

**CHAPTER 2 «DEVELOPMENT OF MATHEMATICAL MODELS FOR DESIGNING EDUCATIONAL RESOURCES IN HIGHER SCHOOL»**

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**Abstract.** *Purpose.* An urgent issue in many branches of the modern economy is the possibility of various uses of large-volume containers of various manufacturing geometries for the purpose of storage, transportation and use of various types of technical liquids, water, etc. The most common geometry for the manufacture of such containers is cylindrical, hemispherical, parabolic tanks. In most cases, the spatial arrangement of such containers during their direct use has a vertical orientation. But as the review and trends of modern uses of such cylindrical structures show, there is a trend towards their horizontal or combined location methods during practical operation. The volume of the filler in the conditions of constant use of such containers is also constantly changing, and therefore the main energy characteristics of the filler materials themselves change accordingly, to which we can attribute the volume of liquid, the set of static moments, as a result, the coordinates of the center of gravity, the moments of inertia relative to those or other coordinate axes, planes, etc. The issue of changing certain geometric characteristics when changing the location of such cylindrical containers from horizontal to vertical or vice versa can be interesting and relevant. A significant number of sectors of the economy show interest in plastic horizontal containers for water storage, and this is understandable, because the ease of use and variety of products attract an increasing number of consumers. Plastic types of cylindrical containers have been gaining popularity in various industries for a long time due to their useful features. Innovative technologies and materials have made it possible to create plastic products of various shapes, sizes and purposes, and this has many obvious advantages. The use of plastic materials in the manufacture of various containers allows you to significantly reduce the initial weight of

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such a product in comparison with its metal counterparts or glass products, which means that when establishing the main energy characteristics, the weight of the container itself can be neglected. In addition, the use of this type of container is convenient for its transportation, installation, etc.

*The results* of scientific analytical studies are also presented, which ensure the determination of the pressure force on underwater hydroelectric structures, such as, for example, dams of hydroelectric power plants, liquid organic fertilizer and wastewater sedimentation tanks, coastal fortifications of lakes, rivers, ponds, reservoirs, etc.

*The practical experience* of industrial and pedagogical activities and the analysis of inventive and research works of domestic and foreign scientists have shown that the structural, technological and economic feasibility, operational safety of underwater hydraulic structures requires further analytical and computational studies based on mathematical optimization methods. The achievements of Chinese scientists are of particular great scientific interest. Their achievements in the calculation, design, construction and operation of small and large hydroelectric power plants are recognized by scientists, designers and manufacturers in the world.

The work is based on research aimed at the mathematical substantiation of the parameters of a rectangular, trapezoidal lock and a circular hatch, which is located at an arbitrary, optimally structurally justified depth.

It is taken into account that every year new students are enrolled in universities who need to acquire knowledge on the calculation of the strength of structures of objects with the involvement of innovative methods based on fundamental mathematical apparatus. The formation of the professional competences of a modern agricultural engineer should ensure the ability to apply the theoretical foundations and basic methods of physics and mathematics, hydrostatics, dynamics, fluid mechanics for the calculation and design of technical objects and, of course, underwater hydraulic structures. An agricultural engineer must be able to perform calculations on the strength of the walls of containers of any geometric profiles for storing liquids, as well as the side walls and bottoms of hydraulic structures, bodies that are placed in liquids.

The work also presents and investigates calculations of volumes in the form of geometric shapes of containers of a standard shape, such as a cone, sphere, and cylinder, using curvilinear coordinates, including cylindrical

and spherical ones. The comparative method of calculating a specific mathematical characteristic, namely volume, confirms the effectiveness of using cylindrical coordinates in the case of separation of one of the independent variables, and spherical coordinates, if all three variables describing the corresponding surface are equal.

### **Introduction**

An actual issue when using containers with different spatial geometry filled with certain liquids is the possibility of establishing their main mechanical characteristics, such as determining the geometric coordinates of the center of gravity, static moments, various moments of inertia of such volumes with homogeneous fillers. Of practical interest is also the possibility of setting the filling height of containers of cylindrical manufacturing geometry when changing their spatial location from a vertical position to a horizontal position or vice versa. Despite the rather difficult conditions for the development of many branches of the current economy, many different trading companies offer the modern market a fairly large palette of products in the form of containers of any geometric shape, made mainly of polymer materials for storing or transporting various types of liquids. When operating, installing or transporting such products, it is often important to know and take into account the main characteristics of such containers. For example, in the scientific research [1], the question of calculation and ratio of numerical values of the work of pumping liquid from a tank of cylindrical geometry has been investigated in cases where such a tank has a vertical or horizontal spatial location. Solving the issues of evaluating and finding the main mechanical characteristics (size of the volume, static moments, coordinates of the center of gravity, moments of inertia) of certain volumes filled with liquid and limited by the corresponding spatial surfaces, has been established in many modern scientific researches, for example, in [2–7]. In essence, these scientific works present theoretical basic approaches to the possibility of establishing and calculating the above relevant indicators. Any final finding of such characteristics is impossible without the use of appropriate modern mathematical apparatus, the basics of integral calculus, presented in many scientific works, for example [8–9]. In the research [10], it is proposed to use not only Cartesian spatial coordinates, but also curvilinear (cylindrical) coordinates with some separate independent

variable. The practical application of such curvilinear coordinates can be performed even when such a separable variable is absent. This applies, for example, to the issue of calculating the corresponding triple integrals over the volume bounded by the sphere or its part, here the equality of all three independent variables is obvious. One of the problems solved in the research is the question of establishing the analytical correspondence of the partial liquid filling of a cylindrical container, which in one of the possible cases of location is in a vertical position, i.e. all elements of the cylindrical surface are vertical, and another position of this cylindrical container with the same geometric by the characteristics of the radius of the base circle and the height (creator), it is possible to have the opposite horizontal location of such a vessel, when the creators of this figure are horizontal. The research concerns the prospects of establishing the basic geometric mechanical characteristics, such as static moments, coordinates of the center of gravity, moments of inertia relative to certain coordinate axes and planes for the volume capacity in the form of a hemisphere, conical and parabolic containers. The use of curvilinear coordinates, such as cylindrical, spherical or other, allows to simplify mathematical solutions and perform complex calculations, especially when objects have complex geometric shapes. For example, to calculate the volumes of volumetric bodies that have a cylindrical, conical or parabolic shape, the use of appropriate curvilinear coordinates helps to find analytical relations that reflect the volume of these bodies depending on their geometric parameters. Also, the calculation of mechanical properties such as static moments, moments of inertia and center of gravity coordinates can also be simplified and performed more efficiently using curvilinear coordinates.

Another part of the research is devoted to modern hydraulic structures. Underwater hydraulic structures have different designs. Various forces act on them, which cause internal stresses, among which static ones prevail, which change slowly in time, and, in this case, inertial forces are neglected. Static loads are usually assumed to be constant [11]. Methods of solving applied problems of mathematical analysis [12] using integral functions for various geometric profiles should be calculated by several methods, and the obtained results should be compared with each other to obtain the most objective assessment of the state of the object. To obtain an optimal objective result, it is important to formulate an applied mathematical problem [13],

integrate and calculate this integral adequately to the real design. Not all engineering and technical workers are able to do this. And therefore, finding the magnitude of the pressure force on underwater hydraulic structures with the use of a mathematical apparatus is important both in the design production activity and in the educational process in innovative pedagogical technologies of training engineering and technical specialists.

Many research works (scientific articles, monographs, training manuals and textbooks, patents, etc.) in our country and abroad are devoted to the problematic issues of general and subject-oriented theory, calculation, design, construction and construction of underwater hydraulic structures. Thus, problems of analytical and applied higher mathematics of mechanics are presented in the works of V. A. Balash [11], B. P. Demidovicha [12], V. M. Dubchaka, V. M. Pryshlyaka, L. I. Novytska [13], N. S. Pyskunov [27] and many other scientific and pedagogical specialists. The basics of engineering methods of strength and stiffness calculations have been considered by the authors H. M. Kaletnikov, M. G. Chausov, V. M. Shvayko, A. V. Pylypenko, A. H. Kutsenko, M. M. Bondar, V. M. Prishlyakom, L. S. Shymko [18; 19; 25; 28] and others. A number of works [14; 15; 16; 20–24; 26; 39] by A. G. Kutsenko, S. M. Bondar, H. M. Kaletnika, M. G. Chausova, M. M. Bondar, V. M. Prishlyaka, V. L. Kuryla, L. V. Ostapchuk, V. E. Dyshlyuka, E. H. Degodyuk, Salieva Enver, Liu Zhong and others. It is noted that hydromechanics is a branch of mechanics that studies the laws of movement and equilibrium of fluids, as well as the laws of interaction between fluids and bodies. Sometimes, with some assumptions in hydromechanics, classical Newtonian mechanics can be used to solve applied problems [15].

