THE METHODOLOGY OF SYNTHESIS OF INFORMATION TECHNOLOGIES FOR DECISION SUPPORT UNDER COMPLEX FORMS OF IGNORANCE

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DOI: https://doi.org/10.30525/978-9934-26-355-2-11

Abstract. An important problem of system analysis is the disclosure of uncertainties due to the variety of purposes, properties and features of the studied objects and processes. The analysis and management of various types of ignorance is of primary importance, since the processes of intelligent technologies creating always proceed under contradiction, incompleteness, inaccuracy, uncertainty connected with processes of obtaining and processing of datasets and expert knowledges. *The purpose* of the paper is to improve the theoretical and methodological foundations of the synthesis of information technologies for decision-making under complex forms of ignorance. *Methodology* of the study is based on system analysis methods, decision-making theory, theory of evidence (Dempster-Shafer theory, DST), theory of plausible and paradoxical reasoning (Dezert-Smarandache theory, DSmT). *Results*. The methodology for the synthesis of information technologies for the generation of individual and group management decisions under multi-criteria, multi alternatives and different types of ignorance has been proposed in this paper. In addition to certain types of ignorance, the situations occurring under the influence of complex forms of ignorance modeled by framework of Dempster-Shafer and Dezert-Smarandache theories have been analyzed. In this case, conditions of mutual exclusivity and/or mutual exhaustion of elements of the initial dataset are imposed. The paper proposes a procedure for choosing an expert evidence combination rule, which is based on the determination of quantitative

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indicators of uncertainty. The proposed technique allows to filter out from the initial set of combination rules acceptable for solving the problem under consideration that part of rules that does not satisfy the imposed restrictions and conditions. Next, for each pair of expert evidence that are combined, choose a rule that minimizes the value of the contradiction measure and maximizes the value of specificity measure for the result of combination. *Practical implications.* The proposed methodology and information technology provide a theoretical and practical foundations for intelligent support of processes of acquisition (synthesis), processing, and analysis data and expert knowledges under complex types of ignorance in decision support systems. *Value/originality*. The proposed methodology for synthesis of information technologies provides intellectual support for the processes of preparation and support of effective and optimal management decisions when solving non-criteria and multi-criteria problems under complex forms of ignorance generated by the simultaneous presence of two or more types of ignorance, such as importance + inconsistency (conflict), uncertainty + contradiction, uncertainty + inaccuracy.

1. Introduction

There are many different definitions of "ignorance". However, all of them to one degree or another indicate that ignorance is associated with an insufficient amount (absence) of knowledge or information about object or phenomenon under consideration.

Despite the existing variety of methods for modeling different types of ignorance, based on traditional as well as new mathematical theories, they are used without proper analysis of the nature of the analyzed types of ignorance. Which in turn leads to them being unreasonable when modeling relevant subject and problem areas of knowledge.

In these conditions, there is a need to find ways to solve an actual scientific and practical problem aimed at creating and developing the foundations of the methodology for synthesis of information technologies focused on the intellectual support of decision-making processes under various types of ignorance. The development of which requires solving the following tasks:

– formalization and identification of various types of ignorance, which can affect the processes associated with the acquisition and analysis of a set of initial data;

– comparative analysis of modern methods of their modeling with the aim of their justified choice;

– formation of a system of conditions, criteria, rules, restrictions, which allows to forn a unified algorithms for the synthesis of IT for decision-making support, with the aim of obtaining effective results when modeling relevant subject and problem areas of knowledge.

The purpose of the paper is to improve the theoretical and methodological foundations of the synthesis of information technologies for decision-making under complex forms of ignorance.

2. Decision support technology under complex forms of ignorance

Let's assume that a group of experts $\mathrm{E} = \{E_j \mid j = \overline{1,t}\}$, evaluating a given initial set of examination objects (alternatives) $\mathrm{A} = \{A_i \mid i = \overline{1,n}\}$, formed experts' profiles $\mathrm{B} = \{B_j \mid j = \overline{1,t}\}$. The profile B_j formed by an expert E_j reflects his / her preferences over all the analyzed elements of the set A.

If the set of initial data A is subject to the restriction of mutual exclusion and mutually exhaustiveness of elements, then the results of expert evaluation can be presented in the form $B_j = \{X_l \mid l = \overline{1,k}\}$ ($k = 2^{|\mathrm{A}|} - 1$, $X_l \subseteq \mathrm{A}$), where B_j is a 2^{A}-dimensional vector reflecting the priorities (choice) of the expert E_j, each element of which is built on the basis of next rules:

1. $X_j = \{\varnothing\}$;

2. $X_j = \{\omega_i\}$ – one element $\omega_i \in \Omega$ has been selected (assessed) by an expert.

3. $X_j = \{\omega_i \mid i = \overline{1,k}\}$, $k < n - k$ elements $\omega_i \in \Omega$ have been selected by an expert. (1)

4. $X_j = \Omega = \{\omega_i \mid i = \overline{1,n}\}$ – expert had difficulties with evaluation / selection (all elements of the set Ω are equivalent).

At the same time, the subset X_l can contain the empty set $\varnothing$, elements $A_i \in \mathrm{A}$, as well as possible combinations of the original elements constructed using the operator $\cup$.

If A is considered only as a set of mutually exclusive elements, then the results of an expert survey can be presented in the form $\mathrm{B} = \{B_j \mid j = \overline{1,t}\}$, $B_j = \{X_l \mid l = \overline{1,k}\}$, where B_j is represented by D^{A} dimensional vector that reflects the choice of E_j, each element of which is built on the basis of rules:

1. A set of conditions that correspond to (1).

2. If $X_i, X_j \subset D^{\mathrm{A}}$, then $X_i \cap X_j \in D^{\mathrm{A}}$ and $X_i \cup X_j \in D^{\mathrm{A}}$. (2)

In this case, the subset X_l may contain the empty set ∅, elements $A_i \in \mathrm{A}$, as well as possible combinations of the initial elements $A_i \in \mathrm{A}$, built on the basis of the operators $\cup$ and $\cap$.

