CHAPTER «PHYSICAL AND MATHEMATICAL SCIENCES»

THEORETICAL ASPECTS AND EXPERIMENTAL SEARCHES FOR PHYSICS BEYOND THE STANDARD MODEL

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Abstract. The application of continuous and discrete groups to the theory of nuclei, elementary particles, in the theoretical physics of high energies leads to the need of systematization of knowledge and methods to obtain new information about phase transitions at high energies. Attempts to find a theory of the experiment on the scale of QCD led to the discovery of String Theory. The development of superstring theory as applied to cosmology led to the concept of the "Universe on brane". D-brane theory asserts that gravity and quantum mechanics are integral principles of the construction of the Universe. D-branes are used to solve three serious problems of the Big Bang: flatness problem; the problem of the horizon; the dark energy problem. The application of a complex approach in the study of new physics at high energies is an effective way of studying all four types of interaction within the framework of the theory of D-branes and superstrings, which are necessary for solving the current problems of modern theoretical and experimental physics. *The purpose* of the work is to clarify the properties of physical objects such as vibrational modes of superstrings located between D-branes for purposeful search on modern accelerators in the form of Kaluza-Klein modes of gauge particles, microscopic black holes and other exotic particles. The study of black holes through the prism of the geometry of extra-dimensional space, superstring theory, and the holographic principle is an important component of understanding its nature. *Methodology* of the study is based on general research methods of analysis and synthesis, induction and deduction,

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mathematical and computer modeling of systems of particles and forces, use of group theory and the analogy method. Also, new theoretical models were proposed by Hawking in the aspect of treating particles near the event horizon, by Maldacena in the aspect of AdS/quantum field theory correspondence, by Witten in the aspect of the Thermal Phase Transition. *Results* of considered material presents a theoretical model of a phase transition in the early stages of the evolution of the Universe, associated with a heterotic string model, identical to the phase transition of a black hole. Calculation of the entropy of a black hole through string excitations at its AdS boundary, cross sections of the production of a rotating and non-rotating microscopic black hole in the ADD model, as well as the production and decay of a graviton at different energies within RS model, are represented. In the framework of the space of extra dimensions, calculations of the energy of BH are presented and the proportionality of the energy to the number of degrees of freedom of the strings located between the branes, N, which has the meaning of the level of energy excitation of a soliton object of the D-brane type, is shown. Since the entropy increases and this corresponds to an increase in the excitation level of the soliton state, the decay of BH would be accompanied by an energy release greater than the explosion of a hydrogen bomb.

1. Introduction

On December 7, 2023, a draft version of the regular report of the P5 committee was published on the formulation of a ten-year strategy for US research in the field of particle physics, which takes into account the expected financial, economic and social factors until the end of the 2030s. The draft was signed by 32 authoritative specialists in this field of physics, who represent both American and foreign scientific centers.

The main challenges facing particle physics are the investigation of the properties of neutrinos and the Higgs boson, the search for new particles such as axions, superparticles, and the clarification of the nature of dark matter and the nature of the evolution of the Universe. They strongly recommend expanding the participation of the United States in the international program to significantly increase the luminosity of the LHC, which will create the prerequisites for the discovery and study of interactions between dark matter and the Higgs boson, as well as