In the study [14], the issue of small hydropower was considered by authors such as H. M. Kaletnik. and V. M. Prishlyak, in particular: technical, economic and ecological aspects of the use of small hydropower stations and installations, the state of small hydropower in Ukraine and in the world. In this work [14] it is noted that increasing the role of small hydropower as an alternative (to the usual coal, oil, gas) source will contribute to the development of the country's energy security. The list of potential sources of electricity for small hydropower is extremely large: these are rivers, elevation differences on irrigation channels of irrigation systems, etc. In general, more than 20,000 small rivers with a total length of

about 200,000 km flow through the territory of Ukraine [14]. And therefore, the potential opportunities for electricity production in our country are significant.

The calculation of underwater hydraulic structures in agro-industrial production can be carried out for the calculation, design, construction and construction of special containers, for the production and storage of organic-mineral fertilizers based on sewage sludge for cultivated soils [10]. Thus, scientists and inventors V. L. Kurylo, L. V. Ostapchuk, V. E. Dyshlyuk, E. G. Degodyuk propose a method of producing organic-mineral fertilizer based on sewage sludge for soil cultivation, which includes the separation of liquid sediment in the process purification of urban wastewater, dewatering of sewage sludge by keeping it on sludge platforms, which can be considered when solving mathematical problems as underwater hydraulic structures, in which the formation of dehydrated sewage sludge takes place on the side for composting. It is worth noting that to decontaminate dehydrated sewage sludge from pathogens of infectious and parasitic diseases and to sterilize weed seeds, the necessary mass of carbon ammonium salt is added to it when forming into burts for composting in accordance with the degree of contamination, and to bind and transfer mobile forms of chemical pollutants in a fixed state, the necessary, scientifically based mass of adsorbent is added to it [20]. The analysis of inventive and research works of domestic and foreign scientists showed that the structural, technological and economic feasibility, operational safety of underwater hydraulic structures requires further analytical and computational studies based on mathematical optimization methods [17].

The achievements of Chinese scientists cause especially great scientific interest in the development of hydropower [29]. Their achievements in the calculation, design, construction and operation of small and large hydroelectric power plants are recognized by scientists, designers and manufacturers all over the world. It should be noted that the largest hydroelectric power plant in the world with an annual electricity production of about 100 million kW·h. electricity has been built specifically in China. This hydroelectric power plant is located in Hubei Province on the Yangtze River.

The development of a mathematical methodology for substantiating the parameters of locks and hatches in hydraulic structures is a key

stage for ensuring their efficiency and reliability. Such studies can help in the development of more accurate and efficient models of hydraulic structures, which is of great importance for various fields – from energy to environmental protection. Undoubtedly, the development of such a methodology will be important for the educational process. Engineering education, built on modern research and practical exercises, prepares future specialists for challenges and innovations in their field. The creation of a mathematical methodology for determining the parameters of locks and hatches in hydraulic structures is pivotal for ensuring their functionality and dependability. This advancement holds significant promise for refining models of hydraulic structures, impacting diverse sectors from energy production to environmental conservation. The implications extend far beyond immediate practical application. This development can significantly augment educational curricula in engineering. By integrating cutting-edge research and practical exercises, educational programs can equip aspiring engineers with the skills and knowledge needed to tackle challenges and drive innovations in the field.

These methodologies not only improve the design and efficiency of hydraulic structures but also serve as a foundation for educating future engineers. The hands-on experience and theoretical understanding gained through these studies are invaluable for shaping well-rounded professionals capable of addressing the evolving demands of the industry. Such advancements in engineering education foster a generation of problem solvers ready to contribute to technological advancements and sustainable solutions.

The analysis of the latest research and publications gives reason to conclude that it is expedient to conduct scientific analytical studies that will ensure the determination of the pressure force on underwater hydraulic structures, such as, for example, dams of hydroelectric power stations, coastal fortifications of lakes, rivers, ponds, reservoirs, settling tanks, sewage, liquid organic fertilizers, etc. It is advisable to direct the strategy of applied research to the mathematical substantiation of the parameters of a rectangular and trapezoidal lock and a circular hatch located at an arbitrary depth. It is worth noting that the calculation method is also important for the educational process during the preparation of future engineers for innovative project activities.

The possibility of effective volume calculation is one of the main characteristics of many technical tasks of modern science. Effective calculation of such a feature for standard geometric spatial figures, such as a cylinder, cone, sphere, is an urgent task, especially with the use of the mathematical apparatus of curvilinear spatial coordinates. Thus, calculating the volume of geometric bodies, such as a cylinder, cone, sphere, and others, is an important task in many scientific fields and technical applications. This is of great importance in physics, engineering, computer graphics, architecture and other fields. For complex shapes that may have curvilinear shapes or unusual areas, calculating the volume can become a more difficult task. In such cases, numerical methods, such as integration methods or numerical simulations, can be used to approximate the volume by dividing the figure into smaller, simpler components. Such methods are often used in calculating the volumes of complex three-dimensional objects in scientific research and engineering calculations. Calculating volumes of standard geometric shapes like cylinders, cones, and spheres is foundational across numerous fields. These shapes are prevalent in various applications, from engineering to architecture. However, when dealing with more intricate or irregular shapes, determining volume becomes more challenging. For complex shapes lacking straightforward formulas, numerical methods step in. Integration methods, like calculus-based techniques, can be employed to find volumes by breaking down a shape into infinitesimally small components and summing their contributions. This method is particularly useful for irregular shapes and those with curvilinear boundaries. Additionally, numerical simulations, utilizing computational power to model and analyze complex geometries, offer another approach. These simulations involve dividing a shape into discrete elements and applying mathematical algorithms to approximate its volume.

In scientific research and engineering, these numerical methods play a crucial role in accurately determining the volumes of intricate three-dimensional objects that don't conform to standard geometric figures. Their applications extend to fields like fluid dynamics, material science, medical imaging, and more, where precise volume calculations are vital for analysis and design.



### **2.1. Determination of the Main Mechanical Characteristics of Classic Geometric Containers with Homogeneous Fillers**

Horizontal arrangement and combined arrangements of cylindrical containers can have several advantages and additional applications compared to vertical arrangement. The main advantages of these approaches are:

1. Saving space. Horizontal tanks can be used effectively in areas with limited space. This can be important for industrial enterprises that need to store large volumes of liquids or other substances.

2. Reduction of pressure on the walls. Horizontal tanks can distribute fluid pressure evenly across the entire wall, which can help reduce stress on the tank material. This can increase the life and reliability of the container.

3. Heat preservation. Horizontal tanks can be more efficient in retaining heat, especially in cases where the liquid must be maintained at a certain temperature. This can save energy and reduce heating or cooling costs.

4. Reduction of product loss. Horizontal tanks can reduce product loss due to accidental spills, as the liquid is distributed along the bottom of the tank rather than vertically.

5. Easier access and maintenance. Horizontal tanks can be more accessible for maintenance and cleaning because the work space is located at ground level, allowing easy access to all components.

Horizontal arrangements of cylindrical containers indeed offer several advantages over vertical arrangements:

1) Space Efficiency: horizontal tanks optimize space, making them suitable for areas with limited vertical clearance. Industries dealing with significant liquid storage requirements can benefit from this space-saving design.

2) Even Pressure Distribution: distributing fluid pressure evenly across the container's walls reduces stress concentration points, potentially enhancing the tank's durability and lifespan.

3) Heat Retention: horizontal tanks are efficient at maintaining consistent temperatures for stored liquids, which is particularly advantageous for substances requiring specific temperature control. This can lead to energy savings and decreased heating or cooling expenses.

4) Reduced Product Loss: spills and losses due to accidental leakage are minimized in horizontal tanks as the liquid is spread along the tank's base rather than having a significant vertical height where spillage can occur.

5) Ease of Access and Maintenance: accessibility is improved in horizontal tanks as they typically have ground-level access for maintenance and cleaning. This makes inspection, repair, and routine upkeep more convenient compared to vertically oriented tanks.

Combined arrangements of cylindrical containers can further amplify these benefits by allowing for customized storage capacities while maintaining the advantages of horizontal placement. Additionally, these combined setups can offer more flexible storage solutions tailored to specific spatial constraints and operational requirements.

The issue of changing the geometric characteristics of cylindrical containers when they change from a horizontal to a vertical position and vice versa is very relevant and has practical importance in many industries. When changing the position of a cylindrical container from horizontal to vertical or vice versa, the volume and content of the container remain unchanged, because they depend only on geometric parameters, such as the radius and height of the cylinder. Changing the position can affect the stability of the container. For example, a vertically positioned cylindrical container may be less resistant to peeling or tipping compared to a horizontal container, especially if it does not have additional stability elements or supports, and also affect the ease of access to the contents of the container, for example, in a vertical position it may be more difficult to remove whether to add items, especially if the content is not evenly distributed. The repositioning of cylindrical containers can have significant implications beyond just their geometric alterations. Stability, accessibility, and ease of use are crucial factors affected by such changes. Regarding stability, the distribution of weight plays a key role. A horizontally positioned cylinder typically has a lower center of gravity, which provides greater stability compared to a vertically positioned one. This difference becomes more pronounced when considering potential external forces or uneven weight distribution within the container. Accessibility is another critical aspect. In a horizontal position, accessing the contents might be more straightforward, especially if it involves manual handling or adding/removing items. However, in a vertical orientation, particularly with poorly distributed contents, accessing or manipulating the contents becomes more challenging. Additionally, considering safety measures like supports or stabilizing elements becomes necessary when containers are placed vertically to prevent tipping or spilling. Understanding these practical implications is vital in

industries where cylindrical containers are frequently used, influencing not only the design of the containers themselves but also their handling, storage, and transportation methods.

