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PECULIARITIES OF THE NET PRESENT VALUE AND PROFIT INDEX CALCULATIONS USING CONTINIOUS FUNCTIONS AND DIFFERENTIAL EQUATIONS MODELS

Summary

Estimation of the investment business projects efficiency is a key stage in business planning. Considering this, there is a problem of ensuring the forecast accuracy of business project implementation from the start to a certain time when profitability reaches the expected value. Given the stochastic nature of the discount index components in the modern economic environment, one of the approaches for improving their forecast accuracy at a certain time interval is to use continuous functions. In turn, mathematical models that use such functions are developed on the basis of differential equations with stochastic variables. Taking it into account, the paper is devoted to the study of the peculiarities of calculating the coefficients of net present value and the profitability index using differential equations and continuous functions. A comparison with the calculations based on traditional methods is made, and advantages and disadvantages are determined. Models have been developed to refine the calculation of the net present value and profitability index coefficients using continuous functions.

Introduction

Forecasting the indicators of investment business projects implementation efficiency is a key task in making management decisions. Given the high level of stochasticity of the modern external economic environment (EEE) [1; 2], this task is important at every stage of business, from the startup phase to the end of the project [3]. This is due to the significant impact of the EEE on a business entity that carries out financial and economic operations within a particular EEE. In turn, the peculiarities of the stochasticity of certain macroeconomic and microeconomic EEE factors depend on many factors, among of which the most important ones can be identified:

1) regional factors are the factors of stochasticity related to the geographical location of the business entity (island, mainland, border zone, offshore tax-free zone, “porto franco” free trade zone, etc.);

2) *geopolitical factors* are the stochastic factors related to the business entity's affiliation with a particular geopolitical space (country, group of countries or alliance), transnational corporation, etc.);

3) *financial and economic* are the stochastic factors related to the level of uncertainty in the financial and economic sector of a particular region (frequency of crises, economic stagnation, currency risks, peculiarities of the banking system, etc.);

4) *political local factors* are the stochastic factors related to the local legislative and administrative apparatus in financial and economic spheres in a particular region (probability of changes in the legislative framework, tax policy, level of state protection of business from criminal structures, dependence of changes in the country's political course on the change of government, etc.);

5) *business sectors (industry-specific)* are the stochastic factors related to the risk factors of a particular business sector (for example, risks in the delivery of components and raw materials for a manufacturing business, pandemics and lockdowns for the tourism sector).

The listed above factors are also components of the total risk factor, which can be used to measure the level of EEE stochasticity. In business planning, when calculating the indicators of efficiency of the investment project implementation, the discount rate r is such an aggregate risk factor [4–5]. In the case of classical approaches, r is determined empirically using various expert calculation methods [6]. In turn, work [6] shows that the use of a particular methodology can lead to a discrepancy in determining the discount rate of up to 15%-16%. It also shows the problematic aspects associated with the lack of a methodology for choosing the most appropriate calculation method for r that would ensure maximum accuracy in forecasting economic indicators such as present value *PV*, net present value *NPV*, profitability index *PI*, etc.

Given the above shortcomings and the peculiarities of determining the degree of stochasticity of the modern market environment, it is proposed in [7] to calculate r using continuous functions and Fourier series. In turn, the calculation of *NPV* and *PI* was carried out using differential equations [7]. This approach made it possible to increase the accuracy of forecasting the main indicators of the investment business projects implementation effectiveness by defining the discounting process as a function that continuously changes over time under the influence of the EEE factors. Considering this, the task is to refine and verify the developed economic and mathematical apparatus to increase the level of numerical analysis adequacy with real economic and business processes.

Chapter 1. Current state of the problem of business efficiency forecasting

The problem of improving the accuracy of forecasting economic indicators in making managerial decisions is one of the most pressing in the modern economy. The study of this problem attracts the interest of both domestic scientists [3; 7–10] and scientists from other countries [11–15]. Thus, in [3], the author analyzes the experience of theoretical and methodological substantiation of the accepting investment projects feasibility, as well as the rational allocation of resources, taking into account their characteristics, which allows achieving the set goals. According to [3], the justification of an investment project should be carried out taking into account seven aspects of analysis, including: technical analysis (feasibility of technical implementation), analysis of the project's ecological impact during its implementation, commercial analysis (analysis of demand for a product or service), financial analysis (financial viability of the project and return on investment), analysis of the project's economic efficiency, analysis of social interests in the case of project implementation and institutional analysis (analysis of the degree to which the external environment facilitates project implementation). It has been also provided criteria for determining the type of project by time duration, type of investment, amount of investment, number of jobs, type of financing, scale, etc. The use of these criteria and analysis methods can significantly reduce the time for the business planning stage and structure all project information well, which is a kind of "road map" for project management. In addition, the authors of [3] also substantiate the expediency of analyzing the impact of the EEE factors and market factors on the expected result when implementing an investment business project. However, for financial and economic analysis of business project implementation, it has been presented classical approaches for calculating *NPV* and *PI* indicators [3]. In turn, these calculation methods are problematic when modeling the risk factors of the EEE impact (discount rate) using continuous functions [4–5; 7].