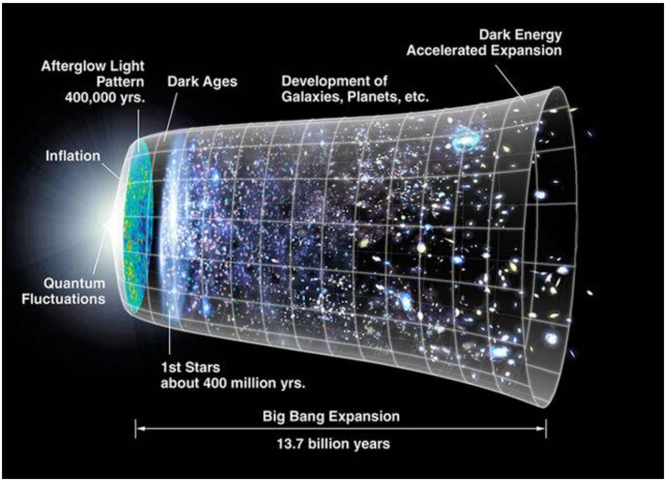


Figure 1. Stages of the evolution of the Universe

for the detection of supersymmetric partners of ordinary particles. The report also emphasizes the importance of completing preparations for the Deep Underground Neutrino Experiment (DUNE), which should begin around 2034. It is recommended to ensure substantial US participation in international projects to build accelerators capable of producing a large number of Higgs bosons (so-called Higgs factories). These are the Future Circular Electron-Positron Collider (Future Circular Collider) and the International Linear Collider (ILC). Touching on the "illumination" of the dark Universe, experts emphasize the importance of experiments on the detection of dark matter particles and offer a wide range of candidates for this role, including, in particular, ultralight axions and massive particles (wimps). In addition, the authors of the report have high hopes for the implementation of the CMB-S4 experiment, designed for long-term precession mapping of microwave relic radiation. This task will be solved with the help of twelve telescopes with an aperture of up to six meters, installed at the South Pole and the Chilean Atacama Desert. These data will make it possible to specify the rates of expansion of space at different stages of the evolution of the Universe (Figure 1) and thus contribute to a better understanding of the phenomenon of dark energy.

2. The ideology of string and D-brane theory

Phase transitions at the early stages of the Universe's evolution are represented by the reduction of the structural group of D-branes to a subgroup that has an analogue in the physics of elementary particles called gauge symmetry breaking. Symmetries, both discrete and continuous, play a leading role in physics. Wigner considered the application of group theory to the problems of quantum mechanics, especially the theory of atomic spectra. He paid the most attention to the symmetric group, the rotation group, and the most important section for applications – representation theory. The general theory of groups and their representations in the form of multiplets, developed by Wigner, is applied to atomic spectra in a form that allows it to be used for a wider range of problems – nuclear spectra, field theory and elementary particles, etc. [1]. The development of nuclear theory is harmoniously connected with the development of its classification with group theory. Thus, the Sakata model of hadrons was the predecessor of the quark model. He proposed that protons, neutrons and lambda baryons were elementary particles and that all other known hadrons were built from them. The success of the Sakata model is due to the fact that there is an equivalence between the proton, neutron and lambda baryon and the top, bottom and strange quarks according to SU(3) symmetry, which allows to reproduce the quantum numbers of the flavor of all hadrons [2]. As a prototype of the superfluid nuclear model of Flowers, Shpykovsky, Ichimura with the Sp(4) symmetry group [3], Malyuta Yu.M. proposed to use the new Sp(6) group for the quark model [4]. The connection between different groups of symmetries is shown in Figure 2 [5].

One of the fundamental problems of modern physics, which concerns the early stages of the development of the Universe, is the unification of four types of interactions. Historically, the theoretical prerequisites for the creation of such a theory were associated with the emergence of string theory and then theory of D-branes. Attempts to find a coherent theory of the experiment on the scale of QCD led to the discovery of string theory. The emergence of string theory is associated with the analysis of experimental data on pion scattering, which was described in 1968 by J. Veneciano and M. Suzuki using beta functions. In 1970, Nambu J., Goto T., Nielsen H. and. Sasskind L. put forward the idea of the interaction between pi-mesons through "an infinitely thin thread that oscillates." Thus, the theory of

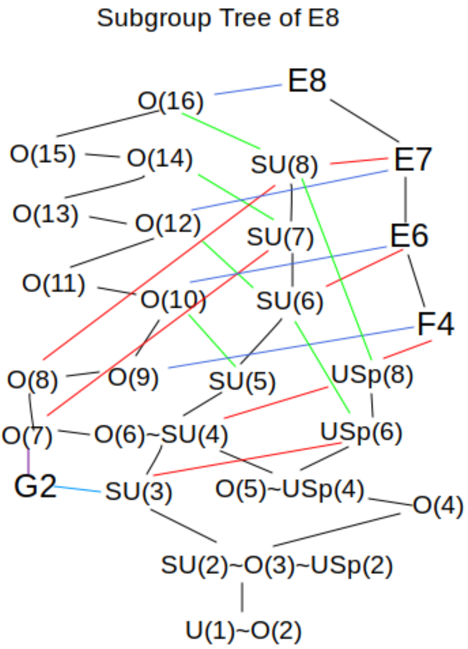


Figure 2. Relationship between groups of symmetries

superstrings appeared, which describes elementary particles and the interactions between them. Scientists such as Green M., Schwartz J., Volkov D.V., Kazakov D.I. associated with the development of this theory. In the 1990s, Witten E., Polchinsky J. and others discovered evidence for combining superstring theories into an 11-dimensional M-theory. Thanks to the application of the principles of supersymmetry for the calculation of masses, charges of the Bogomolny-Prasad-Sommerfeld (BPS) states, scientists Strominger E., Horowitz G.T., Polchynsky J. discovered multidimensional objects – D-branes. The advantage of such objects is the possibility of including quantum gravity in the known gauge theories of elementary particles.