The evaluation of the elements of the set A can be carried out both by individual properties based on the vector of criteria and independently.

In the case of criterial evaluation of alternatives, it is necessary to construct a set of criteria $\mathrm{K} = \{K_q \mid q = \overline{1,s}\}$ in relation to which the evaluation is carried out. In this case, the system of subsets reflecting the results of the expert survey will have the form $\mathrm{B} = \{B_j^{(q)} \mid j = \overline{1,t}\}$, $B_j^{(q)} = \{X_l^{(q)} \mid l = \overline{1,k}\}$, where $B_j^{(q)}$ represents a subset of focal elements formed by the expert E_j according to criterion K_q, i.e., the expert E_j will form a system of subsets representing the profile $\mathrm{P}_j = \{B_j^{(q)} \mid q = \overline{1,s}\}$ of his assessments.

Each $X_l \subseteq B_j$ represents a focal element, based on which a degree of probability will be assigned that the best choice is in that subset. The formed experts' assignments can be expressed in numerical scales or in the scale of relations.

Based on the assigned degrees of preference, for each subset B_j, $j = \overline{1,t}$ a vector $\mathrm{m}_j = \{m_i \mid i = \overline{1,s}\}$, $s = \Lambda$ will be obtained. The elements of this vector satisfy next conditions

$$0 \le m(X_j) \le 1, \quad \forall (X_j \in \Lambda), \quad m(\varnothing) = 0, \quad \sum_{X_j \in \Lambda} m(X_j) = 1, \tag{3}$$

where Λ corresponds to 2^{A}, if Xl is built on the basis of the system of rules (1), which corresponds to the Shafer model; Λ corresponds to D^{A}, if X_l is built on the basis of the system of rules (2), corresponding to the Dezert-Smarandache model.

Aggregation of experts' assignments is performed on the basis of the combination rule: within the framework of Dempster-Shafer (DS) model [3, p. 325; 4, p. 3; 8, p. 15; 10, p. 5; 11, p. 9]; within the framework of Dezert-Smarandache (DSm) model [10, p. 11; 11, p. 20]. In DSm model it is recommended to use one of the conflict redistribution rules [10, p. 11; 11, p. 20].

The construction of the effective ranking of the studied objects R_{rez} is carried out on the basis of the values of belief $Bel(B)$ and plausibility $Pl(B)$ functions within the selected model, respectively.

The procedure for forming an effective ranking R_{rez}, for the case of solving the multi-criteria decision-making problem, can be presented in the form (Figure 1):

1. Definition of a set of criteria $\mathrm{K}=\{K_q \mid q=\overline{1,s}\}$, relative to which the choice is made.

2. Calculation of the vector of criteria priorities $\Omega=\{\omega_q \mid q=\overline{1,s}\}$, the elements of which meet the following conditions:

$$0 \le \omega_q \le 1, \quad \forall q=\overline{1,s}; \ \sum_{q=1}^{s}\omega_q = 1. \tag{4}$$

3. Identification of the model of expert information analysis (DS model, DSm model, Smets model), in the framework of which the focal elements are formed.

4. Extraction of experts' preferences. Expert E_j, according to selected model (DS, DSm, Smets), forms for each criterion K_q, $q=\overline{1,s}$ a system of subsets $B_j^{(q)}=\{X_l^{(q)} \mid l=\overline{1,k}\}$, $X_l^{(q)} \subseteq \mathrm{A}$, in accordance with (1) or (2), and a vector $S_j^{(q)}=\{s_l^{(q)} \mid l=\overline{1,k}\}$ containing numerical values of the degrees of preference of selected focal elements $X_l^{(q)}$.

If the DSm model is chosen, then at this stage various limitations on the interaction of the elements of the frame of discernment (initial set of analyzed objects) are introduced. As a result, the free DSm model is transformed into a hybrid one.

If the expert E_j selects (forms) not all possible subsets of D^{A}, then the remaining unselected (but possible) subsets are considered as introduced limitations of the model (such subsets are recognized as non-existent).

5. Determination of the values of basic probability mass function $\mathrm{M}_j^{(q)}=\{\mathrm{m}_l^{(q)} \mid l=\overline{1,k}\}$ of selected subsets, in accordance with the selected model.

6. Determination of the order of expert evidence combination.

For combination, a pair of $B_j^{(q)}, B_j^{(h)} \in \mathrm{P}_j$ is chosen, such that $q \ne k$: $\min d_J(\mathrm{m}_j^{(q)}, \mathrm{m}_j^{(h)}) \in [0;1]$ in accordance with one of the metrics [1, p. 100; 2, p. 531; 5, p. 94].

7. Selection of combination rule. The algorithm for choosing the expert evidence combination rule is given below.

8. Aggregation of experts' assignments by combination the obtained mass function $\mathrm{M}_j^{(q)}=\{\mathrm{m}_l^{(q)} \mid l=\overline{1,k}\}$ and $B_j^{(q)}=\{X_l^{(q)} \mid l=\overline{1,k}\}$, formed by the expert E_j according to all criteria K_q, $(q=\overline{1,s})$.

