:

Stage 1. Formation of the frame of discernment $\Omega = \{\omega_1, \omega_2\}$ of the problem, where ω_1 – the event $X_j \in X$ is implemented; ω_2 – the event $X_j \in X$ is considered as not significant (not implemented).

Stage 2. Formation of the vector of probability estimates of realization of events: $\forall X_j \in X$: $\mathrm{M}_j = \{\mathrm{m}_i^{(j)} \mid i = \overline{1,n}\}$, $\mathrm{m}_i^{(j)} = \{m(\acute{\mathrm{E}}_1), m(\acute{\mathrm{E}}_2)\}$.

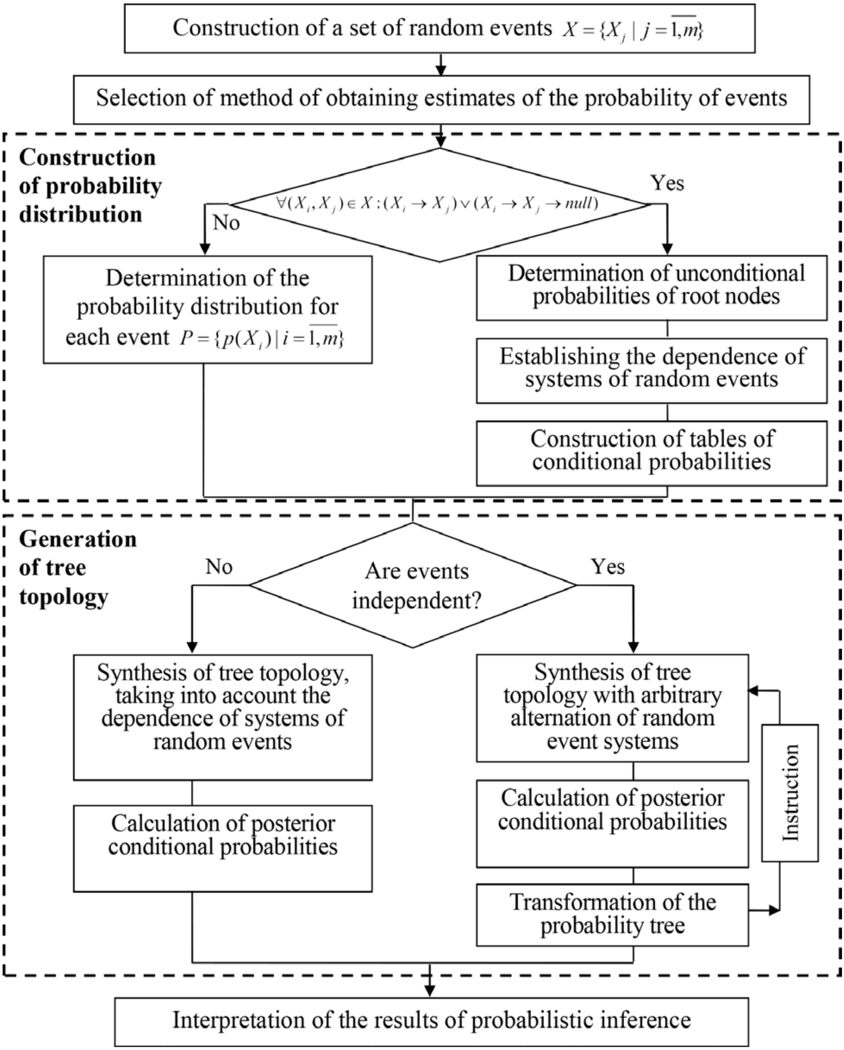

Figure 2. Algorithm for probability tree construction

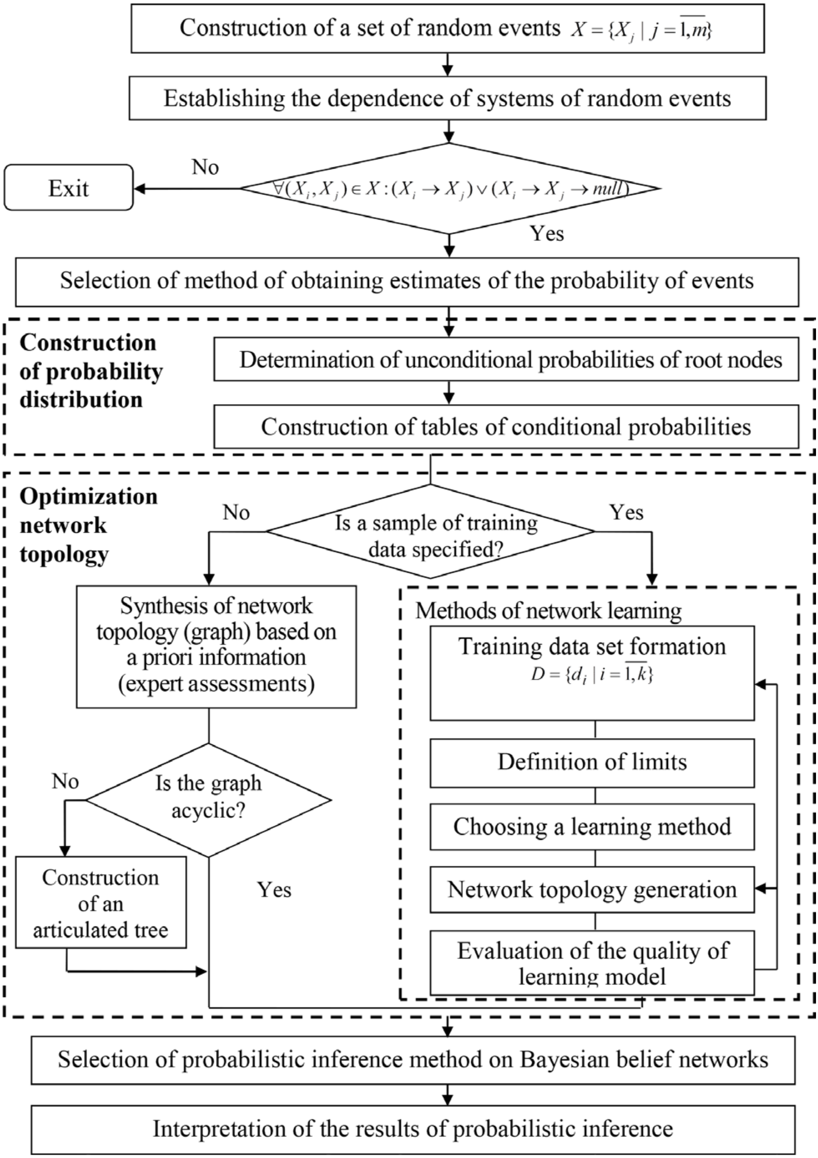

Figure 3. Algorithm for BN constructiont

If $m(\omega_1)$ is the probability of the occurrence of the event $X_j \in X$, then, accordingly, the probability of the event is not occurring $X_j \in X$ can be expressed as $m(\omega_2)=1-m(\omega_1)$.

Thus, for each event $X_j \in X$, a set $\mathrm{M}_j = \{\mathrm{m}_i^{(j)} \mid i = \overline{1,n}\}$ will be obtained, where $\mathrm{m}_i^{(j)} = \{m(\acute{\mathrm{E}}_1), m(\acute{\mathrm{E}}_2)\}$ is the vector of probability estimates of the event $X_j \in X$, obtained on the basis of individual assessments of the expert E_i.

Stag 3. Aggregation of values of the vector of probability estimates of $X_j \in X$ realization for $\forall E_i \in \mathrm{E}$: $\mathrm{m}_{\mathrm{rez}}^{(j)} = \mathrm{m}_1^{(j)} \oplus \mathrm{m}_i^{(j)} \oplus \ldots \oplus \mathrm{m}_n^{(j)}$, $\mathrm{m}_{\mathrm{rez}}^{(j)} = \{m(\omega_1), m(\omega_2)\}$.