The advantages of plastic cylindrical containers are felt in both positions. Horizontal containers can be more stable when transported on flat surfaces, while vertical ones can save space when stored.

Plastic cylindrical containers have a wide range of applications in different industries including agriculture, industry, food industry, chemical industry and many others. The change in position may be due to the specific needs and requirements of the relevant industry. Absolutely, the versatility of plastic cylindrical containers is one of their strongest attributes. Their adaptability in different positions caters to diverse needs across various industries. The flexibility to use them horizontally for stability during transportation or vertically for space-saving storage makes them incredibly practical. In agriculture, these containers can hold fertilizers, seeds, or even liquids like pesticides. For industrial use, they might store chemicals, lubricants, or other raw materials. In the food industry, they're handy for storing liquids, powders, or even semi-solid products like sauces or syrups. The adaptability in positioning can indeed be industry-specific. For instance, in the chemical industry, stability during transport to prevent spills might be paramount, thus favoring horizontal positioning. Meanwhile, in a warehouse or retail setting, vertical stacking could optimize space. The ability to cater to various needs by simply adjusting their orientation makes plastic cylindrical containers a versatile and indispensable asset in numerous fields. This adaptability to different industries and their specific demands underscores their practicality and utility.

In general, the issue of changing the position of cylindrical containers has many practical applications and can be investigated from the viewpoints of engineering, safety, usability and other aspects. It can also become the subject of research and development to improve the design and functionality of such containers in various sectors of the economy. In many industries, such as oil refining, chemical industry, water supply and others, horizontal and combined tanks can be effective solutions in terms of spatial arrangement and functionality. However, choosing the optimal type of tank should be done taking into account the specific needs, constraints and requirements of the industry or project.

Horizontal and combined methods of arrangement of cylindrical tanks and containers can have several advantages and practical applications compared to vertical arrangement, depending on the specific task and area of use. Here are some possible reasons and benefits behind this trend:

1. Efficient use of space: horizontally arranged containers can be more efficient use of limited space. They can be placed near the ground or in underground rooms, which allows you to save space and use it for other purposes.

2. Reduction of product loss: horizontal tanks can be useful for liquid or product storage where it is important to reduce pressure gradient loss. They can ensure uniform distribution of pressure over the entire length of the tank and reduce turbulence.

3. Reduced foundation requirements: horizontal tanks may require less massive foundations compared to vertical tanks, which can reduce construction and maintenance costs.

4. Portability: horizontal tanks can be more easily transported and installed in different locations, which can be important for some activities, such as construction or oil and gas extraction.

5. Aesthetic appearance: for some additional uses, such as volumetric aquariums or architectural structures, horizontal tanks can have a more attractive appearance.

The choice between horizontal and vertical placement depends on the specific needs and requirements of a specific project. The decision must take into account factors such as liquid volume, available space, technical requirements and economic feasibility. The choice between horizontal and vertical placement depends on the specific needs and requirements of a specific project. In our opinion, it is appropriate to consider some key factors that can influence this:

1. Liquid volume. If you need to store or process large volumes of liquid, then a vertical tank may be a more practical choice as it can provide more storage volume in the same area.

2. Available space. Horizontal tanks can be useful if you have limited horizontal space, such as in an underground or confined area. Vertical tanks can save the space of the transverse arrangement.

3. Technical requirements. Some specific technical requirements of the project, such as pressure, temperature, agitation, or the need for heating

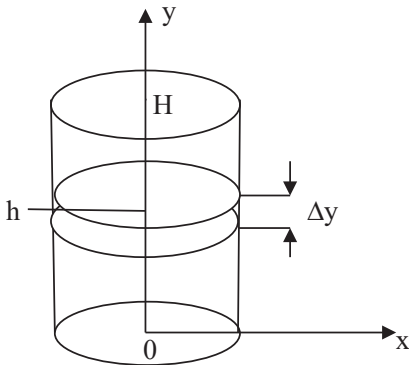
or cooling, may influence the choice between vertical and horizontal arrangement.

4. Economic feasibility. Estimating the cost of construction, maintenance and operation of both options can be a deciding factor. It is important to compare the costs and benefits of both options before making a decision.

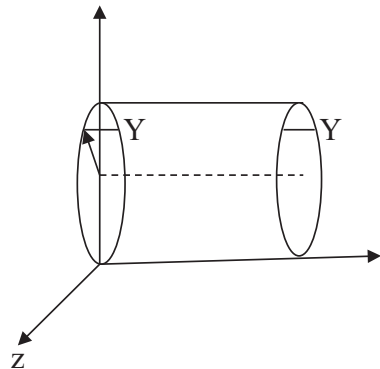
5. The need for anti-corrosion protection. Vertical tanks may have an advantage in terms of corrosion protection, as their main surface faces upwards and is less exposed to ground factors.

In general, it is important to conduct a detailed analysis of all these factors and take them into account in the context of a specific project before choosing between horizontal and vertical tank placement.

Consider the issue of arbitrary filling of a cylindrical container with some homogeneous liquid. At the same time, the spatial location of this figure can be twofold, as shown in Figure 1 or Figure 2, i.e. vertical or horizontal position. In both cases, we consider the geometric parameters of such a container to be constant:  $R$  – radius of the circle of the base of such a geometric figure,  $H$  – its height,  $h$  – the height of the liquid level when the container is placed vertically,  $Y$  – the height of the liquid level when the container is placed horizontally.



**Figure 1. A cylindrical container filled with a homogeneous liquid with a vertical arrangement of its height**



**Figure 2. A cylindrical container filled with a homogeneous liquid with a horizontal arrangement of its height**

Let us set the goal to investigate and establish an analytical relationship between the values  $h$  and  $Y$ , where  $0 \leq h \leq H, 0 \leq Y \leq 2R$ . To do this, we first set the volume of the liquid directly filling the container in Figure 1. This volume is defined as

$$V_1 = \pi R^2 h. \quad (1)$$

To establish a similar volume of liquid, a cylindrical container (Figure 2) is placed horizontally at an arbitrarily chosen height. To establish a similar volume  $V_2$  of the amount of liquid horizontally located in a cylindrical container (Figure 2) initially at an arbitrary height  $y$  ( $0 \leq y \leq Y \leq 2R$ ) set the elementary volume of the liquid  $\Delta V_2$ :

$$\Delta V_2 = 2xH\Delta y, \quad (2)$$

here  $x$  is determined from the ratio  $x^2 + (y - R)^2 = R^2$ , from where  $x = \sqrt{R^2 - (y - R)^2}$ ,  $y \in [0; 2R]$ . Therefore, the value of the entire volume  $V_2$  is defined as follows:

$$\begin{aligned} V_2 &= \int_0^Y 2H\sqrt{R^2 - (y - R)^2} dy = \int_{y=0 \rightarrow t=-R}^{y=Y \rightarrow t=Y-R} 2H\sqrt{R^2 - t^2} dt = \\ &= 2H \left( \frac{t\sqrt{R^2 - t^2}}{2} \Big|_{-R}^{Y-R} + \frac{R^2}{2} \arcsin \frac{t}{R} \Big|_{-R}^{Y-R} \right) = \\ &= H \left( (Y - R)\sqrt{R^2 - (Y - R)^2} + R^2 \arcsin \frac{Y - R}{R} + \frac{\pi R^2}{2} \right). \quad (3) \end{aligned}$$

Equating volumes  $V_1$  and  $V_2$ , we can establish the desired analytical dependence between the quantities  $h$  and  $Y$ :

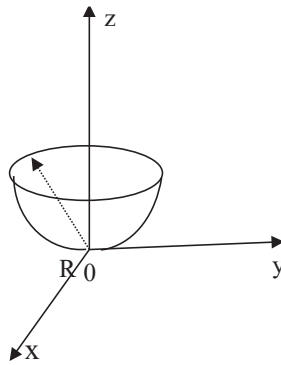
$$h = \frac{H}{\pi R^2} \left( (Y - R)\sqrt{R^2 - (Y - R)^2} + R^2 \arcsin \frac{Y - R}{R} + \frac{\pi R^2}{2} \right). \quad (4)$$

We can establish the fairness of the obtained result by a partial case, which corresponds to the possibility of completely filling the containers of Figure 1 and Figure 2, namely:

$$\text{if } Y = 2R, \text{ then } \frac{H}{\pi R^2} \left( (2R - R) \cdot 0 + R^2 \arcsin 1 + \frac{\pi R^2}{2} \right) = \frac{H}{\pi R^2} \left( \frac{\pi R^2}{2} + \frac{\pi R^2}{2} \right) = H.$$

The next research question is to establish the numerical mechanical characteristics of the most common volumes of homogeneous liquids, limited in space by standard containers with classical geometry. Since the assumption is made about the homogeneity of the filler material, in the following we will talk about the geometric characteristics in the absence of the density function of a specific liquid in the corresponding formulas, i.e.  $\rho = const \equiv 1$ .