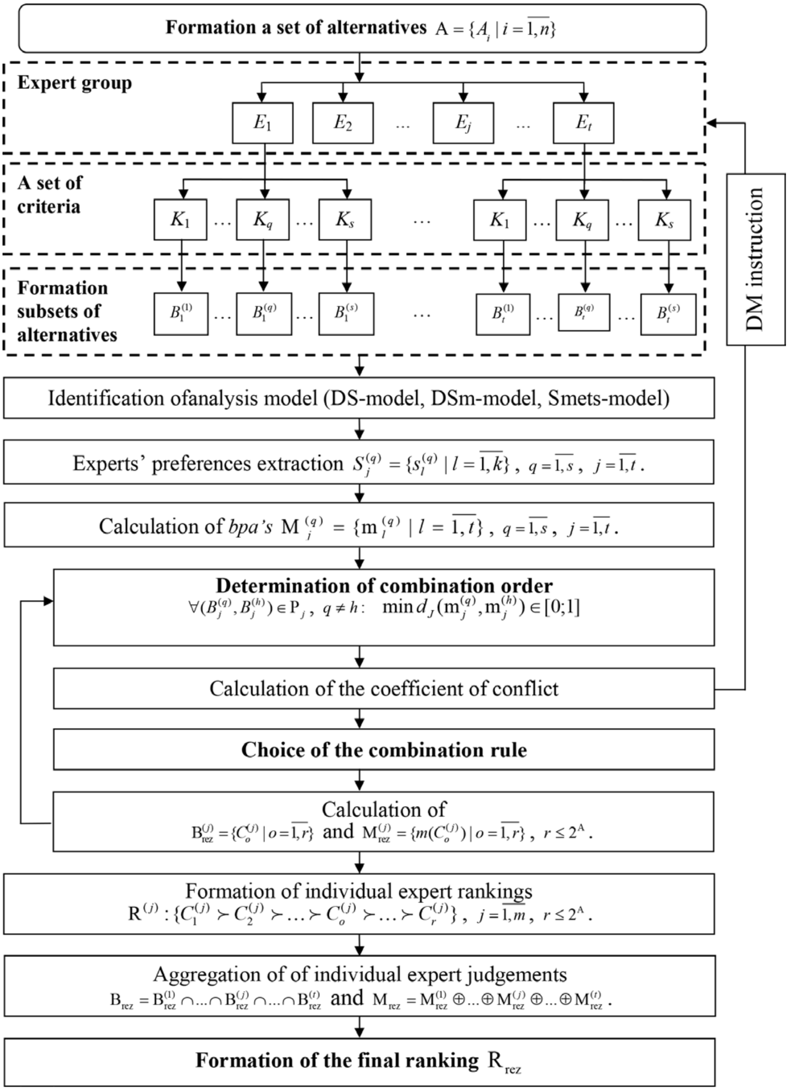

Figure 1. Generalized structure of the experts' assignments structuring technology under complex forms of ignorance in multi-criteria decision-making problem

The result of combination is vector $\mathrm{B}_{\text{rez}}^{(j)}=\{C_1^{(j)},C_2^{(j)},...,C_o^{(j)},...,C_r^{(j)}\}$ and vector $\mathrm{M}_{\text{rez}}^{(j)}=\{m(C_1^{(j)}),m(C_2^{(j)}),...,m(C_o^{(j)}),...,m(C_r^{(j)})\}$, respectiveli.

9. Calculation the values of the belief $Bel(\cdot)$ and plausibility $Pl(\cdot)$ functions for each obtained subset $C_o^{(j)}\subset\mathrm{B}_{\text{rez}}^{(j)}$:

– belief function:

$$Bel(B)=\sum_{X_j\subseteq B,\,X_j\in\Lambda} m(X_j)\,. \tag{5}$$

– plausibility function $Pl:\Lambda\to[0,1]$:

$$Pl(B)=\sum_{X_j\cap B\neq\varnothing,\,X_j\in\Lambda} m(X_j)\,, \tag{6}$$

where Λ corresponds to 2^{Ω}.

10. Formation of $[Bel(\{C_o^{(j)}\}),Pl(\{C_o^{(j)}\})]$ intervals for resulting subsets $C_o^{(j)}$ ($C_o^{(j)}\subseteq\mathrm{P}_{\text{rez}}^{(j)}$), calculation of crisp estimates for resulting subsets $C_o^{(j)}$.

11. Formation of individual expert rankings

$$\mathrm{R}^{(j)}:\{C_1^{(j)}\succ C_2^{(j)}\succ\ldots\succ C_o^{(j)}\succ\ldots\succ C_r^{(j)}\},\ j=\overline{1,m}\,.$$

12. Aggregation of individual expert preferences into a group expert assignment is carried out in accordance with steps 6–8.

Formation

$\mathrm{B}_{rez}=\mathrm{B}_{\text{rez}}^{(1)}\cap\mathrm{B}_{\text{rez}}^{(2)}\cap...\cap\mathrm{B}_{\text{rez}}^{(i)}\cap...\cap\mathrm{B}_{\text{rez}}^{(l)}$ and $\mathrm{M}_{rez}=\mathrm{M}_{\text{rez}}^{(1)}\oplus\mathrm{M}_{\text{rez}}^{(2)}\oplus...\oplus\mathrm{M}_{\text{rez}}^{(i)}\oplus...\oplus\mathrm{M}_{\text{rez}}^{(l)}$.

13. Formation of the rezult ranking R_{rez} of the analyzed objects, which reflects the collective opinion of the expert group, in accordance with steps 9–11.

14. Choosing the best alternative.

In the case of solving a non-criteria decision-making problem, the considered procedure for forming the resulting ranking R_{rez} is reduced to the implementation of the following steps:

1. Identification of the expert information analysis model (DS model, DSm model, Smets model), within which focal elements are formed.

2. Extraction of experts' preferences. The expert E_j, in accordance with the selected analysis model, forms a system of subsets $B_j=\{X_l\mid l=\overline{1,k}\}$, $X_l\subseteq B_j$, and a vector $S_j=\{s_l\mid l=\overline{1,k}\}$ that contain numerical values of the degrees of preference of selected focal elements X_l. Entering of various limitations on the interaction of the elements of the frame of discernment (for the DSm model).

3. Determination of the values of basic probability mass function

$\mathrm{M}_j = \{\mathrm{m}_l \mid l = \overline{1,k}\}$, according to the selected model.

4. Determination of the order of expert evidence combination.

For combination, a pair of $B_j, B_h \in \mathrm{B}$ is chosen, such that $q \neq k$: $\min d_J(\mathrm{m}_j, \mathrm{m}_h) \in [0;1]$ in accordance with one of the metrics [1, p. 100; 2, p. 531; 5, p. 94].

5. Selection of combination rule. The algorithm for choosing the expert evidence combination rule is given below.

6. Aggregation of experts' assignments by combination the obtained mass function $\mathrm{M}_j = \{\mathrm{m}_l \mid l = \overline{1,k}\}$ and $B_j = \{X_l \mid l = \overline{1,k}\}$, according to all experts E_j, $(j = \overline{1,t})$.