Aggregation of obtained probabilistic estimates is carried out on the basis of the mathematical apparatus of Dempster-Shafer theory (DST) [5, p. 325; 6, p.3; 15, p. 15; 18, p. 5; 19, p. 9; 22, p. 110], Dezert-Smarandache Theory (DSmT) [18, p. 11; 19, p. 20]. Aggregation of individual EPs into a group expert assessments is carried out by combination of obtained main probability masses $\mathrm{m}_i^{(j)} = \{m(\acute{\mathrm{E}}_1), m(\acute{\mathrm{E}}_2)\}$ for each random event $X_j \in X$ according to all experts $\mathrm{m}_{\mathrm{rez}}^{(j)} = \mathrm{m}_1^{(j)} \oplus \mathrm{m}_i^{(j)} \oplus \ldots \oplus \mathrm{m}_n^{(j)}$. To obtain aggregated estimates, it is recommended to use conflict redistribution rules [19, p. 20]. Since when using any of them, the resulting combined probability masses are obtained by adding parts of the total conflict mass or partial conflict masses to the corresponding value of m(·), while the resulting subsets correspond to the original ones, new subsets are not formed. In order to improve the quality of the results of combination, it is recommended to determine the order of expert assessment combination, for example, based on metrics [1, p. 100; 4, p. 531; 9, p. 94].