The vibrational modes of the string can be represented in the form of particles, the possible spectrum of which for a superstring of type IIB is presented in Table 1, [6]. Spectra of closed superstrings are calculated in terms of the SU(4) × U(1)-formalism of eight-dimensional representations of the group SO(8).

The content of these representations the by subgroup SU(4) × U(1) is the following

$$8_V = 6_0 + 1_1 + 1_{-1}$$

$$8_C = 4_{1/2} + \underline{4}_{-1/2}$$

$$8_S = 4_{-1/2} + \underline{4}_{1/2}$$

Table 1

The spectrum of massless modes of a type IIB closed superstring

1_2
$4_{3/2} + 4_{3/2}$
$10_1 + 6_1 + 6_1 + 6_1$
$20_{1/2} + 20_{1/2} + \underline{4}_{1/2} + \underline{4}_{1/2} + \underline{4}_{1/2} + \underline{4}_{1/2}$
$20_0 + 15_0 + 15_0 + 15_0 + 1_0 + 1_0 + 1_0 + 1_0 + 1_0$
$\underline{20}_{-1/2} + \underline{20}_{-1/2} + 4_{-1/2} + 4_{-1/2} + 4_{-1/2} + 4_{-1/2}$
$\underline{10}_{-1} + 6_{-1} + 6_{-1} + 6_{-1}$
$\underline{4}_{-3/2} + \underline{4}_{-3/2}$
1_{-2}

where the subgroup SU(4) describes isospin, strangeness and charm, and the subgroup U(1) describes helicity.

One can imagine the interaction of strings with branes, which is shown in Figure 3.

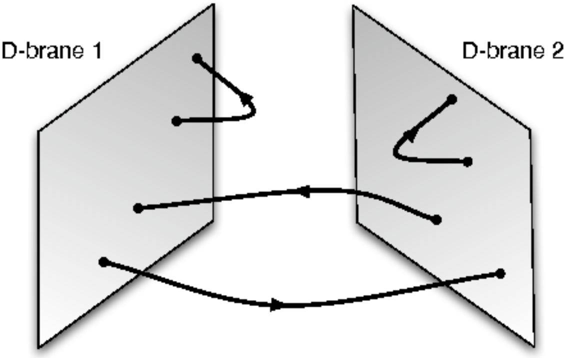


Figure 3. Dp-branes as two hypersurfaces with strings between them

The development of superstring theory as applied to cosmology led to the concept of the "Universe on brane". In it, the Universe is represented as a (3+1)-dimensional D-brane immersed in an 11-dimensional space with 7 compactified dimensions. This space is called inner space, or the space of extra dimensions. Among the many properties of D-brane theory, the following three are particularly important. First, gravity and quantum mechanics are integral principles of the construction of the Universe, and therefore any project of a unified theory must include both. This is realized in the theory of D-branes. Second, research over the past century has shown that there are other key ideas – many of which have been tested experimentally – that are central to our understanding of the Universe. Among these ideas, we will mention the existence of generations of matter and carrier particles of interaction, gauge symmetry, the principle of equivalence, symmetry breaking and supersymmetry. All these ideas follow naturally from the theory of D-branes. Third, unlike more conventional theories such as the SM (Standard Model) with its 19 free parameters that can be adjusted to ensure agreement with experiment, the D-brane theory has no free parameters. In cosmology, the theory of D-branes is also widely used, since such extensions of the SM are necessary to solve three serious problems of the Big Bang:

1. Flatness problem.
2. The problem of the horizon.
3. The dark energy problem (BOOMERanG, MAXIMA, WMAP experiments).

3. Phase transitions from D-branes to elementary Standard Model particles

We will take as given that the gauge groups unify into a Grand Unification Theory and that the matter content of the SM descends from the representation content of an exceptional gauge group that arises in string theory as the $E_8 \times E_8$ heterotic string. The phase transition from D-branes to elementary SM particles is associated with the subsequent reduction to the SM group [7]

$$E_8 \to E_6 \to \ldots \to SU(3) \times SU(2) \times U \qquad (1).$$

We will represent the sequential breaking of symmetry through embeddings of the matter content of group representations into groups of lower rank and proceed iteratively until we arrive at the SM gauge group [8]. Starting from representations of E_8, we will rule out breaking patterns as possible according to gauge invariance. For SO gauge groups, we assume the matter presented by the fundamental, spinor or adjoint representations. For SU gauge groups, we deal with the adjoint representation and one, two or three anti-symmetric representations.