The result of combination is vector $\mathrm{B}_{rez} = \{C_1, C_2, ..., C_o, ..., C_r\}$ and $\mathrm{M}_{rez} = \{m(C_1), m(C_1), ..., m(C_o), ..., m(C_r)\}$ vector obtained by $\mathrm{B}_{rez} = B_1 \cap B_2 \cap ... \cap B_j \cap ... \cap B_t$, $\mathrm{M}_{rez} = \mathrm{M}_1 \oplus \mathrm{M}_2 \oplus ... \oplus \mathrm{M}_j \oplus ... \oplus \mathrm{M}_t$, respectively.

7. Calculation of the values of the belief $Bel(\cdot)$ and plausibility $Pl(\cdot)$ functions for each subset $C_o \subset \mathrm{B}_{rez}$, based on (5) and (6).

8. Formation of $[Bel(\{C_o\}), Pl(\{C_o\})]$ intervals for resulting subsets C_o ($C_o \subset \mathrm{B}_{rez}$), calculation of crisp estimates for resulting subsets C_o.

9. Formation of the resulting ranking R_{rez} of the analyzed objects, reflecting the collective opinion of the expert group.

10. Choosing the best alternative.

Currently, within the framework of the Dempster-Shafer theory, a significant number of combination rules have been proposed [3, p. 325; 4, p. 3; 8, p. 15; 10, p. 5; 11, p. 9]. Each of such rule has a number of advantages, but also has certain disadvantages. The comparative analysis of the considered combination rules is quite difficult, since there are no unified criteria by which each rule can be reasonably evaluated.

A procedure for choosing a combination rule based on the principle of minimal uncertainty has been proposed [13, p. 23; 14, p. 163]. A schematically generalized algorithm for choosing a combination rule is presented in Figure 2.

Suppose a set of combination rules $\mathrm{P} = \{P_i \mid i = \overline{1,k}\}$ is given. Based on the principle of minimum uncertainty (minimum entropy), it is necessary to choose a rule $P \in \mathrm{P}$, $\mathrm{m}_{combP} = \mathrm{m}_i P\, \mathrm{m}_j$ that minimizes the value of total uncertainty of combined probability mass function $\min\left(T(\mathrm{m}_{combP})\right)$.

Formally, the procedure for choosing a combination rule can be presented in the form of two consecutive stages. At the first stage, subset $\mathrm{P}' \subseteq \mathrm{P}$ is selected from the set of available combination rules $\mathrm{P} = \{P_i \mid i = \overline{1,k}\}$, which corresponds to given set of criteria $C = \{c_i \mid i = \overline{1,q}\}$.

In advance, it is necessary to select a number of criteria against which this or that rule of combination will be evaluated. The analysis model (DS model, DSm) can be considered as criteria for choosing the combination rule; information about data sources (competence of experts); the nature of the analyzed data (information about conflicts and consensus both between individual expert evidences); information about the degree of interaction and the structure of expert evidences, etc.

As a result, a set $\mathrm{P}' = \{P_i \mid i = \overline{1,z}\}$, $z \le k$ will be formed, which is obtained by removing from $\mathrm{P} = \{P_i \mid i = \overline{1,k}\}$ rules that do not satisfy the specified criteria for choosing a rule.

The second stage is to select a combination rule based on the analysis of the quantitative characteristics of uncertainty within the framework of the DS notation.

The rule is chosen based on the next recommendations:

1. According to principle of maximum specificity, a combination rule $P_l \in \mathrm{P}'$ is selected that maximizes the degree of specificity of combined mass function max($\delta_S(\mathrm{m}_i P_l\ \mathrm{m}_j)$), $\delta_S(\mathrm{m}_i P_l\ \mathrm{m}_j) \ne 1$. Where $\delta_S(m) \in [0,1]$ is the "degree of specificity" of evidences [12, p. 477]:

$$\delta_S(m) = 1 - d(m, m_S),\ \forall m \quad d(m, m_X) = d(m, m_Y), \quad m(X) = m(Y). \tag{7}$$

2. According to principle of minimum conflict, a combination rule $P_r \in \mathrm{P}'$ is chosen that minimizes the value of contradiction measure of combined mass function min($Contr(\mathrm{m}_i P_r\ \mathrm{m}_j)$), $Contr(\mathrm{m}_i P_r\ \mathrm{m}_j) \ne 0$:

$$Contr_m = \sum_{B_j \in 2^\Omega} m(B_j) d(m, m_{B_j}) \tag{8}$$

3. IF $P_l \ne P_r$, THEN a combination rule is chosen that satisfying the next conditions

$$P = \begin{cases} P_l, & T\left(\mathrm{m}_i P_l\ \mathrm{m}_j\right) < T\left(\mathrm{m}_i P_r\ \mathrm{m}_j\right); \\ P_r, & T\left(\mathrm{m}_i P_l\ \mathrm{m}_j\right) > T\left(\mathrm{m}_i P_r\ \mathrm{m}_j\right). \end{cases} \tag{9}$$

The proposed approach to the selection of the combination rule was investigated for combination an experts' evidences with different structure.