As a result, a set $\mathrm{m}_{\mathrm{rez}}^{(j)} = \{m(\omega_1), m(\omega_2)\}$ will be obtained for each original event $X_j \in X$.

Stage 4. Analysis and calculation of the constructed probability tree for independent systems of random events.

Next, the analysis and calculation of the obtained probability tree for independent systems of random events is carried out, with the corresponding probability estimates of the realization $m_{\mathrm{rez}}^{(j)}(\omega_1) \in \mathrm{m}_{\mathrm{rez}}^{(j)}$ and non-realization (implementation) $m_{\mathrm{rez}}^{(j)}(\omega_2) \in \mathrm{m}_{\mathrm{rez}}^{(j)}$ of the event $X_j \in X$, Figure 4.

The transformation of the tree, provided that the random events are independent, causes a new redistribution of the probability estimates between the events. This makes it possible to analyze and determine the probability of realization of each of the possible scenarios that are formed by different combinations of systems of random events.

Let us consider an example that illustrates the proposed technique for analysis of some organizational problems of ship repair using probability trees and probabilistic inference.

The shipping company is faced with the task of selection of a ship repair company to enter into a contract for ship repairs. Consider the three most important risk factors affecting on the contract: r_1 is the risk of increased a repair cost (increasing the total project costs); r_2 is the risk of increased duration of repair; r_3 is the risk of decreasing quality of repair [11, p. 113].

Five experts were asked to assess the probability of occurrence each of the events (risks). The results of expertise are reported in Table 2.

Table 2

Basic probability assignments of risk factors

		E_1	E_2	E_3	E_4	E_5	m_{comb}
r_1	$m(\omega_1)$	0.4	0.3	0.5	0.2	0.6	0.34
	$m(\omega_2)$	0.6	0.7	0.5	0.8	0.4	0.66
r_2	$m(\omega_1)$	0.2	0.4	0.7	0.3	0.4	0.29
	$m(\omega_2)$	0.8	0.6	0.3	0.7	0.6	0.71
r_3	$m(\omega_1)$	0.7	0.5	0.4	0.6	0.8	0.78
	$m(\omega_2)$	0.3	0.5	0.6	0.4	0.2	0.22

To aggregate the individual expert's assessments the proportional conflict redistribution rule PCR5 [19, p. 36] has been used. According to the conflict redistribution rule PCR5 combined basic probability assignments $m_{PCR5}(C)$ is calculated according to expression:

$$m_{PCR5}(C) = m_{12}(C) + \sum_{\substack{Y \in D^{\Omega} \setminus \{X\} \\ X \cap Y = \varnothing}} \left[\frac{m_1(X)^2 \cdot m_2(Y)}{m_1(X) + m_2(Y)} + \frac{m_2(X)^2 \cdot m_1(Y)}{m_2(X) + m_1(Y)} \right], \qquad (3)$$

where $m_{12}(C)$ is the combined mass of probability for the subset $C = X \cap Y$, calculated on the basis of conjunctive consensus.

Figure 5 shows the distribution trees that constructed on the basis of aggregated expert's assessments. The probability tree represents 8 possible scenarios (each scenario is indicated in Figure 6 as sequence of numbers in parentheses).