The efficiency of the neural network approach in the STATISTICA package for forecasting the financial performance of the enterprise "Ukrtransnafta" is shown in the paper [8]. Based on the results of the regression neural network analysis, the key factors influencing the economic efficiency of this enterprise were identified and ranked, and the optimal neural network model was developed to predict the resulting indicator (efficiency of the enterprise business model). According to [8], this can allow the company's management to rationally manage and increase the economic efficiency at relatively low costs. However, the application of such a methodology requires initial statistical data on financial and economic indicators for previous periods of the enterprise's activity. In view of this, the use of neural network forecasting is problematic for startups, especially in specific innovative industries, where there is no even estimated general information on the financial and economic activities of analogous companies in the market.

In turn, for such specific startups, in the absence of a priori information, probabilistic modeling methods can be applied. Thus, one of the methods based on the synthesis of "probability trees" is proposed in [9]. This approach is convenient and allows to estimate the impact of EEE factors for different scenarios using the apparatus of probability theory by introducing indicators of the expected *NPV* value, its variance, standard deviation and coefficient of variation. However, the issue of determining the exact probability distribution for different scenarios of the EEE behavior remains problematic. Given this, domestic business is constantly faced with the task of refining and adapting mathematical models for predicting the effectiveness of investment projects or the activities of existing enterprises to make the right management decisions, as also noted in [10].

Also, the problems of analyzing the effectiveness of business projects arise not only in the Ukrainian economic space. The article [11] analyzes the main elements of business analysis and defines the metrics of efficiency criteria. The author of the article pays special attention to the interpretation and justification of the introduced indicators such as mixed profit, profit requirements and required risk premium rate, and their direct application to assess the business efficiency. Based on the analysis, the author notes that there is no comprehensive literature on the category of business efficiency. This is due to the lack of development of basic principles and methods for its analysis [11]. The author of the article also concluded that there is no coherent developed concept on this issue, but only separate methods that have certain limitations in their application. Taking it into account, the author of the paper [11] substantiates the expediency of further research in the field of methodology development for forecasting and estimating the business efficiency, as well as their practical verification.

The urgency of improving methods for determining business efficiency is also noted in the works of international organizations such as the European Central Bank [12]. The paper investigates the relationship between profit efficiency, access to finance and innovation activity of companies. Using the methods of stochastic frontier analysis to estimate the financial and economic indicators of a large sample of European firms, the impact of technical innovations on productivity gains of micro and small enterprises is shown. A similar methodology was previously used to assess the profit efficiency of small and medium-sized enterprises in Spain [13]. Using stochastic profit frontier function and the inefficiency effects function it has been estimated the profit efficiency and its determinants for 599 small and medium-sized enterprises in Spain during the period of 2008–2015. These estimations were done using a single-stage procedure following Battese and Coelli (1995). The obtained results have shown the dependences of profit efficiency on the labor productivity, size of enterprise, export activity and innovation, public aid and the years of activity. In turn, according to authors conclusion [13] the increase in the first four variables affect the small and medium enterprises profit efficiency positively, while an increase in the fifth variable (years of activity) impacts negatively. According to [13], the results and conclusions of the research have important implications for both policy makers and the managers of small and medium enterprises. However, despite the appeal of using stochastic frontier analysis modeling, there is also no one-size-fits-all methodology for its application in different cases. Taking it into account, this area of econometric analysis is also in the process of continuous improvement and refinement.

With the development of machine learning algorithms, these artificial intelligence tools have become popular in economic applications. So, in the research [14] it has been proposed to predict *NPV* from well placement binary data using machine learning algorithms. The evidence of the possibility to efficiently infer the *NPV* in oil production using only well placement binary data through data-driven machine learning methods was presented in the paper. The obtained results showed that multi-layer perceptron produced a robust model, able to estimate the *NPV* in a wide range with a small constant error [14]. These results showed the feasibility of this approach, as well as directions for further research and improvement of the prediction model.