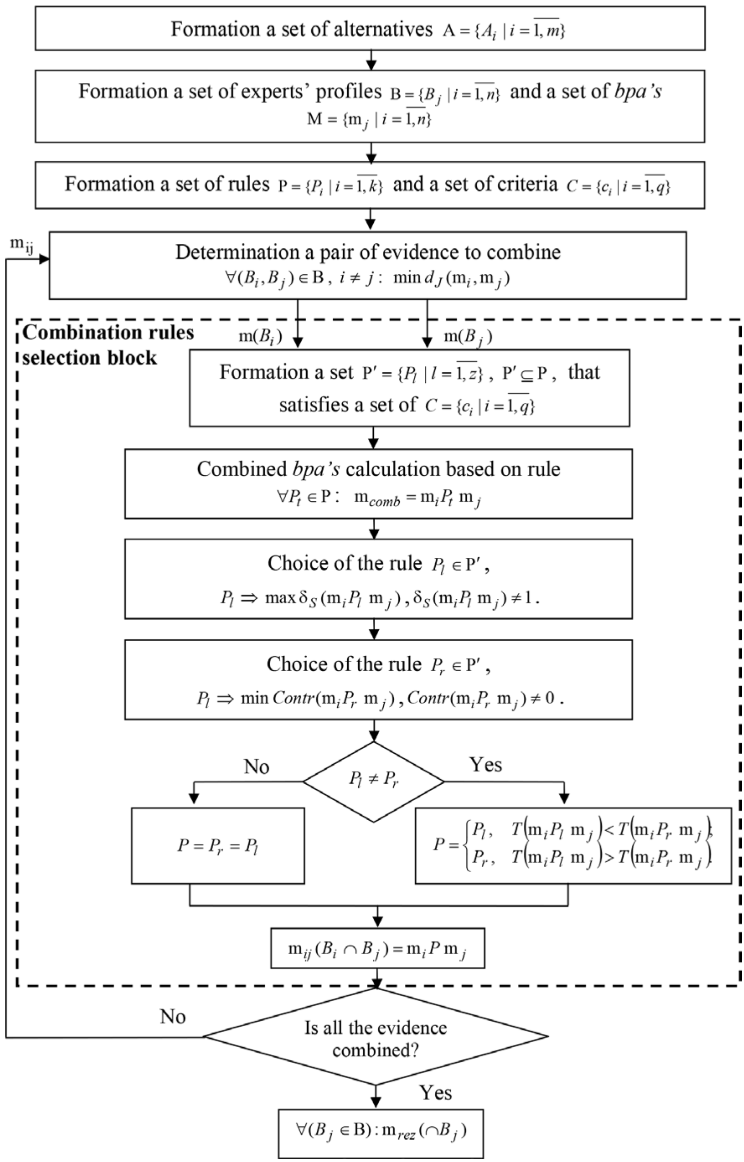

Figure 2. Generalized combination rule selection algorithm

3. Synthesis of information technology for structuring expert assessments under complex forms of ignorance

The generalized structure of IT for structuring of expert information under complex forms of ignorance and synthesis of a group solution is shown in Figure 3. Let's consider the main ideas of IT for analysis of expert information, formed under various forms of ignorance.

Let $\mathrm{A} = \{A_i \mid i = \overline{1,n}\}$ be an initial set of objects (alternatives) under consideration, on which certain restrictions may be imposed: mutual exclusion and/or mutual exhaustiveness, which determines the type of model within which expert evidences (preferences) will be formed.

Suppose a group of experts $\mathrm{E} = \{E_j \mid j = \overline{1,t}\}$, evaluating a certain initial set of alternatives $\mathrm{A} = \{A_i \mid i = \overline{1,n}\}$, formed experts' profiles $\mathrm{B} = \{B_j \mid j = \overline{1,t}\}$. The profile B_j formed by the expert E_j reflects his/her priorities relative to all the analyzed elements of the set A, and corresponds to one of the systems of rules (1) or (2), respectively (depending on the selected analysis model).

Experts are presented with the same instruction, which states what they should do with the elements of the set A.

The profile B_j formed by the expert E_j reflects his/her preferences, expressed within the given scale, regarding the elements of the set A. Expert E_j himself / herself decides which elements (or selected groups of elements) of the set he/she will evaluate. Thus, the profile B_j formed by the expert E_j may contain: assessments expressed in relation to all elements of the set A; assessments expressed in relation to the desired elements of set A; assessments expressed in relation to selected groups of desired elements of set A.

Next, the set of profiles $\mathrm{B} = \{B_j \mid j = \overline{1,t}\}$ enters the input of ignorance type nf_i identification block. In this case we are talking about such types of ignorance as uncertainty, inconsistency / conflict, contradiction, combinations of which can be simultaneously present in input dataset. In this block, a set of identifying parameters (criteria) of the analyzed complex forms of ignorance (combinations of uncertainty, inaccuracy, inconsistency / conflict, contradiction) $C_i = \{c_j^{(i)} \mid j = \overline{1,z}\}$ is formed, on the basis of which a system of decisive rules $SR_i = \{R_l^{(i)} \mid l = \overline{1,h}\}$ for ignorance type identification is formed.

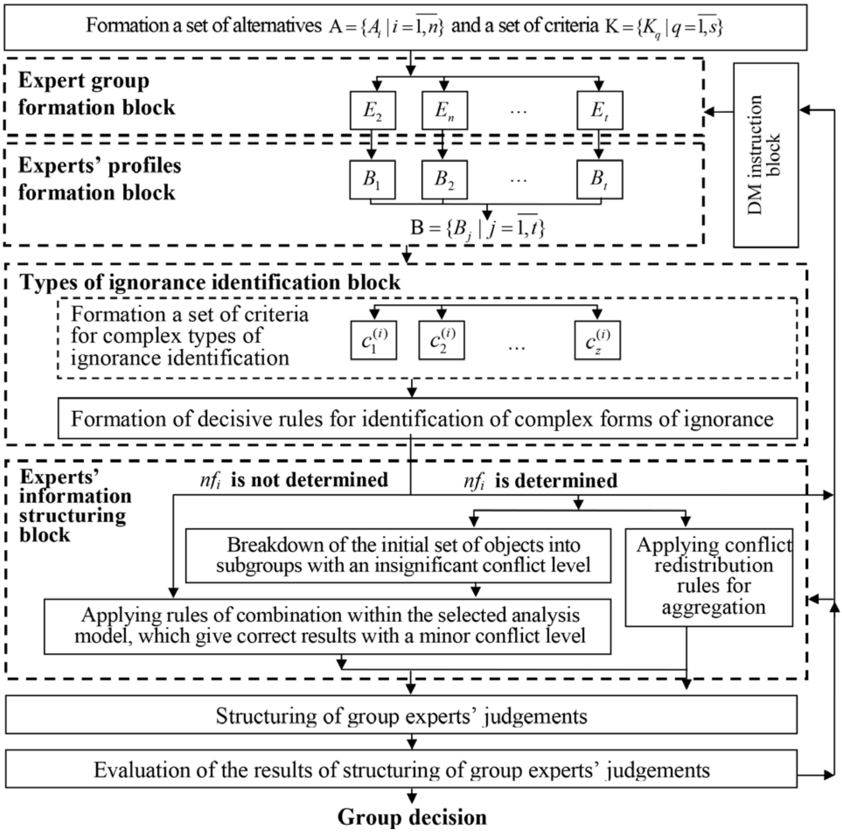

Figure 3. Structure of IT for analysis of experts' judgements under complex forms of ignorance