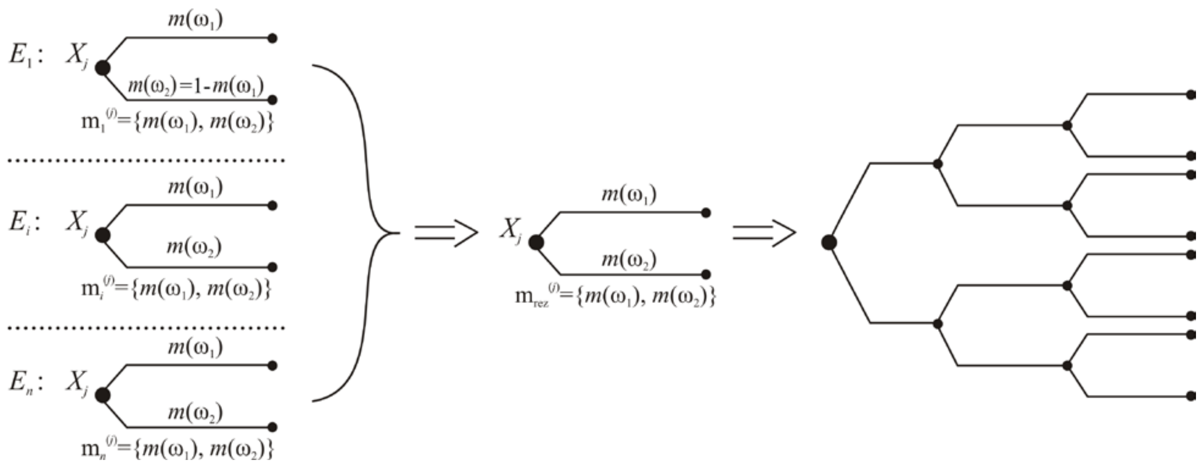

Figure 4. Graphical interpretation of the process of synthesis of group expert assessments for solving the problems of probabilistic inference in probability trees

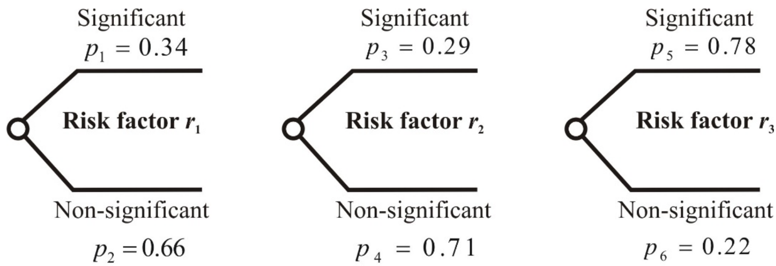

Figure 5. Disrtibution trees for systems of analuzed ramdom events

Let us make all necessary calculations in a tree diagram. First of all, the probability of each scenario defined (taking into account all obtained probabilities of events that are included in scenario:

$$P_1(a_1, a_3, a_7) = p(a_1)\cdot p(a_3)\cdot p(a_7) = 0.34\cdot 0.29\cdot 0.78 = 0.077;$$
$$P_2(a_1, a_3, a_8) = p(a_1)\cdot p(a_3)\cdot p(a_8) = 0.34\cdot 0.29\cdot 0.22 = 0.022;$$
$$P_3(a_1, a_4, a_9) = p(a_1)\cdot p(a_4)\cdot p(a_9) = 0.34\cdot 0.71\cdot 0.78 = 0.188;$$
$$P_4(a_1, a_4, a_{10}) = p(a_1)\cdot p(a_4)\cdot p(a_{10}) = 0.34\cdot 0.71\cdot 0.22 = 0.053;$$
$$P_5(a_2, a_5, a_{11}) = p(a_2)\cdot p(a_5)\cdot p(a_{11}) = 0.66\cdot 0.29\cdot 0.78 = 0.149;$$
$$P_6(a_2, a_5, a_{12}) = p(a_2)\cdot p(a_5)\cdot p(a_{12}) = 0.66\cdot 0.29\cdot 0.22 = 0.042;$$
$$P_7(a_2, a_6, a_{13}) = p(a_2)\cdot p(a_6)\cdot p(a_{13}) = 0.66\cdot 0.71\cdot 0.78 = 0.366;$$
$$P_8(a_2, a_6, a_{14}) = p(a_2)\cdot p(a_6)\cdot p(a_{14}) = 0.66\cdot 0.71\cdot 0.22 = 0.103;$$
$$P = \sum_{i=1}^{8} P_i = 0.077 + 0.022 + 0.188 + 0.053 + 0.149 + 0.042 + 0.366 + 0.103 = 1.$$

Based on the calculations above, it can be concluded that scenario (7) is the most negative, and scenario (2) at acceptable rates of risk factors has the minimal probability of its realization.

The transformation of tree diagram causes to redistribution probability estimates between events, thus making it possible to analyze and evaluate each of the possible scenario.

2. There is a dependence between the events of the set *X*.

In the case of the existence of dependence between the events of the *X* set, it is necessary to determine the a priori probabilities of their realization and conditional probabilities of occurrence of events.