The next type of modeling economic processes is based on the differential equations usage [15–17]. The economical processes in this frame are considered as dynamic systems with time-varying parameters. Given the in-depth study and application of this mathematical apparatus for modeling various dynamic processes in other sciences, it has become interesting as a tool for modeling economic processes. So, the approach of market efficiency estimation through a forecasting model based on differential equations has been proposed in the paper [15]. The differential equation based mathematical model for forecasting shares of the stock exchange in Brazil was created. Using this model, it has been possible to forecast stock rates exchange at separate short intervals with good accuracy. In turn, the usage of differential equations with stochastic components allowed to forecast indicators of the eBay auction [16]. It has been shown the advantages of stochastic modeling of auction bidder behavior and price fluctuations and also their integration into the stochastic components of differential equations for predicting prices at the end of the auction.

The difficulty in this case is connected with the selection of the most appropriate stochastic model that closely describes the stochastic process. In turn, in [17], the use of stochastic correlation calculations in time series forecasting allowed us to somewhat simplify the determination of the stochastic component of the differential equations and improve the accuracy of the forecast. In turn, given the convenience and attractiveness of using differential equations for modeling economic processes, the main problematic task is the correct definition of parameters included in these equations, the mathematical description of their relationships, as well as the right-hand sides of the equation (in the case of ordinary differential equations).

Considering this, the authors of this paper proposed a methodological framework for creating a mathematical model for predicting the parameters of business project implementation efficiency based on the use of ordinary differential equations [7]. Using this methodology, a generalized test mathematical model was developed to forecast the efficiency indicators of the portable power generator store startup. Test calculations have shown good prediction accuracy using this model [7]. However, the task of refining the methodology for calculating the *NPV* and *PI* indicators when modeling them by continuous functions in the right-hand sides of ordinary differential equations still remains.

Chapter 2. The refining of the *NPV* and *PI* indicators modeling in the right-hand sides of ordinary differential equations

The ordinary differential equations (ODE) for forecasting the efficiency indicators that was developed in previous research [7] has the next form:

$$\begin{aligned}
&\frac{dV}{dt} = V_{cur},\\
&\frac{dCF}{dt} = V_{cur} \cdot P \cdot k_{n.} - V_{cur} \cdot CP + CP \cdot V_{cur} \cdot ka,\\
&\frac{dNPV}{dt} = CF \cdot (-r) \cdot e^{-rt} - I_{oi} \cdot \varphi \cdot e^{\varphi \cdot t},\\
&\frac{dPI}{dt} = \frac{-CF \cdot (r+\varphi) \cdot e^{-(r+\varphi)t}}{I_{oi}},
\end{aligned} \tag{1}$$

where V is the production volume (number of manufactured products) or volume of services, goods (for non-manufacturing sectors and retail trade);

V_{cur} is the rate of production of goods, units/day;

CP is the cost of production (goods, services);

CF is the cash flow;

P is the price of goods, services, etc.

I_{oi} is the initial investment from the investor / self-investment (or investment from the investor / self-investment at the beginning of the forecast period);

$k_{n.}$ is the expected sales ratio for the enterprise for the forecast period, which is selected by the method of expert estimates;

ka is the depreciation coefficient, which is selected by expert estimates in accordance with the characteristics of the equipment, type of enterprise and method of calculating depreciation;

r is the discount rate;

φ is investor's interest rate (has non-zero value in the case of external investment and equal to zero in the case of self-investment or preferential type of investment);

t is the forecasting time;

In turn, there are 3 strategies for the V_{cur}. According to [7] these strategies are:

a) linear production;

b) production with an increase according to the parabolic law;

c) production with an extension according to the law of the cubic parabola.

Taking into account these three types of production strategies, the first equation of the system (1) can be written as [7]:

$$V_{cur} = \text{switch} \rightarrow \begin{array}{r|l} \text{A-str} & (k_1 - k_2)V_{norm}, \\ \text{B-str} & (k_1 - k_2)V_{norm} \cdot t, \\ \text{C-str} & (k_1 - k_2)V_{norm} \cdot t^2, \end{array} \quad (2)$$

where switch is the function of choosing a strategy for the production of goods or the volume of services, goods (for non-production areas and retail);

V_{norm} is the norm of the production rate, units/day;

k_1 is the coefficient of arrival of funds (repair of equipment, purchase of new production units, renewal or repair of transport and logistics system, etc.);

k_2 is the coefficient of disposal of funds (depreciation of equipment, failures of certain production units, transport for the supply of goods, etc.);

A-str, B-str, C-str – types of strategy for the production of the function V_{cur}.