For identification (confirmation) of the presence of the specified type ignorance, either one parameter (criterion) that allows to unambiguously establish the presence of nf_i, or their combination can be used:

1. the absence of some type of ignorance: $nf_i: \ \forall j : c_j^{(i)} \to no\ nf_i$;
2. the presence of some type of ignorance: $nf_i: \ \exists j : c_j^{(i)} \to nf_i$.

In order to identify complex forms of ignorance (combined types of ignorance), it is proposed to use the following parameters (criteria):

1. the structure of expert judgements $X_l \subseteq B_j$;

2. the conflict level;

3. indicators of the quality of received judgements: level of auto-conflict (conflict within a group of evidences); the degree of specificity of the generated evidences, and etc.;

4. the degree of inconsistency of the generated evidences;

5. limitations imposed on the frame of discernment A.

The next step is the formation of a system of decisive rules for identifying complex forms of ignorance $SR_i = \{R_l^{(i)} \mid l = \overline{1,h}\}$.

On the basis of the formed decisive rules $R_l^{(i)}$, a rule for choosing a method of modeling complex forms of ignorance can be obtained:

$$B_j \in \begin{cases} P_1, & if \quad \forall l : R_l^{(i)} \to no \quad nf_i; \\ P_2, & if \quad \exists l : R_l^{(i)} \to nf_i; \end{cases} \tag{10}$$

where P_1 denotes that expert evidences are not contradictory, characterized by high quality and consistancy; P_2 denotes that represent conflicting judgements.

If $B_j \in P_1$, then the assumption of consistancy of experts' judgements (characterized by close evidence, the presence of a low / insignificant level of conflict) is accepted, and may indicate a high (or acceptable) quality of experts' judgements. In this case, for obtaining an aggregated estimates can be recommended the combination rules wthin the DS model [3, p. 325; 4, p. 3; 8, p. 15; 10, p. 5; 11, p. 9]; within the framework of the DSm model can be recommended the classic and/or hybrid DSm rule, depending on the restrictions of the constructed model [10, p. 11; 11, p. 20].

If $B_j \in P_2$, then it is found that there is an inconsistency (conflict) in experts' judgements, as a result of which three problems arise:

1. detection and exclusion of conflicting (contradictory) experts' judgements;

2. division (clustering) of the original set of experts' judgements into homogeneous (with a permissible level of conflict) subgroups;

3. aggregation of conflicting (contradictory) experts' judgements in order to generate a group assessment.

To solve the first problem, various measures can be used, which allow to quantitatively assess the similarities and differences in experts' judgements. These measures can use distance metrics [1, p. 100; 2, p. 531; 5, p. 94]; assess the degree of conflict between focal elements of several groups of experts' judgements [5, p. 94]. At the same time, both the nature of the subsets that distinguished by experts (including singletons), and the values of the mass function (3) of corresponding subsets can be taken into account.

For example, suppose a given frame of discernment $\Omega = \{a, b, c, d\}$, thus,

– experts' E_1 and E_2 evidences:

E_1 : $m\{a\} = 0.1$; $m\{b\} = 0.9$; E_2 : $m\{a\} = 0.9$; $m\{b\} = 0.1$;

are contradictory (the same elements of the frame of discernment are evaluated, but they are assigned a conflicting assessment);

– experts' E_1 and E_2 evidences:

E_1 : $m\{a\} = 0.4$; $m\{b\} = 0.6$; E_2 : $m\{c\} = 0.6$; $m\{d\} = 0.4$;

are also contradictory (there are no jointly selected and evaluated elements of the frame of discernment, when combined they give empty intersections).

To solve the second problem, a procedure for structuring of group experts' judgements under uncertainty and inconsistency is proposed [7, p. 73]. The proposed technique allows to select from the initial set of experts' judgements agreed subgroups $E \Rightarrow \{G_1\}, \{G_2\},\ldots, \{G_q\}, \ldots, \{G_p\}$ ($G_q \subseteq E$, $\{G_q\} = \{E_1,\ldots,E_r\}$, $t \geq r \geq 1$, $t \geq p \geq 1$). Further, within each of the formed subgroups, aggregated group estimates can be obtained.

To solve the third problem, it is proposed to use one of the conflict redistribution rules [11, p. 36], each of which, using different mechanisms, allows redistributing partial conflict probability masses or the total conflict probability mass (depending on the rule) to subsets involved in conflicts. Thus, aggregated estimates can be obtained even on the basis of completely contradictory evidence.

The next stage is the choice of a mathematical apparatus for the analysis of experts' judgements.

If the absence of nf_i is confirmed, then the procedure for structuring of experts' judgements is reduced to solving the problem of finding an aggregated (generalized) solution. If the presence of nf_i is established during the analysis (the set of experts' judgements is characterized by low consistency, inconsistency and conflict), then the procedure for structuring of consistency is reduced to solving the task of dividing the expert group into several subgroups (clusters) of experts with close (agreed, non-contradictory) assessments, for their further analysis and search for an aggregated assessment within each of the selected groups.

If the division of the initial set of experts' judgements into a number of agreed subgroups and the search for aggregated estimates within the selected subgroups is inadmissible, it is advisable to determine the reason for the dispersion of experts' estimates, identify experts whose estimates violate the consistency of the overall set of evidences, and conduct a repeat survey (perhaps with corrections to the composition of the expert group, changing the procedure of the expert survey, the form of presentation of experts' judgements, etc.) in order to obtain the agreed experts' judgements.