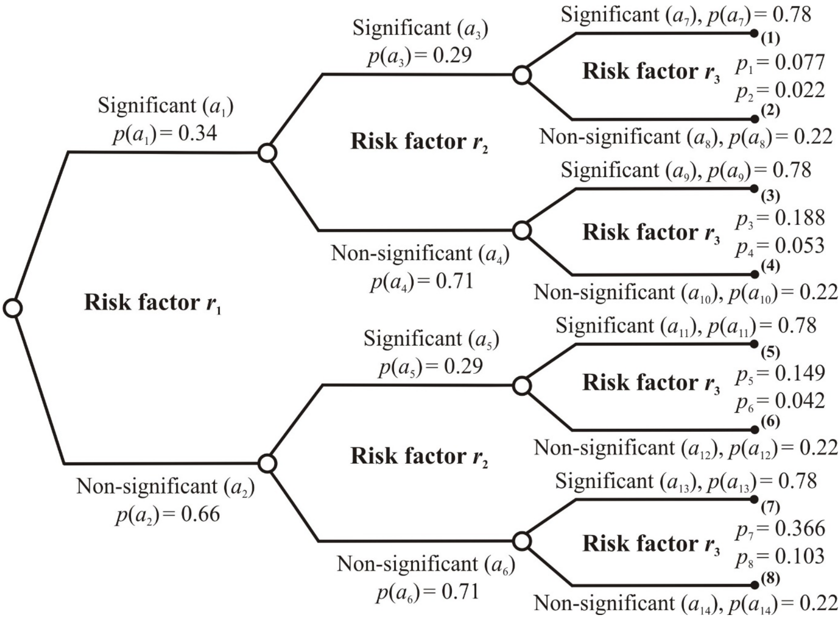

Figure 6. Tree diagram for systems of random risk factors for problem of concluding a contract for ship repair solving

The approach proposed above can be used to obtain aggregated (generalized) values of a priori probabilities of realization of events of set X, formed by a group of experts.

4. Conclusions

A decision support technology using probabilistic inference methods has been proposed. The methods of probabilistic inference that have become widely used recently are analyzed: probabilistic inference in probability trees, the method of condensation of probability distributions, probabilistic inference in Bayesian network, abductive inference, probabilistic inference in algebraic Bayesian networks. Their advantages and disadvantages are analyzed.

A structures of information technology for the analysis of expert assessments, formed under uncertainty using probabilistic inference methods, is proposed. In the framework of which an approach for aggregation of individual probability estimates of experts for solving problems of probabilistic inference in probability trees is proposed. This approach allows synthesizing generalized estimates of the probability of the realization of random events. Aggregation of the obtained individual probability assessments is carried out on the basis of the mathematical apparatus of DST and DSmT. The obtained values of the probability of realization of random events are used in the construction of probability trees and the calculation of the ratios of the probability output in them. This approach allows processing expert assessments obtained under of specific forms of uncertainty, as well as conflict (contradictory) expert judgments.

References:

1. Bhattacharyya A. (1943). On a measure of divergence between two statistical populations defined by their probability distribution. *Bulletin of the Calcutta Mathematical Society*, vol. 35, pp. 99–110.

2. Bidyuk P. I., Terentev A. N., Gasanov A. S. (2005). Postroenie i metodyi obucheniya baesovskoy seti [Constructing and methods of learning of Bayesian networks]. *Kibernetika i sistemnyiy analiz – Cybernetics and Systems Analysis*, vol. 4, pp. 133–147. (in Russian)

3. Cooper G. F. (1990). Computational complexity of probabilistic inference using bayesian belief networks. *Artificial Intelligence*, vol. 42, pp. 393–405.

4. Cuzzolin F. (2007). A geometric approach to the theory of evidence. *Transactions on Systems, Man, and Cybernetics* (*Part C: Applications and Reviews*), vol. 38(4), pp. 522–534. DOI: https://doi.org/10.1109/TSMCC.2008.919174

5. Dempster A. P. (1967). Upper and lower probabilities induced by a multivalued mapping. *Annals of Mathematical Statistics*, vol. 38(2), pp. 325–339. DOI: https://doi.org/10.1214/aoms/1177698950

6. Florea M. C., Bosse E. (2009). Crisis management using Dempster-Shafer theory: using dissimilarity measures to characterize sources' reliability. *Proceedings Symposium on C3I for Crisis, Emergency and Consequence Management*, RTO-MP-IST-086. (Bucharest, Romania, May 11-12, 2009).