In turn, for correct usage of these differential equations (1) for calculations it is necessary to evaluate third and fourth equation analytically without taking into account first two equations and considering that CF is independent variable. This is needed to avoid double integration of cash flow that would be in the case of integration ODE (1) simultaneously as an ODE system. So, if we separate the variables in the third differential equation of (1) and then analytically integrate it, it will be got the following:

$$\begin{aligned} &\int dNPV = \int\left((-r)\cdot CF \cdot e^{-rt} - I_{oi} \cdot \varphi \cdot e^{\varphi \cdot t}\right)dt, \\ &NPV = CF \cdot e^{-rt} - I_{oi} \cdot e^{\varphi \cdot t} + const. \end{aligned} \quad (3)$$

Setting $const = 0$ it can be obtained the expression for NPV analytical calculation: $NPV = CF \cdot e^{-rt} - I_{oi} \cdot e^{\varphi \cdot t}$. The same operation of analytical integration has been done with fourth equation for *PI* (1). After this evaluation, the formulas (1) will be written in the next form:

$$\begin{aligned} &\frac{dV}{dt} = V_{cur}, \\ &\frac{dCF}{dt} = V_{cur} \cdot P \cdot k_{n.} - V_{cur} \cdot CP + CP \cdot V_{cur} \cdot ka, \\ &NPV = CF \cdot e^{-rt} - I_{oi} \cdot e^{\varphi \cdot t}, \\ &PI = \frac{CF \cdot e^{-(r+\varphi)t}}{I_{oi}}. \end{aligned} \qquad (4)$$

Analyzing obtained ODE (4) it can be observed that cash flow *CF* is an integrable parameter. It means that in each step of integration this parameter changes taking into account its previous values. Taking into account a type of the ODE (3) it can be seen, that *CF* is integrated at first in the second equation, and then the integral value of *CF* is discounted in the third equation. Considering the integral form of third equation (3)-(4) it can be seen that this type of discounting is close to traditional continuous discounting of present value *PV* [4,5]. In this case, the total amount of cash flows is discounted, as opposed to discounting the fractions of cash flows for certain periods in *NPV* calculations. Thus, in the first case we have the discounted sum of all cash flows for the period (closer to *PV*), and in the second case we have the sum of discounted flows for all periods (*NPV* calculation). Considering these differences in formulas, it is proposed to carry out the calculations for preliminary numerical estimation analysis of discrete and continuous types of discounting for both cases. The comparison of formulas for two calculation methods is presented in the Table 1.

Table 1

The formulas for two calculation methods of *NPV* for discrete and continuous types

Type	**First method (closer to the traditional *PV* calculation)**	**Second method (traditional calculation of *NPV*)**
Discrete	$NPV_1^d = \frac{CF}{(1+r)^t} - I_o$	$NPV_2^d = \sum_{t=1}^{n} \frac{CF_t}{(1+r)^t} - I_o$
Continuous	$NPV_1^c = CF \cdot e^{-rt} - I_o$	$NPV_2^c = \int_0^n CF_t \cdot e^{-rt} dt - I_o$, [18]

Table 1 contains the following designations: NPV_1^d is *NPV* that calculates using first method for discrete type; NPV_2^d is *NPV* that calculates using second method for discrete type; NPV_1^c is *NPV* that calculates using first method for continuous type; NPV_2^c is *NPV* that calculates using second method for continuous type [18]; CF is the sum of cash flows for determined period *t* (for discrete method this sum is calculated as $CF = \sum_{t=1}^{n} CF_t$, for continuous method this sum is calculated as $CF = \int_0^n CF_t dt$; CF_t is the cash flow for determined period *t*; *n* is the number of periods for forecasting (years, days, etc.); r is discount rate which is adjusted to a specific discount period (years, days, etc.).