As the first such container, consider a volume of liquid limited by a hemisphere of radius  $R$  (Figure 3).



**Figure 3. The volume of a homogeneous liquid bounded by a hemisphere of radius  $R$**

We have:  $x^2 + y^2 + (z - R)^2 = R^2, z = R, 0 \leq z \leq R$ .

In cylindrical coordinates, the same equation will take the form:

$$r^2 + (z - R)^2 = R^2, \text{ або } r = \sqrt{R^2 - (z - R)^2}.$$

The volume of a homogeneous liquid squeezed by the surface of such a hemisphere is defined as

$$\begin{aligned} V &= \iiint dV = \iiint r d\varphi dz dr = \\ &= \int_0^{2\pi} d\varphi \int_0^R dz \int_0^{\sqrt{R^2 - (z - R)^2}} r dr = \pi \int_0^R (R^2 - (z - R)^2) dz = \frac{2}{3} \pi R^3. \end{aligned} \quad (5)$$

The static moment relative to the  $0z$  axis is set as follows:

$$\begin{aligned}
 M_z &= \iiint z dV = \iiint r z d\varphi dz dr = \int_0^{2\pi} d\varphi \int_0^R z dz \int_0^{\sqrt{R^2-(z-R)^2}} r dr = \pi \int_0^R z (R^2 - (z-R)^2) dz = \\
 &= \pi \left( \frac{2}{3} R^4 - \frac{1}{4} R^4 \right) = \frac{5}{12} \pi R^4 .
 \end{aligned} \tag{6}$$

Then the  $z$  coordinate of the center of mass of the volume of such a homogeneous liquid (Figure 3) is determined as

$$\bar{z} = \frac{M_z}{V} = \frac{15}{24} R = \frac{5}{8} R . \text{ At the same time, it is obvious, } \bar{x} = \bar{y} = 0 .$$

The moment of inertia  $J_z$  relative to the coordinate axis  $z$  can be set by the formula:

$$\begin{aligned}
 J_z &= \iiint (x^2 + y^2) dV = \iiint r \cdot r^2 d\varphi dz dr = \int_0^{2\pi} d\varphi \int_0^R dz \int_0^{\sqrt{R^2-(z-R)^2}} r^3 dr = \\
 &= \frac{\pi}{2} \int_0^R ((R^2 - (z-R)^2)^2 dz = \frac{\pi}{2} \int_0^R (2Rz - z^2)^2 dz = \frac{\pi R^5}{2} \left( \frac{4}{3} - 1 + \frac{1}{5} \right) = \\
 &= \frac{4}{15} \pi R^5 - 1 + \frac{1}{5} = \frac{4}{15} \pi R^5 .
 \end{aligned} \tag{7}$$

The moment of inertia  $J_{xy}$  relative to the coordinate plane  $oxy$  is calculated by the formula:

$$\begin{aligned}
 J_{xy} \iiint z^2 dV &= \iiint r \cdot z^2 d\varphi dz dr = \int_0^{2\pi} d\varphi \int_0^R z^2 dz \int_0^{\sqrt{R^2-(z-R)^2}} r dr = \\
 &= \pi \int_0^R z^2 ((R^2 - (z-R)^2) dz = \pi \int_0^R z^2 (2Rz - z^2) dz = \pi R^5 \left( \frac{1}{2} - \frac{1}{5} \right) = \frac{3}{10} \pi R^5 .
 \end{aligned} \tag{8}$$

Containers of a less common scale of their uses are conical or similar containers for storing or transporting homogeneous liquids. Let's set the equation of some conical limited surface of real space, which has the form:  $z^2 = x^2 + y^2, z \geq 0, z = H$ , here  $H$  is the height of a container of this kind. We set the amount of liquid volume clamped by this surface accordingly:

$$V = \iiint dV = \iiint r d\varphi dz dr = \int_0^{2\pi} d\varphi \int_0^H dz \int_0^z r dr = \pi \int_0^H z^2 dz = \frac{1}{3} \pi H^3 . \tag{9}$$

The static moment about the  $oz$  axis can be found as



$$M_z = \iiint z dV = \iiint r z d\varphi dz dr = \int_0^{2\pi} d\varphi \int_0^H z dz \int_0^z r dr = \pi \int_0^H z^3 dz = \frac{1}{4} \pi H^4. \quad (10)$$

Thus, the coordinate of the center of mass along the oz axis of the entire volume of a homogeneous filler – liquid is determined as follows:

$$\bar{z} = \frac{M_z}{V} = \frac{3}{4} H. \quad (11)$$

And again, it's obvious that  $\bar{x} = \bar{y} = 0$ .

Finally, there are many reservoirs bounded by parabolic or near-parabolic spatial surfaces. The appearance of such a surface is shown in Figure 4. The equation of the surface of a circular paraboloid in the form is set:

$$z = k(x^2 + y^2), k > 0, z = H. \quad (12)$$

Here,  $k$  is some parameter that has the appropriate dimension to equalize the overall dimension of the right and left parts of the last formula. The dimension of this parameter is set as (linear unit)<sup>-1</sup>.

In cylindrical coordinates, the lateral surface of a circular paraboloid is defined as follows:

$$r^2 = \frac{z}{k}, \text{ and the given surface is bounded by the area } z = H.$$

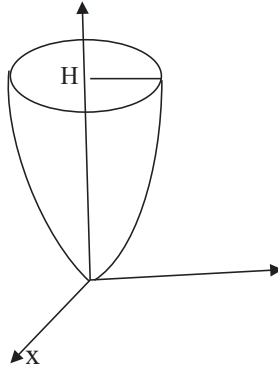
Regarding the issue of establishing the mechanical characteristics of the volume of liquid bounded by the surface of a circular paraboloid, we will first find the volume of liquid squeezed by this surface. Such a volume is established through the addition and calculation of the corresponding triple integral, the further calculation of which is expedient to be carried out using spatial cylindrical coordinates:

$$V = \iiint dV = \iiint r d\varphi dz dr = \int_0^{2\pi} d\varphi \int_0^H dz \int_0^{\sqrt{\frac{z}{k}}} r dr = \pi \int_0^H \frac{z}{k} dz = \frac{\pi H^2}{2k}. \quad (13)$$

The static moment of this volume relative to the z axis is determined in a similar way:

$$M_z = \iiint z dV = \iiint r z d\varphi dz dr = \int_0^{2\pi} d\varphi \int_0^H z dz \int_0^{\sqrt{\frac{z}{k}}} r dr = \pi \int_0^H z \cdot \frac{z}{k} dz = \frac{\pi H^3}{3k}. \quad (14)$$

Thus, the third coordinate of the center of mass point inside the volume bounded by the surface of the circular paraboloid is defined as  $\bar{z} = \frac{M_z}{V} = \frac{2}{3} H$ , at the same time we again have  $\bar{x} = \bar{y} = 0$ .



**Figure 4. Capacity of height H, limited by the surface of a circular paraboloid**

The moment of inertia of this volume relative to the oz axis is set as

$$J_z = \iiint (x^2 + y^2) dV = \iiint r^3 d\varphi dz dr = \int_0^{2\pi} d\varphi \int_0^H z dz \int_0^{\sqrt{\frac{z}{k}}} r^3 dr = \pi \int_0^H \frac{z^2}{2k^2} dz = \frac{\pi H^3}{6k^2}. \quad (15)$$

The moment of inertia of this volume relative to the 0xy coordinate plane is set as follows:

$$J_{xy} = \iiint z^2 dV = \iiint rz^2 d\varphi dz dr = \int_0^{2\pi} d\varphi \int_0^H z^2 dz \int_0^{\sqrt{\frac{z}{k}}} r dr = \frac{\pi}{k} \int_0^H z^3 dz = \frac{\pi H^4}{4k}. \quad (16)$$

## **2.2. Establishing the Force of Pressure on Underwater Hydraulic Structures in Design Practice and Agricultural Engineering Training of Students of Higher Educational Institutions**

The accumulated scientific and practical experience, a review of literary sources and the results of a patent search showed that highly productive agricultural production, a developed social infrastructure is based on a systematic, integrated approach to the formation of the landscape environment, the rational use of water resources using the achievements of scientific and technical progress, including in the field construction of hydroelectric power plants, strengthening of riverbanks, creation of water reservoirs, sedimentation tanks for liquid organic waste from animal

husbandry, processing industry, wastewater, etc. During the construction of underwater hydraulic structures, modern reclamation machines and equipment are used. Without the use of modern machine complexes, it is impossible to build reliable, long-lasting underwater hydraulic structures. Regarding the expansion of the development of small hydropower, the construction of dams as the basic basis of hydropower, it is worth noting such a feature as the choice of location.