7. Gorodetsky V. I., Drozdgin V. V., Jusupov R. M. (1984). Application of attributed grammar and algorithmic sensitivity model for knowledge representation and estimation. *Artificial Intelligence and Information, Control Systems of ROBOTSA*, pp. 232–237.

8. Heckerman D., Geiger D., Chickering D. M. (1995). Learning bayesian networks: the combination of knowledge and statistical data. *Machine Learning*, vol. 20, pp. 197–243. DOI: https://doi.org/10.1023/A:1022623210503

9. Jousselme A. L., Grenier D., Boss´e E. (2001). A new distance between two bodies of evidence. *Information Fusion*, vol. 2(2), pp. 91–101. DOI: https://doi.org/10.1016/S1566-2535(01)00026-4

10. Kovalenko I. I., Shved A. V. (2013). Informatsionnaya tehnologiya podderzhki prinyatiya resheniy s ispolzovaniem metodov veroyatnostnogo vyivoda

[Information technology for decision support using probabilistic inference methods]. *Naukovi pratsi: Naukovo-metodychnyi zhurnal Chornomorskoho derzhavnoho universytetu im. P. Mohyly. Seriia: Kompiuterni tekhnolohii*, vol. 217 (229), pp. 51–58. (in Russian)

11. Kovalenko I. I., Shved A. V., Melnik A. V. (2014). Veroyatnostnyiy analiz riskoobrazuyuschih faktorov v organizatsionnyih zadachah sudoremonta [Probabilistic analysis of risk factors in organizational tasks of ship repair]. *Sudostroenie i morskaya infrastruktura – Shipbuilding & marine infrastructure*, vol. 2(2), pp. 111–121. (in Russian)

12. Mosleh A., Bier V. M. (1996). Uncertainty about probability: a reconciliation with the subjectivist viewpoint. *Transactions on Systems, Man, and Cybernetics, Part A: System and Humans*, vol. 26(3), pp. 303–310.

13. Neapolitan R. E. (1996). Is higher-order uncertainty needed? *IEEE Transactions on Systems, Man, and Cybernetics, Part A: System and Human,* vol. 26(3), pp. 294–302.

14. Pearl J. (1988). *Probabilistic reasoning in intelligent systems: networks of plausible inference*. California: Morgan Kaufman Publishers.

15. Sentz K., Ferson S. (2002). *Combination of evidence in Dempster-Shafer theory: tech. report SAND 2002-0835*. Albuquerque: Sandia National Laboratories.

16. Shafer G. A. (1976). *Mathematical Theory of Evidence*. Princeton: Princeton University Press.

17. Shved A. V. (2017). Probabilistic risk analysis of investment projects under uncertainty. *Proceedings of the 9th IEEE International Conference on Intelligent Data Acquisition and Advanced Computing Systems: Technology and Applications* (IDAACS 2017). (Bucharest, Romania, September 21st–23rd, 2015). DOI: https://doi.org/10.1109/IDAACS.2017.8095072

18. Smarandache F., Dezert J. (2004). *Advances and applications of DSmT for information fusion. Collected Works*. Rehoboth: American Research Press.

19. Smarandache F., Dezert J. (2006). *Advances and applications of DSmT for information fusion*. *Collected works*. Rehoboth: American Research Press.

20. Tulupev A. L., Nikolenko S. I., Sirotkin A. V. (2006). *Bayesovskie seti: logiko-veroyatnostnyiy podhod* [Bayesian networks: a logical-probabilistic approach]. SPb.: Nauka. (in Russian)

21. Uzhga-Rebrov O. I. (2004). Upravlenie neopredelennostyami. Chast 1 "Sovremennyie kontseptsii i prilozheniya teorii veroyatnostey" [Uncertainty management. Part 1 "Modern concepts and applications of probability theory"]. Rezekne: Izdevnieciba. (in Russian)

22. Yager R. R. (1987). On the Dempster-Shafer framework and new combination rules. *Information Science*, vol. 41(2), pp. 93–137. DOI: https://doi.org/10.1016/0020-0255(87)90007-7

23. Zgurovskiy M. Z., Bidyuk P. I., Terentev A. N. (2008). Metodyi postroeniya bayesovskih setey na osnove otsenochnyih funktsiy [Methods for constructing Bayesian networks based on scoring functions]. *Kibernetika i sistemnyiy analiz – Cybernetics and Systems Analysis,* vol. 44(2), pp. 81–88. (in Russian)