For estimating calculations, it is proposed to take the example of a project with investments for 5 years with steady cash flows by years: $CF_t = 5000$ \$ per each year. Initial investment I_o is equal 7000 \$, discount rate is constant and equal 15% per year. The obtained results are following:

$$NPV_1^d = \frac{25000}{(1+0.15)^5} - 7000 = 5429.418\$,$$

$$NPV_1^c = 25000 \cdot \mathrm{e}^{-0.15 \cdot 5} - 7000 = 4809.164\$,$$

$$NPV_2^d = \sum_{t=1}^{5} \frac{5000}{(1+0.15)^t} - 7000 = \frac{5000}{(1+0.15)^1} +$$

$$+\frac{5000}{(1+0.15)^2} + \frac{5000}{(1+0.15)^3} + \frac{5000}{(1+0.15)^4} + \frac{5000}{(1+0.15)^5} -$$

$$-7000 = 9760.775\$,$$

$$NPV_2^c = \int_0^5 5000 \cdot \mathrm{e}^{-0.15 \cdot t}\, dt - 7000 = 10587.782\$.$$

Analyzing obtained results, it can be seen that $NPV_2^d > NPV_1^d$ and $NPV_2^c >$ NPV_1^c which means that first method of calculation is more stringent than second and also significantly influences on other efficiency indicators estimations. This peculiarity can be explained by the next property of calculations:

In the case of discounting the sum of whole cash flows the increase of discount denominator with the time influence on all cash flows including that obtaining in previous times. In turn, when discounting the components of cash *flows for each time interval, only the last few components are more discountable.*

Given this feature, the first method is rationally used to estimate the efficiency of business projects with long preparation and development phases and a short sales phase (realization phase). The examples of such business projects are: scientific and innovative projects, innovative IT-project, aerospace engineering project, etc. These projects are characterized by significant costs for the development and innovations at the first stages without any income cash flows and profit. In turn, the second type of *NPV* calculation is advisable for business projects with time-periodic income cash flows, such as delivery, logistics, trade, mass production, etc.

Taking it into account, the authors of the paper propose to adapt the ODE (4) that take into account the variability of the discount rate closer to the second type of NPV calculation. So, the ODE is proposed to write in the next form:

$$\begin{aligned} &\frac{dV}{dt} = V_{cur}, \\ &\frac{dCF}{dt} = CF_t, \\ &\frac{dPVI}{dt} = CF_t \cdot e^{-rt}, \\ &NPV = PVI - I_{oi} \cdot e^{\varphi \cdot t}, \\ &PI = PVI / I_{oi} \cdot e^{\varphi \cdot t}, \end{aligned} \qquad (5)$$

where $CF_t = V_{cur} \cdot P \cdot k_{n.} - V_{cur} \cdot CP + CP \cdot V_{cur} \cdot ka$ is the instantaneous value of cash flow; PVI is the integral value of discount cash flow.

Thus, an updated ODE for forecasting indicators of business project implementation efficiency in the case of periodic in time cash flows incomes is obtained. An analysis of this ODE usage and its verification is proposed in the next section.

Chapter 3. Analysis of NPV and PI forecast calculation using the updated system of ordinary differential equations

To analyze the peculiarities of the application of the finalized models, it is proposed to consider the variant of forecasting the enterprise at a variable discount index and the influence of external market factors on the price of products, as in the previous research [7]. It is proposed to present the discount rate as the sum of four components [7] in the next form:

$$r = r_{pol}^{USA}(t) + r_{pol}^{UA}(t) + r_{inf} + r_{bi}, \qquad (6)$$

where $r_{pol}^{USA}(t)$ is the political risk component of discount rate which is connected with the President elections in the USA; $r_{pol}^{UA}(t)$ is the political risk component of discount rate which is connected with the President elections in the Ukraine; r_{inf} is the inflation index; r_{bi} is the industry risk ratio.

The political risks $r_{pol}^{USA}(t)$ and $r_{pol}^{UA}(t)$ are calculated using Fourier series in accordance with the methodology presented in [7]. In turn, inflation index and risk ratio are estimated by experts according to the current economic condition of the country.

To analyze the impact of the external market environment on pricing and enterprise activity, it has been used the following indicators which described in [7]. These coefficients are: the coefficient of current demand *q* and coefficient of current supply *s*.

These coefficients are calculated using next formulas [7] with updated thresholds:

$$q = \frac{N}{N_1}, \quad 1 < q \le tr, \qquad s = \frac{V_p}{N_1}, \quad 1 < s \le tr. \tag{7}$$

where N is number of potential consumers for the predefined forecasting period; N_1 is the sales volume for the predefined forecasting period; V_p is the volume of similar products on the market from competitors for the predefined forecasting period; tr is the upper value of *q* and *s* coefficients threshold, which shows the degree of influence on the price (set by expert judgment, but not less than 1.0). Parameters N, N_1 and V_p are calculated from statistical data using approximation that proposed in [7]. So, taking into account (6) price is proposed to calculate as:

$$P = P_1 \frac{q}{s}, \quad 0.5 \cdot P_1 \le P \le 1.5 \cdot P_1 \tag{8}$$

where P_1 is the price of products calculated at the time of the start of forecasting, taking into account profits and taxes; P is the current value of price.