The requirements for the system of internal quality assurance of higher education provide for the training of scientific and pedagogical researchers [30], both in our country and abroad. One such form of training was an innovative renewable energy training course in China for participants from developing countries. During this training course, issues of mathematical calculation of dams and locks of various structures were also studied.

We will conduct, using a mathematical apparatus, a theoretical analysis and calculation of some structures of underwater hydraulic structures according to the following algorithm.

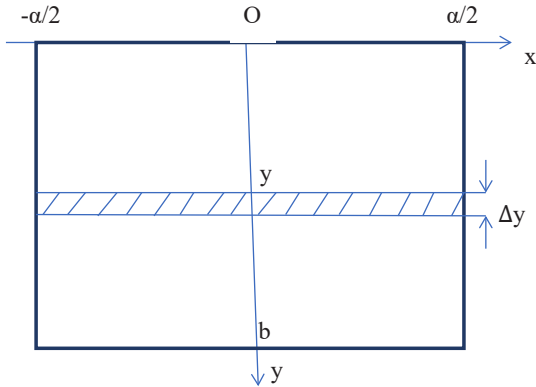
First, let's draw a diagram of the equivalent model of a rectangular sluice with a selected coordinate system and a selected elementary platform, on which the elementary pressure force from the liquid side is calculated (Figure 5). It can also be a rectangular container filled with liquid. Let's calculate the pressure  $P$  of this liquid on one of the walls of the capacity or on the side wall of the lock, which is similar.

Let  $a$  and  $b$  be the geometric dimensions of the given rectangular gateway ( $a$  – is the horizontal component, and  $b$  – is the vertical component). If we determine the pressure on the side wall of the container containing the liquid, then  $a$  is the base of the wall, and  $h$  is the height [31]. It is worth noting that the pressure on the entire wall consists of the sum of the pressures on its individual sections. We select at an arbitrary depth an elementary platform  $\Delta S$  with its height  $\Delta y$ , and set the value of the elementary force of pressure from the side of the liquid on this platform. So we just get:

$$\Delta P \approx \gamma y \Delta S = \gamma a y \Delta y ,$$

where  $\gamma$  – density or specific mass of water or any liquid acting on the wall of the lock,  $n/m^3$ .

We select at an arbitrary depth an elementary platform  $\Delta S$  with its height  $\Delta y$ , and set the value of the elementary force of pressure from the side of the liquid on this platform. So we just get:



**Figure 5. A rectangular sluice with a selected coordinate system and a selected elementary platform on which the elementary pressure force from the liquid side is calculated**

$$\Delta P \approx \gamma y \Delta S = \gamma a y \Delta y ,$$

where  $\gamma$  – density or specific mass of water or any liquid acting on the wall of the lock,  $n/m^3$ .

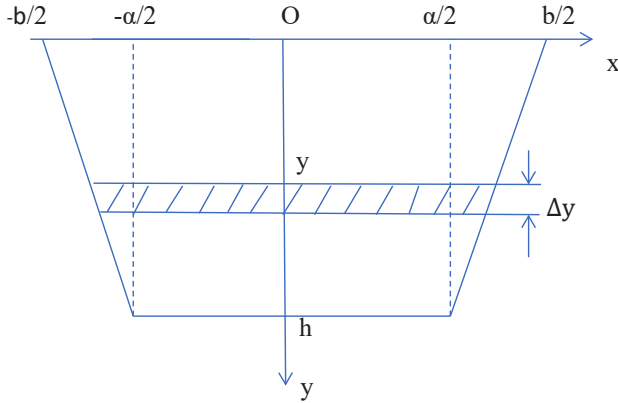
If we direct  $\Delta y \rightarrow 0$  in the limit transition, then we obtain with the help of the integral the total value of the lateral pressure force on the entire vertically located gateway, which was supposed to be determined in the data of theoretical scientific research by an analytical method with the involvement of mathematical apparatus methodologies. And so, the elementary force of pressure on the part of the liquid is equal to:

$$P = \int_0^h \gamma a y dy = \gamma a \frac{y^2}{2} \Big|_0^h = \frac{\gamma a h^2}{2} . \quad (17)$$

It is important to note that this pressure is equal to the pressure of a liquid on a platform of the same size immersed horizontally to the depth of its center of gravity.

Dams of hydroelectric power plants are mostly trapezoidal in shape. Let us determine the elementary force of pressure from the side of the liquid on the wall of the dam, the equivalent scheme of which is presented in Figure 6.

Here  $a$  and  $b$  – bases of an equilateral trapezoid ( $a < b$ ),  $h$  – its height. The solution to this problem is based on Pascal's well-known law [26]:



**Figure 6. A dam in the form of an equilateral trapezoid with a selected coordinate system and a selected elementary platform on which the elementary pressure force from the liquid side is calculated**

$$P = \gamma h S ,$$

where  $\gamma$  – specific gravity of the liquid,  $h$  is the immersion depth of some platform of area  $S$ ,  $P$  is the value of the required pressure force.

Having selected the elementary area  $\Delta S$  of this trapezoid, located at the depth of immersion in the liquid  $y$  with its elementary height  $\Delta y$ . The amount of pressure exerted by the liquid on this elementary platform is equal to:

$$\Delta P \approx \gamma y \Delta S = 2\gamma x y \Delta y . \quad (18)$$

Let  $x = \frac{a}{2} + t$ , then  $\frac{2t}{b-a} = \frac{h-y}{h}$ , or

$$t = \frac{1}{2}(b-a)\left(1 - \frac{y}{h}\right), \quad x = \frac{a}{2} + \frac{b-a}{2}\left(1 - \frac{y}{h}\right) = \frac{1}{2}\left(b - \frac{y}{h}(b-a)\right).$$

And here we get:

$$\Delta P \approx \gamma y \left(b - \frac{y}{h}(b-a)\right) \Delta y , \quad (19)$$

and under the condition that  $\Delta y \rightarrow 0$ , we get the value of the required pressure force using the definite integral of the following form:

$$\begin{aligned}
 P &= \int_0^h \left( by - (b-a) \frac{y^2}{h} \right) dy = \gamma \left( \frac{1}{2} by^2 - \frac{1}{3} (b-a) \frac{y^3}{h} \right) \Big|_0^h = \\
 &= \gamma h^2 \left( \frac{b}{2} - \frac{b}{3} + \frac{a}{3} \right) = \gamma h^2 \left( \frac{a}{3} + \frac{b}{6} \right) = \frac{1}{6} \gamma h^2 (2a+b).
 \end{aligned}
 \tag{20}$$

Corollary: for a rectangular dam ( $a = b, h$  – height)

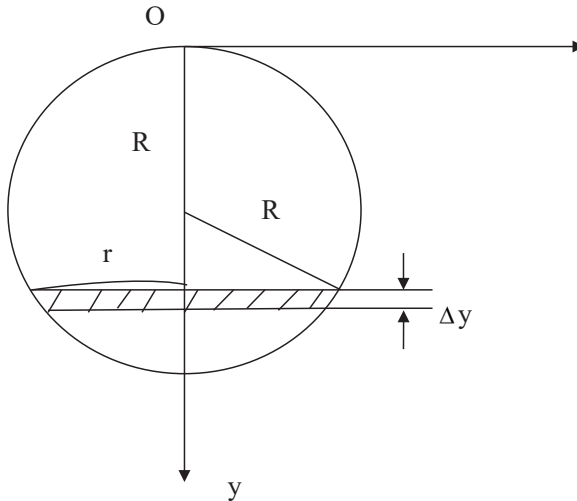
$P = \frac{1}{6} \gamma h^2 (2a+a) = \frac{\gamma a h^2}{2}$ , which coincides with the result of the previous point of setting the pressure force on the airlock.