The result of the processes taking place in this block is the information prepared (structured) for decision-making that meets the goals of the analysis.

The resulting stage is the interpretation of the received structuring results and the development of a group solution.

The developed IT can be applied to solve various problems of choice under complex forms of ignorance, characterized by multi-criteria and multi-alternativeness.

Let's consider an example of the synthesis of IT for decision-making support under complex forms of ignorance.

Formulation of the problem. Let's assume a given set of analyzed variants of the technological process of cutting and welding $\mathrm{A} = \{A_i \mid i = \overline{1,n}\}$, a set of criteria $\mathrm{K} = \{K_l \mid l = \overline{1,s}\}$ for their selection and a group of experts $\mathrm{E} = \{E_j \mid j = \overline{1,t}\}$ conducting the examination.

The expert group may include representatives of headquarters units (the department of the chief designer, the department of the chief technologist, the planning and production department, the department of material and technical supply, etc.), as well as representatives of line units (production shops, divisions, etc.).

As criteria for the selection of welding technologies, the following can be considered: technical capabilities, operational reliability, ease of maintenance, types and amount of energy required for the operation of the device, equipment maintenance expenses, welding quality, etc.

It is necessary to determine the optimal, from the point of view of the considered criteria and the obtained technical and economic indicators, the variant of the technological process (TP) of cutting and welding $\mathrm{A} = \{A_i \mid i = \overline{1,n}\}$.

Input data:

A set of input data	initial data set $\mathrm{A} = \{A_i \mid i = \overline{1,n}\}$; a set of criteria $\mathrm{K} = \{K_l \mid l = \overline{1,s}\}$; a group of experts $\mathrm{E} = \{E_j \mid j = \overline{1,t}\}$; the corresponding values of the vector of expert competence coefficients $\Omega = \{\omega_j \mid j = \overline{1,t}\}$; expert preference scale 1 ÷ 9.
Data structure	ST = {binary relations; numbers};
Analyzed task	z_t = {construction of collective ranking of objects}.

The procedure of identification of combined type of ignorance {nf_1 = inconsistency, nf_2 = uncertainty, nf_3 = conflict, nf_4 = vagueness}.

1. Example of identification criteria $C_i = \{c_j^{(i)} \mid j = \overline{1,s}\}$, $C_i \subset CN$:

$c_1^{(i)}$ is a structure of expert evidences: consistent: $\forall (B_j, B_t) \subseteq \mathrm{B}:\ B_i \subseteq B_j$;

$c_2^{(i)}$ is a structure of expert evidences: separate: $\forall (B_j, B_t) \subseteq \mathrm{B}:\ B_i \cap B_j = \varnothing$;

$c_3^{(i)}$ is a structure of expert evidences: compatible: $\forall (B_j, B_t) \subseteq \mathrm{B}$: $B_i \cap B_j \neq \varnothing$;

$c_4^{(i)}$ is a structure of expert evidences: arbitrary: $\exists (B_j, B_t) \subseteq \mathrm{B}$: $B_i \cap B_j \neq \varnothing$;

$c_5^{(i)}$ – $\exists X_l \subseteq B_j:\ X_l = \{A_i \cap A_n\}$;

$c_6^{(i)}$ – $\forall X_l \subseteq B_j:\ X_l = \{A_i \cup A_n\} \vee |X_l| = 1$;

$c_7^{(i)}$ is an estimator of the specificity coefficient according to (7);

$c_8^{(i)}$ is an estimator of the contradiction coefficient according to (8);

$c_9^{(i)}$ is an estimator of the coefficient of auto-conflict;

$c_{10}^{(i)}$ is an estimator of the global uncertainty coefficient [6, p. 139];

$c_{11}^{(i)}$ is an estimator of the conflict between the group of expert evidences 470];

$c_{12}^{(i)}$ is a form of expert evidences (crisp, interval, fuzzy, mixed);

$c_{13}^{(i)}$ is the number of formed focal elements.

2. An example of the decisive rules for the identification of a combined type of ignorance $SR_i = \{R_l^{(i)} \mid l = \overline{1,h}\}$, $SR_i \subset SRN$:

$R_1^{(i)}: \; \left(c_6^{(i)} \wedge ((c_{11}^{(i)} > k_1) \vee (c_9^{(i)} > k_2)) \wedge (c_{13}^{(i)} >| A |), D\right) \to \{nf_2, nf_3\}$, k_1, k_2 are a threshold level of conflict;

$R_2^{(i)}: \; \left(c_6^{(i)} \wedge (c_{11}^{(i)} > k) \wedge (c_{13}^{(i)} >| A |), D\right) \to \{nf_1, nf_2\}$, k is a threshold level of conflict;

$R_3^{(i)}: \; \left(c_6^{(i)} \wedge c_{12}^{(i)} = \{\text{fuzzy}\}, D\right) \to \{nf_2, nf_4\}$;

$$R_4^{(i)}: \; \begin{pmatrix} c_6^{(i)} \wedge ((c_{11}^{(i)} > k_1) \vee (c_9^{(i)} > k_2)) \wedge (c_{13}^{(i)} >| A |) \wedge \\ \wedge (c_{12}^{(i)} = \{\text{fuzzy}\}), D \end{pmatrix} \to \{nf_2, nf_3, nf_4\};$$

$R_5^{(i)}: \; \left(c_1^{(i)}, D\right) \to$ absence of local nf_3;

$R_6^{(i)}: \; \left(c_2^{(i)}, D\right) \to nf_3$;

$R_7^{(i)}: \; \left(c_2^{(i)} \wedge c_6^{(i)} \wedge c_{12}^{(i)} = \{\text{mixed}\}, D\right) \to \{nf_2, nf_3\}$.

Selection of a method of modeling a complex type of ignorance $\{nf_1$ = inconsistency, nf_2 = uncertainty, nf_3 = conflict, nf_4 = vagueness$\}$.