According to [7] it has been introduced two thresholds for price (8). These thresholds mean that the price cannot be increased or decreased by more than 50 %. In turn, these thresholds can be changed and adopted according peculiarities of business and market.

To analyze the peculiarities of the usage mathematical models (4) and (5) for the case of time-periodic discount rate change and the impact of the external market environment on pricing it is proposed to calculate the *NPV* and *PI* for the startup which based on producing aerospace aerodynamic deorbiting systems. The determined indicators of cost and profit in the manufacture of one aerodynamic deorbit system is given in the Table 2.

Table 2

Determination of cost, price, and profit in the production of one aerodynamic deorbit system in USD

Indicator	**Amount**
Total cost	12383.55
Depreciation of equipment	1938.35
Profitability. %	56.53
Unit price	19383.55
Profit per unit	7000

So, high profitability of this good (Table 1) is explained by belonging of this device to the highly technological aerospace goods of the rocket and space industry. In the view of this, it is proposed to analyze the profitability of this startup project using two methods of calculation:

1) the discounting of cash flow integral value (4);

2) the discounting of each cash flow part in each integration step (5).

Main project characteristics are:

– the duration of the project is expected to be 1824 days (aprox. 5 years);

– the total volume of produced good for this period is expected to be 456 units;

– initial investment is 1000000 USD;

– the type of investing – self investments without investor interest rate;

– considering that all financial operations are expected to be in USD the r_{inf} is set to be 2.5%;

– r_{bi} is set to be 4%.

It is proposed to make calculations using both methods (4) and (5) for two variants (the first one – taking into account the change in the price of manufactured products under the influence of the external market environment, the second – at a constant price).

Using the application that was developed in previous studies [7] with the addition of the calculation new type (5), the following results were obtained for the case of a constant price (Figures 1–6).

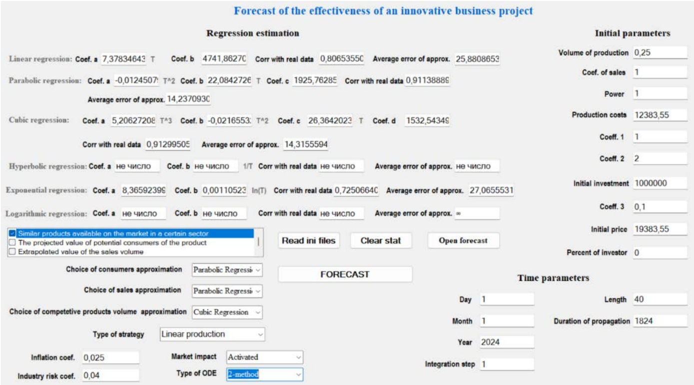

Figure 1. Main interface of the application with the added update “Type of ODE”, where 1 – method is for the first strategy of sales, 2 – method is for the second strategy of sales