Let us find the pressure force  $P$  acting on a vertical circular hatch of radius  $R$ . First, consider the case when the arc of the circle touches the surface of the liquid and we direct the horizontal axis  $OX$  along this surface (Figure 7). We have obvious algebraic relations:  $r^2 + (y-R)^2 = R^2 \Rightarrow$

$$r = \sqrt{R^2 - (y-R)^2}, \tag{21}$$

$$\Delta P \approx \gamma y \Delta S \approx 2\gamma r y \Delta y \approx 2\gamma y \sqrt{R^2 - (y-R)^2} \Delta y, y \in [0, 2R]. \tag{22}$$

Having made a traditional boundary transition in the last equality  $\Delta y \rightarrow 0$ , we get:

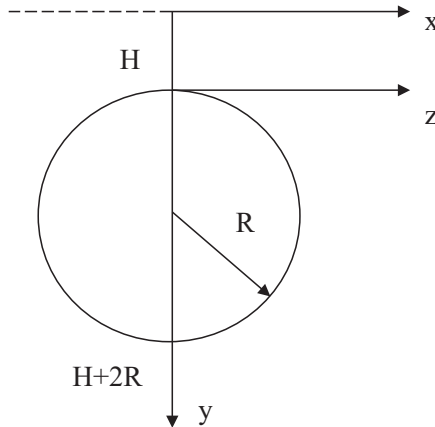


**Figure 7. Vertical circular hatch of radius  $R$**

$$\begin{aligned}
 P &= 2\gamma \int_0^{2R} y \sqrt{R^2 - (y-R)^2} dy = \left\{ \begin{array}{l} t = y - R \\ y = t + R \\ y = 2R \Rightarrow t = R \\ y = 0 \Rightarrow t = -R \end{array} \right\} = 2\gamma \int_{-R}^R (t+R) \sqrt{R^2 - t^2} dt = \\
 &= 2\gamma \int_{-R}^R t \sqrt{R^2 - t^2} dt + 2\gamma R \int_{-R}^R \sqrt{R^2 - t^2} dt = 4\gamma R \int_0^R \sqrt{R^2 - t^2} dt = \\
 &= 4\gamma R \left[ \frac{t\sqrt{R^2 - t^2}}{2} + \frac{R^2}{2} \arcsin \frac{t}{R} \right] \Bigg|_0^R = 2\gamma R^3 (\arcsin 1 - \arcsin 0) = \\
 &= 2\rho\gamma R^3 \left( \frac{\pi}{2} \right) = \pi\gamma R^3. \tag{23}
 \end{aligned}$$

Remarks:  $2\gamma \int_{-R}^R t \sqrt{R^2 - t^2} dt = 0$ , as an integral of odd  $t$  as a function of the interval symmetrically relative to 0.

In Figure 8 shows a generalized model for finding the magnitude of the pressure force on a circular hatch immersed in a liquid to an arbitrary depth  $y \in [H, H + 2R]$ .



**Figure 8. A circular hatch of radius  $R$ , located at an arbitrary depth  $H$  of the liquid**

In this case, the pressure force from the liquid side is determined as follows:

$$\begin{aligned}
 P &= 2\gamma \int_H^{H+2R} y \sqrt{R^2 - (y - R - H)^2} dy = \\
 &= \left\{ \begin{array}{l} z = y - H \\ y = z + H \\ y = H + 2R \Rightarrow z = 2R \\ y = H \Rightarrow z = 0 \end{array} \right\} = 2\gamma \int_0^{2R} (z + H) \sqrt{R^2 - (z - R)^2} dz = \\
 &= \left\{ \begin{array}{l} t = z - R \\ z = t + R \\ z = 2R \Rightarrow t = R \\ z = 0 \Rightarrow t = -R \end{array} \right\} = 2\gamma \int_{-R}^R (t + H + R) \sqrt{R^2 - t^2} dt = \\
 &4\gamma \int_0^R (t + H + R) \sqrt{R^2 - t^2} dt = 4\gamma(H + R) \left[ \frac{t\sqrt{R^2 - t^2}}{2} + \frac{R^2}{2} \arcsin \frac{t}{R} \right] \Bigg|_0^R = \\
 &= \pi\gamma R^2 (H + R). \tag{24}
 \end{aligned}$$

It should be noted, if a circular hatch is immersed in a liquid horizontally to a depth of  $H$ , then the force of pressure from the liquid side on it is determined directly by Pascal's law without additions and calculations of the corresponding integrals, namely:

$$P = \pi\gamma R^2 H. \tag{25}$$

In the case of a moving incompressible fluid, we can tentatively talk about the validity of Pascal's law, because adding an arbitrary constant value to the pressure does not change the form of the fluid motion equation, it is the Euler equation or, if the effect of viscosity is taken into account, the Navier-Stokes equation. In this case, the term Pascal's law does not usually apply. As you know, it is not recommended to apply Pascal's law for compressible liquids.

It is believed that the first ship locks appeared in China in 984, thanks to Qiao Weiyue, who was the assistant minister of transport in Huainan. The gateway was a straight section 76 m long, bounded on two sides by wooden sliding gates. The construction of the lock was carried out on the



Great Chinese Canal, and during the 11th century, similar structures were built along the entire length of this waterway. The pressure on the structures reached 1.5 m. The construction of the canal itself was carried out from 70 to the 13th century. At the present time, one of the world's largest ship lock and lift-type ship lift is being operated at the Three Gorges Hydroelectric Power Plant in China. Later, locks began to be built in Holland and Italy.

A shipping lock is a hydraulic structure on waterways to ensure the passage of ships from one water basin (head water, tailwater) to another with different water levels in them. It is bounded on two sides by gates, between which an adjacent chamber is placed, which makes it possible to change the water level within it. The transfer of crafts using the shipping lock is carried out by successive transfer to the adjacent chamber after leveling the water level in them. The use of locks is mainly aimed at making water spaces with different water levels in them more suitable for navigation. Locks are key hydraulic structures on waterways that allow vessels with different water levels to pass between water basins or river systems. The primary function of a navigation lock is to allow vessels to overcome an obstacle, which may be a difference in water levels between two bodies of water. The sluice consists of a chamber that can be filled or emptied with water, as well as gates on two sides. The ship enters the lock chamber, the gates are closed, and then the water level in the lock chamber is controlled to change to equalize with the water level in the corresponding water basin. After that, the gates on the other side of the lock are opened, and the ship can enter the corresponding water basin. This mechanism allows ships to be transported through water obstacles without the need to adjust the ship itself to the water level in the waterway. Locks allow water to be stored in appropriate reservoirs and water levels to be regulated, ensuring smooth and continuous movement of ships. These structures are an important element in the transport system, allowing efficient use of waterways for the transportation of goods and providing communication between different waterways. The process of passing a ship through a lock is called locking. Locking is a set of technological operations for the vertical movement of ships from the level of the upstream to the level of the downstream.

Each gateway has three main elements.

Gates (Doors): lock gates are large steel structures that open and close to allow vessels to enter or exit the lock.

Sluice chambers: a sluice consists of spillway chambers that can be filled or emptied of water. When the vessel has entered the chamber, water is released or poured in to raise or lower the vessel to the water level at the other maritime stretch.

Control mechanisms: locks are equipped with control mechanisms such as water filling and discharge systems, gate control and security for efficient and safe movement of vessels.

This process is important to ensure the navigability of waterways for navigation and the movement of vessels between different water levels.

Hermetic chamber connecting the upper and lower main parts of the channel and having a volume sufficient to include one or more vessels. The position of the camera is fixed, but the water level in it can change. Hermetic chamber connecting the upper and lower main parts of the channel and having a volume sufficient to include one or more vessels. The position of the camera is fixed, but the water level in it can change.

Gates are metal shields located at both ends of the chamber, which serve to let the vessel in and out of the chamber before locking and seal the chamber during locking.

A plumbing device is a device designed to fill or empty a chamber. As a rule, a flat shield shutter is used for it. Transfer pumps can be used in large locks. The principle of operation of the gateway is as follows: the entrance gate opens and the vessel enters the chamber. The entrance gate is closing and the bypass valve opens, causing the water level in the chamber with the vessel to drop. The inlet gate opens, the ship leaves the chamber. Sluicing lasts, as a rule, from 10 to 20 minutes, depending on the size of the chamber and the drop in the water level. If the vessel moves upstream, the process is reversed: the vessel enters the empty chamber, then the valve opens, filling the chamber with water and lifting the vessel.

When sluicing, as a rule, they try to alternate the direction of passage of vessels: after a vessel going, for example, downstream, has completed sluicing, a vessel moving in the opposite direction can immediately enter a chamber with an already lowered water level in it. Thus, there is no need to overflow or drain the water from the chamber. The use of locks is necessary in cases where an artificial bypass channel is created to bypass such obstacles as rapids, dams, dams, etc., for the passage of ships. The dam increases the depth of the water space in front of it, in this case the

lock can be installed either directly in the dam, being its component part, or in the mouth of the stretch. A river equipped with such structures is called a waterway. Also, by installing a lock at the mouth of the river, it is possible to make the river no longer sensitive to the influence of tides in the large reservoirs into which it flows. Additional sluices are used to improve the waterway on the river: a spillway sluice is installed in the upper part of the bypass channel to prevent floods. The longer the bypass canal, the greater the difference in water level in the river between the beginning and the end of the canal, so a very long bypass canal may require the installation of additional groups of locks at different places in the canal. Sluice gates are also used to raise and lower water in reservoirs. During the construction of the first artificial canals, which were built, as a rule, on fairly flat, non-mountainous areas, their engineers and builders preferred to build bypasses in case of hills or lowlands in the path of the canal. However, the extension of the canal entailed an excessive increase in the cost of its construction and increased the duration of passage of the vessel through it. To solve this problem, gateways began to be used. Further, as technical knowledge and capabilities improved, new solutions were used to overcome obstacles: aqueducts, tunnels, dams. And in each of these structures, locks continued to be used, which have become an integral part of almost any hydraulic structure from those times to this day. To date, many types of locks are known, including swinging, vibrating, magnetic, hydraulic, expanding, narrowing, angular, with continuous discharge of concentrate, with moving stencils, moving mats, multi-deck, overturning, etc.