Parameters of IT synthesis:

Vector of input parameters:	$V=\{D; s_l; ST; z_t\}$;
Parameters of IT synthesis $PS = \{Par_i \mid i = \overline{1,k}\}$, $PS = PS^V \cup PS^P$:	$PS^V = V$, PSP = {experts' profiles; analysis model; structure of experts' evidences; set of identification criteria for nf_i; identified types of nf_i; a set of methods for determining aggregated assessments; a set of parameters for choosing a modeling method; selected modeling method(s)}
IT generation rule:	$\wedge Par_i \to IT_q$, $i \le k$.

The vector of parameters-results contains the following information: the identified type(s) of ignorance (nf_i), or information about its absence; the number of rule (l) by which the type of ignorance was identified; the metric by which the distance between pairs of experts' judgements was estimated;

the chosen method(s) by which the aggregated scores were determined; collective ranking of elements of the set A; intervals [$Bel(\cdot)$, $Pl(\cdot)$] for all resulting focal elements which are formed; the maximum achievable value of the conflict coefficient between experts' evidences. Based on the results of IT synthesis, an IT synthesis protocol has been formed.

4. Conclusions

1. The methodology for synthesis and generalized structure of IT for decision-making support under multi-criteria, multi-alternativeness and complex forms of ignorance have been proposed. The developed IT is constructed to solve the problem of analysis (ranking, clustering, ranking of clusters) of group experts' judgements under multicriteria and complex forms of ignorance (uncertainty, inaccuracy, inconsistency / conflict, contradiction) with the aim of producing a resulting (generalized) assessment. The proposed IT can be used in solving non-criteria decision-making problems under complex forms of uncertainty (combined types of ignorance).

The proposed IT is based on mathematical models for individual and group solutions synthesis, which are based on the mechanism of integrated use of combination rules within the framework of DS and DSm models [3, p. 325; 4, p. 3; 8, p. 15; 10, p. 5; 10, p. 11; 11, p. 9].

This approach makes it possible to select and group different combinations of initial options (objects of examination, alternatives) into clusters, in accordance with the individual choice of the expert, to conduct their analysis, and to obtain the resulting ranking of the group of experts' judgements. At the same time, the limitation on the number of analyzed objects (alternatives), the condition of consistency of experts' judgements, was removed.

2. The procedure for choosing the optimal combination rule is proposed, depending on the nature of the initial data obtained from various sources. The proposed procedure ensures obtaining the combined probability mass with the lowest achievable level of uncertainty. The algorithm provides for cutting off a number of rules that do not satisfy a given set of criteria for combination rules selection. Based on the principle of minimal uncertainty, it is proposed to choose a rule that minimizes the value of inconsistency measure and maximizes the value of specificity measure of results of combination. As criteria for rules selection, the following can be recommended: analysis model, information about data sources; the nature of the analyzed data.

References:

1. Bhattacharyya A. (1943). On a measure of divergence between two statistical populations defined by their probability distribution. *Bulletin of the Calcutta Mathematical Society*, vol. 35, pp. 99–110.

2. Cuzzolin F. (2007). A geometric approach to the theory of evidence. *Transactions on Systems, Man, and Cybernetics* (*Part C: Applications and Reviews*), vol. 38(4), pp. 522–534. DOI: https://doi.org/10.1109/TSMCC.2008.919174

3. Dempster A. P. (1967). Upper and lower probabilities induced by a multivalued mapping. *Annals of Mathematical Statistics*, vol. 38(2), pp. 325–339. DOI: https://doi.org/10.1214/aoms/1177698950

4. Florea M. C., Bosse E. (2009). Crisis management using Dempster-Shafer theory: using dissimilarity measures to characterize sources' reliability. *Proceedings Symposium on C3I for Crisis, Emergency and Consequence Management*, RTO-MP-IST-086. (Bucharest, Romania, May 11-12, 2009).

5. Jousselme A. L., Grenier D., Boss´e E. (2001). A new distance between two bodies of evidence. *Information Fusion*, vol. 2(2), pp. 91–101. DOI: https://doi.org/10.1016/S1566-2535(01)00026-4

6. Klir G. J., Parviz B. (1992). A note on measure of discord. *Uncertainty in Artificial Intelligence* (UAI'92): 8th International Conference (Stanford, CA, USA, July 17th–19th, 1992). DOI: https://doi.org/10.1016/B978-1-4832-8287-9.50023-2

7. Kovalenko I. I., Shved A. V. (2016). Clustering of group expert estimates based on measures in the theory of evidence. *Naukovyi Visnyk Natsionalnoho Hirnychoho Universytetu*, vol. 4(154), pp. 71–78.

8. Sentz K., Ferson S. (2002). *Combination of evidence in Dempster-Shafer theory: tech. report SAND 2002-0835*. Albuquerque: Sandia National Laboratories.

9. Shafer G. A. (1976). *Mathematical Theory of Evidence*. Princeton: Princeton University Press.

10. Smarandache F., Dezert J. (2004). *Advances and applications of DSmT for information fusion. Collected Works*. Rehoboth: American Research Press.

11. Smarandache F., Dezert J. (2006). *Advances and applications of DSmT for information fusion*. *Collected works*. Rehoboth: American Research Press.

12. Smarandache F., Martin A., Osswald C. (2011). Contradiction measures and specificity degrees of basic belief assignments. *Information Fusion* (FUSION 2011): 14th International Conference (Chicago, USA, July 5th–8th, 2011)

13. Shved A. V. (2016). Alhorytm vyboru pravyla kombinuvannia ekspertnykh svidotstv [Algorithm for selection the rule for experts' evidence combination]. *Naukovi pratsi: Naukovo-metodychnyi zhurnal Chornomorskoho derzhavnoho universytetu im. P. Mohyly. Seriia: Kompiuterni tekhnolohii*, vol. 271(283), pp. 19–25.

14. Shved A. V. (2016). One approach to selection of the combination rules. *Intellectual Systems for Decision Making and Problems of Computational Intelligence* (ISDMCI'2016): 12th International conference (Kherson, Ukraine, May 24th–28th, 2016).