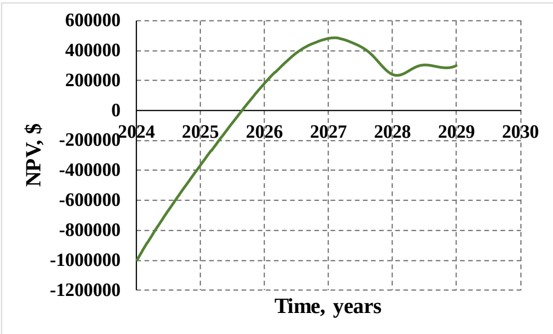

Figure 2. *NPV* forecast using method (4) for the case of constant price

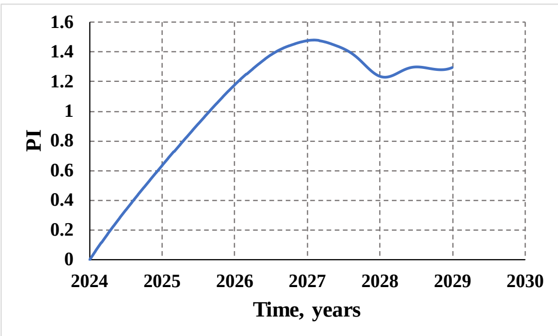

Figure 3. *PI* forecast using method (4) for the case of constant price

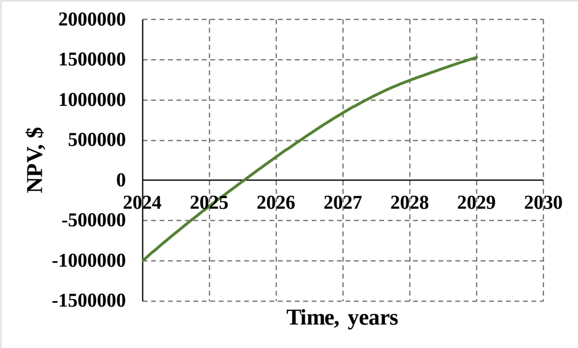

Figure 4. ***NPV*** **forecast using method (5) for the case of constant price**

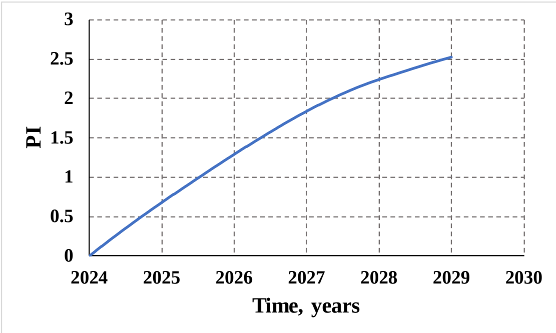

Figure 5. ***PI*** **forecast using method (5) for the case of constant price**

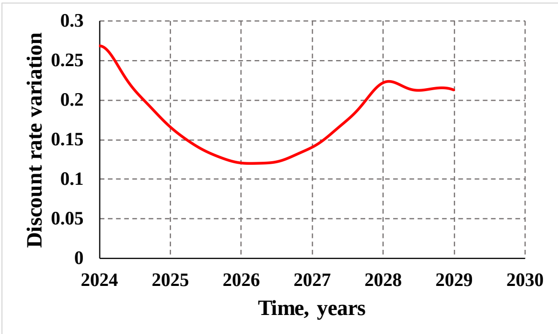

Figure 6. Changes in the discount rate over time

The obtained results of *NPV* and *PI* forecasting (Figures 2–6) have shown the similar tendency of calculation depending on the selected model (4) or (5) as in the case of *NPV* calculation type (Table 1). Thus, it can be observed that discounting all sum of income cash flows gives greater risks when calculating the *NPV* and *PI* than in the case of separate discounting all received income cash flows throughout the life of the entire project. This result can be explained by the impact of discount rate current value to the whole sum of cash flow and by the peculiarities that have been determined in the previous section. Discounting the entire amount of cash flow in each stage of the project is appropriate for high-risk projects with a significant number of reinvestments. These high-risk projects can be in the case when investor wants to return back all investments after determined period taking into account his interest rate.

In turn, in the case of variable discount rate (Figure 6) the choosing of the method of calculation is very important because it can significantly impact the project profitability estimation (Figure 3 and Figure 5) which can be critical when making management decisions.

The results of calculations using two methods in the case of changeable price of manufactured products under the influence of the external market environment are presented on Figures 7–11.

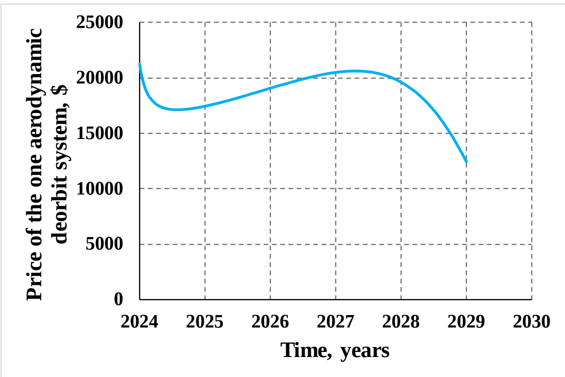

Figure 7. The forecast of possible changes in price due to the external market environment impact