The largest lock at the moment is the structure built at the Eymoiden hydroelectric junction in the Netherlands. The dimensions of the ego chamber are (LxWxH) 500x70x18 m. For comparison, the width of the chambers of the locks of the Panama Canal is 50 m. Information about Kakhovsky gateway. This sluice is part of the complex of structures of the pressure front of the Kakhovsky hydraulic junction, and is the lower stage of the Dnieper Cascade. The gateway was built in the period from 1951 to 1955, put into temporary operation on June 30, 1955, and into permanent operation on April 15, 1958.

The Kakhovsky single-chamber shipping lock is used for ships to overcome the concentrated level difference in the creation of the dam of

the Kakhovsky HPP. The average daily number of locks is 7, the maximum number of locks per day is 12 vessels, the planned number is 32.

The navigable facilities are located in the left bank floodplain of the Dnipro River. The pressure on the structure is 16 m.

The shipping facilities of the Kakhovsky Lock include:

- single-chamber single-threaded gateway;
- upper and lower approaches.

The sluice consists of upper and lower heads and a sluice chamber. The useful dimensions of the camera are 260 by 18 m. Made of reinforced concrete. The walls are lined with reinforced concrete shell plates.

The power supply system of the airlock is distributed with the help of longitudinal galleries located in the bottom of the chamber. Filling is carried out through outlets in the chamber ceiling. The emptying of the chamber takes place with the help of bottom water pipes located in the foundations of the lower head of the lock. The upper head is equipped with a flat working and emergency repair gate, the lower one is equipped with a double-leaf working and repair gate.

The kinetic energy of water is converted into electricity with the help of hydroturbines. Under the influence of water pressure, the blades of the hydro turbine rotate, which activates the generator, which produces electricity. The necessary water pressure is created by building a dam. In the building of the hydroelectric power plant, hydro units are located, which directly convert the energy of the water current into electrical energy. The process of converting the kinetic energy of water movement into electricity through hydro turbines and generators is the basic principle of operation of hydroelectric power plants. Dams are used to generate water pressure, which is then used to drive turbines that turn generators and generate electricity.

Hydro aggregates located in the building of the hydroelectric power plant are responsible for the direct conversion of the energy of water movement into electrical energy. This process occurs thanks to the operation of turbines, which are formed as a result of the action of water pressure, and generators, which convert mechanical energy into electrical energy. Such a system is an effective way of producing electricity due to the use of natural resources – water and its movement. Depending on the capacity, according to the Law of Ukraine "On Electricity", hydroelectric power stations are divided into: powerful – producing from 25 MW to 250 MW and above,

medium – up to 25 MW, small hydroelectric power stations – 1... 10 MW, mini hydroelectric power stations – 200 KW... 1 MW, micro-hydroelectric power stations – the capacity of which does not exceed 200 KW. The power of the hydroelectric power station directly depends on the water pressure, as well as on the efficiency of the generator used. Hydroelectric power plants are also divided depending on the maximum use of water head: high-pressure – more than 60 m, medium-pressure – from 25 m, low-pressure – 3... 25 m. The principle of operation of all types of turbines is similar – pressurized water enters the turbine blades through mainly circular hatches that rotate to generate electricity.

On the same river with many of its tributaries, hydroelectric power plants of various capacities can be built. As the distance from the upper reaches of the river to the mouth increases, the power of the water flow increases – accordingly, the water potential for electricity generation also increases. Water can also store potential energy reserves in reservoirs that are at a certain height. If necessary, this water can be used by special technical systems to obtain electrical or mechanical energy for driving machines, installations, etc. Water-powered wheels have a deep history of their use. Over time, other constructive solutions for transforming the force of the water flow into rotational motion appeared. The scientific and industrial experience of using technical objects using the potential and kinetic energy of water has shown that turbines are more efficient. Water wheels can still be used at small hydroelectric power stations, but turbine heads are more modern, for which special hatches are created in dams. Their use is especially effective in the presence of a powerful flow of water. The construction of structures can have different scales, including, it can be both small and large hydroelectric plants. A significant number of underwater hydraulic structures have been built in Ukraine, and therefore there is a sufficiently large scientific and practical experience of such activities. However, young people come to universities every year, new students are enrolled, who need to acquire knowledge on the calculation of the strength of structures of objects with the involvement of innovative methods of computational and mathematical design training, which is based on the fundamental methodological apparatus. The formation of special professional competences of the future agricultural engineer [30] should ensure his ability to use the theoretical foundations and basic methods of

physics and mathematics, hydrostatics, dynamics, solid and fluid mechanics, strength of materials, etc. for calculation, construction, design of technical objects, and, of course, underwater hydraulic structures. An agricultural engineer must possess knowledge and skillfully apply the basic concepts and methods of mathematical analysis [13], linear algebra and analytical geometry, discrete mathematics, theory of differential equations, theory of probability and mathematical statistics, methods of processing experimental data, etc. in production project activities. [30]. An agricultural engineer must be able to perform calculations on the strength of the capacities of any geometric profiles for storing liquids, as well as the side walls and bottoms of hydraulic structures, parts, nodes, mechanisms, bodies that are placed in liquids [30], and at the same time use a mathematical apparatus for processing and analysis data related to the reliability of technical products, to determine the parameters of the operating modes of hydraulic systems.

### 2.3. Establishing Volumetric Characteristics of Standard Three-Dimensional Geometric Figures by the Method of Using Curvilinear Coordinates

As is known, one of the main applications of the triple integral in higher mathematics is the possibility of finding the numerical value of the volume of some closed region of space  $R^3$  [32–35]. The domain of integration is limited to one or more surfaces in three-dimensional space, that is

$$V = \iiint 1 dx dy dz , \quad (26)$$

here the integral is calculated over the set D – the closed region in the space  $R^3$ , the volume of which is located. If this domain D is bounded by surfaces of the first order, then it is rational to calculate this triple integral in Cartesian coordinates. But if the area D is bounded by one or more surfaces of higher orders, first of all, surfaces of the second order, then it is expedient to switch to the use of curvilinear, namely, cylindrical or spherical coordinates in the calculations. Particularly, in cases where all three variables are equal in surface equations, then it is more effective to use spherical coordinates:

$$\begin{cases} x = r \cos \varphi \cos \theta \\ y = r \cos \varphi \sin \theta \\ z = r \sin \theta \end{cases} \quad (27)$$

A vivid example of such surfaces is a sphere or an ellipsoid. If one of the variables is isolated in the equation of the surface bounding the area  $D$ , then it is advisable to use cylindrical coordinates:

$$\begin{cases} x = r\cos\varphi \\ y = r\sin\varphi. \\ z = z \end{cases} \quad (28)$$

An example of such surfaces is a cylinder, cone, etc. In spherical coordinates, the volume of the area  $D$  is calculated by the formula

$$V = \iiint r^2 \cos\varphi dr d\varphi d\vartheta, \quad (29)$$

where the value  $r^2 \cos\varphi$  is the Jacobian determinant of the transition from Cartesian coordinates  $(x, y, z)$  to spherical  $(r, \varphi, \vartheta)$  [36; 37]. This determinant of the transition from Cartesian coordinates  $(x, y, z)$  to cylindrical  $(r, \varphi, z)$  or spherical  $(r, \varphi, \vartheta)$ , respectively, is determined as follows:

$$\begin{vmatrix} \frac{\partial x}{\partial r} & \frac{\partial x}{\partial \varphi} & \frac{\partial x}{\partial z} \\ \frac{\partial y}{\partial r} & \frac{\partial y}{\partial \varphi} & \frac{\partial y}{\partial z} \\ \frac{\partial z}{\partial r} & \frac{\partial z}{\partial \varphi} & \frac{\partial z}{\partial z} \end{vmatrix} = \begin{vmatrix} \cos\varphi & -r\sin\varphi & 0 \\ \sin\varphi & r\cos\varphi & 0 \\ 0 & 0 & 1 \end{vmatrix} = r(\cos^2\varphi + \sin^2\varphi) = r,$$

$$\begin{vmatrix} \frac{\partial x}{\partial r} & \frac{\partial x}{\partial \varphi} & \frac{\partial x}{\partial \vartheta} \\ \frac{\partial y}{\partial r} & \frac{\partial y}{\partial \varphi} & \frac{\partial y}{\partial \vartheta} \\ \frac{\partial z}{\partial r} & \frac{\partial z}{\partial \varphi} & \frac{\partial z}{\partial \vartheta} \end{vmatrix} = \begin{vmatrix} \cos\varphi\cos\vartheta & -r\sin\varphi\cos\vartheta & -r\cos\varphi\sin\vartheta \\ \cos\varphi\sin\vartheta & -r\sin\varphi\sin\vartheta & r\cos\varphi\cos\vartheta \\ \sin\varphi & \cos\varphi & 0 \end{vmatrix} =$$

$$= r^2 \begin{vmatrix} \cos\varphi\cos\vartheta & -\sin\varphi\cos\vartheta & -\cos\varphi\sin\vartheta \\ \cos\varphi\sin\vartheta & -\sin\varphi\sin\vartheta & \cos\varphi\cos\vartheta \\ \sin\varphi & \cos\varphi & 0 \end{vmatrix} = -r^2 \cos\varphi.$$

The modulus of the last value is equal to  $r^2 \cos\varphi$ . In cylindrical coordinates, the volume of the region  $D$  is defined as

$$V = \iiint r dr d\varphi dz, \quad (30)$$

where  $r$  – the value of the Jacobian determinant of the transition from Cartesian coordinates  $(x, y, z)$  to cylindrical ones  $(r, \varphi, z)$ .

We compare the alternative use of curvilinear coordinates when finding the volumes of the mentioned surfaces of the second order. For example, to find the volume of a cone, we use spherical coordinates, and for the volume of a sphere, cylindrical coordinates. Let a cone be given (Figure 1), for which  $R$  is the radius of the base of the circle,  $H$  is its height. The equation of such a surface is obviously defined as  $z = H$ . Then the corner  $\alpha = \arctg \frac{H}{R}$ , and spherical coordinate  $\varphi \in \left[ \arctg \frac{H}{R}; \frac{\pi}{2} \right]$ . From the equation  $z = H$  we get  $H = r \sin\varphi$ , wherefrom  $r = \frac{H}{\sin\varphi}$ , while  $\vartheta \in [0; 2\pi]$ .

So,

$$V = \int_0^{2\pi} d\vartheta \int_{\arctg \frac{H}{R}}^{\frac{\pi}{2}} \cos\varphi \int_0^{\frac{H}{\sin\varphi}} r^2 dr = \vartheta \left| 2\pi \cdot \int_{\arctg \frac{H}{R}}^{\frac{\pi}{2}} \frac{1}{3} \cos\varphi \cdot r^3 \right|_{\frac{H}{\sin\varphi}} = \frac{1}{3} \pi R^2 H.$$

In cylindrical coordinates, this answer is as follows:

$$V = \iiint r dr d\varphi dz = \int_0^{2\pi} d\varphi \int_0^{\frac{zR}{H}} \int_0^{\frac{zR}{H}} r dr = \frac{1}{3} \pi R^2 H. \quad (31)$$

(here, when determining the boundaries by the variable  $r$ , it is taken into account that in cylindrical coordinates the equation of the cone has the form

$$r^2 = \frac{z^2 R^2}{H^2}, \text{ wherefrom } r = \frac{zR}{H} \text{ (} r \geq 0 \text{)}.$$

Now let us set the volume of the sphere in cylindrical coordinates, the equation of which has the following form:  $r^2 + z^2 = R^2$  ( $r \geq 0$ ).

Then we just get:

$$V = \int_{-R}^R dz \int_0^{\sqrt{R^2 - z^2}} r dr \int_0^{2\pi} d\varphi = 4\pi \int_0^R \frac{1}{2} r^2 \left| \sqrt{R^2 - z^2} \right| dz = \frac{4}{3} \pi R^3. \quad (32)$$

This result is achieved much easier in spherical coordinates:



$$V = \int_0^{2\pi} d\vartheta \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} d\varphi \int_0^R r^2 \cos\varphi d\varphi = \vartheta \left| \begin{matrix} 2\pi \\ 0 \end{matrix} \cdot \sin\varphi \right|_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cdot \frac{1}{3} r^3 \Big|_0^R = \frac{4}{3} \pi R^3. \quad (33)$$

Let us dwell on the approaches to calculating the volume of a cylinder in different coordinates, when the use of cylindrical coordinates is as logical as the use of Cartesian coordinates when finding the volume of a straight parallelepiped. In cylindrical coordinates we get:

$$x^2 + y^2 = R^2, z = H, (z \geq 0).$$

$$V_u = \iiint r dr d\varphi dz = \int_0^{2\pi} d\varphi \int_0^R r dr \int_0^H dz = \pi R^2 H. \quad (34)$$

The advantage of using cylindrical coordinates is that all the limits of the three repeated integrals are constants, which certainly simplifies the calculation. We compare finding this answer in spherical coordinates and get  $V_u = V_1 + V_2$ . The value of the volume  $V_2$  has been already in spherical coordinates when calculating the volume of the cone (in this case, it is inscribed in a cylinder). To find the volume  $V_1$ , given that  $\alpha = \arctg \frac{H}{R}$ , we get:

$$x^2 + y^2 = R^2 \Rightarrow r^2 \cos^2\varphi = R^2 \Rightarrow r = \frac{R}{\cos\varphi}, (r \geq 0). \quad (35)$$

$$V_1 = \int_0^{\arctg \frac{H}{R}} d\varphi \int_0^{\frac{R}{\cos\varphi}} r^2 \cos\varphi dr \int_0^{2\pi} d\vartheta = \frac{2}{3} \pi R^2 H. \quad (36)$$

Since  $V_2 = \frac{1}{3} \pi R^2 H$ , then  $V_u = V_1 + V_2 = \pi R^2 H$ .

And, finally, this work considers the concept and application of generalized

$$\text{cylindrical} \begin{cases} x = a\rho\cos\varphi \\ y = b\rho\sin\varphi \\ z = z \end{cases} \text{ and generalized spherical} \begin{cases} x = a\rho\cos\varphi\cos\theta \\ y = b\rho\cos\varphi\sin\theta \\ z = c\rho\sin\theta \end{cases}$$

coordinates, where  $0 \leq \rho \leq 1$ , at the same time, the value of the Jacobian determinant of such transformations is, respectively, for cylindrical coordinates  $ab\rho$ , and for spherical coordinates –  $abc\rho^2 \cos\varphi$ .

For example, the volume of an elliptical cylinder  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, z=0, z=H$  in generalized cylindrical coordinates is found as

$$V = \iiint ab\rho d\rho d\varphi dz = ab \int_0^{2\pi} d\varphi \int_0^1 \rho d\rho \int_0^H dz = \pi ab. \quad (37)$$

Consequence 1. If  $a = b = R$ , a formula for calculating the volume of a circular cylinder is got [7; 8].

The volume of the ellipsoid  $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$  is found using generalized spherical coordinates:

$$V = \iiint abc\rho^2 \cos\varphi d\rho d\varphi d\vartheta = abc \int_0^{\frac{R}{\cos\varphi}} \rho^2 d\rho \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos\varphi d\varphi \int_0^{2\pi} d\vartheta = \frac{4}{3} \pi abc. \quad (38)$$

Consequence 2. If  $a = b = c = R$ , we get a formula for calculating the volume of a sphere [36; 37].

And the result of it is the advantages in choosing curvilinear coordinates include the calculation of an arbitrary triple integral of a function of three variables  $\iiint_D f(x, y, z) dx dy dz$ , if the volume  $V$  is bounded by surfaces of the second order. A comparison of methods for calculating triple integrals using the example of finding the volumes of known geometric shapes proves the significant effectiveness and feasibility of using spherical coordinates in some cases (limitation of the area  $D$  by the surface of a sphere, ellipsoid), in other cases – cylindrical coordinates, when separation of one variable is observed (cone, cylinder, etc.). In both cases, the calculation of triple integrals in Cartesian coordinates is far from always rational.

### Conclusions

In the above studies, the form of analytical correspondence of the heights of arbitrary filling of a cylindrical container has been established, when such a container in one case is located vertically, and in the other case – horizontally. Another result of the research is the establishment of basic mechanical characteristics, such as static moments, coordinates of the center of gravity, moments of inertia relative to the given coordinate axes and planes of the volumetric capacity in the form of a hemisphere, as well as conical and parabolic containers.

It is obvious that in practice there are quite a lot of containers made in the form of a straight parallelepiped, but in this paper we can only mention such a geometry of the execution of the corresponding tanks,

since from the point of view of establishing such a basic mechanical characteristic as the point of the center of gravity of the liquid filled in such a container is set trivially as the point of intersection of the large diagonals of this parallelepiped. Also, in the conclusions, it is worth noting the potentially possible variation of the corresponding height coordinate of the center of gravity of each of the above-mentioned containers. That is, when each container is filled with liquid, this point logically rises up, and when the liquid leaks out, the volume of useful filling decreases and this point drops vertically. This fact must be taken into account when guaranteeing the necessary safe operation of each such container, to prevent possible cases of overturning of such vessels. For example, in Ukraine and many other countries, in the last 10-15 years, portable pools with multi-ton filling volumes, made of various light materials, the fastening of which requires a virtually ideal horizontal installation of the bottom of such a pool, and over time under the influence of such a large weight, have become popular construction, with slight subsidence of the site of such a pool, all this leads to its overturning with all the negative consequences.

In the result of conducting scientific and practical researches, a methodology for finding the value of the pressure force on underwater hydraulic structures in the design practice and agricultural engineering training of specialists has been developed. On the example of China, the peculiarities of the design and construction of dams of hydroelectric power stations of various power generation capacities have been analyzed.

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