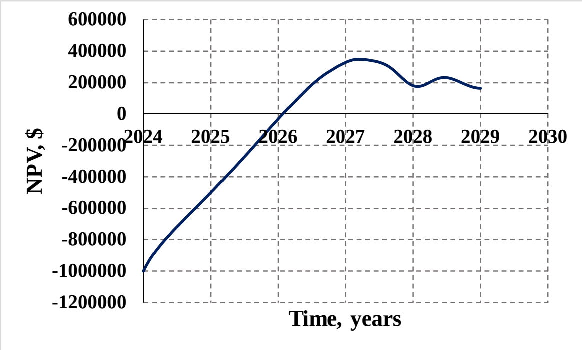

Figure 8. ***NPV*** **forecast using method (4) for the case of variable price**

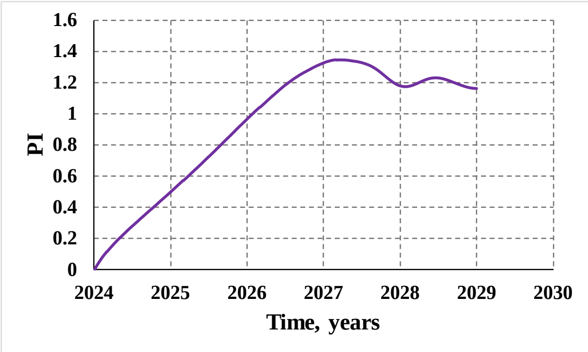

Figure 9. ***NPV*** **forecast using method (4) for the case of variable price**

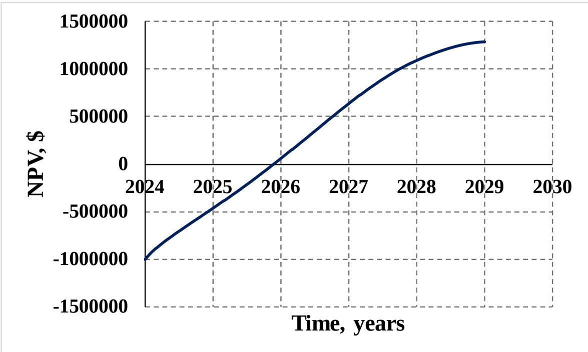

Figure 10. ***NPV*** **forecast using method (5) for the case of variable price**

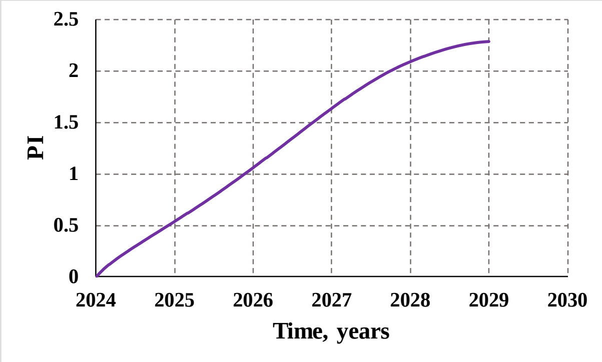

Figure 11. ***PI*** **forecast using method (5) for the case of variable price**

The price variability (fig. 7) forecast has been calculated based on statistical data of the N, N_1 and V_p indicators using approximation which proposed in [7] and formulas (7)-(8). As can be seen from the obtained result of price forecasting, in 2024–2025 there will be a drop in price due to the geopolitical crisis in Europe and as a consequence a drop in demand. Further, there is an increase in price to market saturation in 2028 with a subsequent fall in 2029.

The obtained results (Figures 8-11) show the similar difference in *NPV* and *PI* calculations as in the case of constant price. Considering these aspects, the choice of ODEs type methodology affects the calculation of *NPV* and *PI* in about the same way in both cases: constant price and variable price.

Conclusions

The peculiarities of calculation of net present value and profitability index using differential equations and continuous functions has been described in the chapter. Two types of discounted cash flow calculation are considered:

1) discounting the whole sum of cash flows in each period including cash flows from previous periods;

2) discounting each cash flow in each period and then summing it up.

The difference in calculation *NPV* and *PI* when using first or second discounted cash flow calculation type has been determined. Based on these estimations it has been confirmed that using the discounting of cash flows whole sum is more stringent condition when calculating *NPV* and *PI* than separate discounting of each cash flow with further summing up. Taking this into account it has been refined the differential equations for calculation *NPV* and *PI* indexes in the frames of forecasting the business project implementation considering external economic environment impact.

The calculations performed using two types of differential equations (discounted sum and discounted separate cash flows) showed a significant difference in the profitability of a business project. The usage differential equations model with discounting of whole cash flows sum has performed the approximately half as much PI value than model with separate discount of each cash flow. Considering these peculiarities, it has been proposed to use the first model for high-risk projects with a significant number of reinvestments. In turn, the second type of discounting has been offered for business projects with time-periodic income cash flows, such as delivery, logistics, trade, mass production, etc.

Thus, it has been expanded the usage of differential equations models for forecast the efficiency of business projects implementation depending of discounting strategy and external market environment impact.

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