The non-trivial representations of E6 which can descend from the adjoint representation of E_8 are the multiplets 27 and 78 of E_6. The maximal subgroups of E_6 are:

$$E_6 \supset SO(10) \times [U(1)]$$

$$27 \rightarrow 1_4 + 10_{-2} + 16_1$$

$$78 \rightarrow 1_0 + 16_{-3} + \underline{16}_3 + 45_0.$$

The matter content of SO(10) organizes into the representations 10, 16, and 45. The maximal subgroups of SO(10) are:

$$SO(10) \supset SU(5) \times [U(1)] \supset SU(3) \times SU(2) \times [U(1)_a] \times [U(1)_b]$$

$$10 \rightarrow (1,2)_{3,2} + (3,1)_{-2,2} + (1,2)_{-3,-2} + (\underline{3},1)_{2,-2}$$

$$16 \rightarrow (1,1)_{0,-5} + (1,2)_{-3,3} + (\underline{3},1)_{2,3} + (1,1)_{6,-1} + (\underline{3},1)_{-4,-1} + (3,2)_{1,-1}$$

$$45 \rightarrow (1,1)_0 + (1,1)_{6,4} + (\underline{3},1)_{-4,4} + (3,2)_{1,4} + (1,1)_{-6,-4} + (3,1)_{4,-4}$$

$$+ (\underline{3},2)_{-1,-4} + (1,1)_{0,0} + (1,3)_{0,0} + (8,1)_{0,0} + (3,2)_{-5,0} + (\underline{3},2)_{5,0}.$$

The maximal subgroup of SU(5) which contains the SM gauge group, has the representation content given by the Georgi-Glashow model:

$$SU(5) \supset SU(3)_C \times SU(2)_L \times U(1)_Y \equiv G_{std}$$

$$5 \rightarrow (1,2)_3 + (3,1)_{-2}$$

$$10 \rightarrow (1,1)_6 + (\underline{3},1)_{-4} + (3,2)_1$$

$$24 \rightarrow (1,1)_0 + (1,3)_0 + (3,2)_{-5} + (\underline{3},2)_5 + (8,1)_0.$$

Within the framework of this theory, it is assumed that matter particles are localized on D-branes (ultraviolet branes at Planck energies and D3-branes, or SM-branes in the infrared range), and gauge bosons are located between D-branes in the space of extra dimensions and take part in phase transition from higher energies with E_8, E_6 symmetry groups to low energies, which leads to a change in the multiplet state of the particles. The change in the multiplet state of particles for models with different calibration groups (for n=2,4,6,8,10,12) is shown in Table 2, [9].

The weakness of gravity in our world can be explained by the existence of extra dimensions. At modern particle colliders, these effects are sought through the formation of Kalutz-Klein (KK) resonances in the space of extra dimensions and a radical change in gravitational forces in the submillimeter range, Figure 4.

Table 2

The content of the fields of models with extended calibration groups (for n=2,4,6,8,10,12)

Group	Matter content
SU(2)	(6n+16)2
SU(3)	(6n+18)3
SU(4)	(n+2)6+(4n+16)4
SU(5)	(3n+16)5+(2+n)10
SO(10)	(n+4)16+(n+6)10
E_6	(n+6)27
E_7	(n/2+4)56

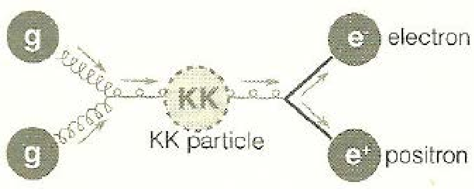


Figure 4. Formation of KK particles due to the interaction of gluons and its decay into an electron-positron pair

Graviton detection signals can be obtained from such processes

a) Drell-Jahn processes

$$q\bar{q} \to G^{(1)} \to l^+l^-$$

$$gg \to G^{(1)} \to l^+l^-$$

b) excitation in two-jets channels

$$q\bar{q},\ gg \to G^{(1)} \to q\bar{q},\ gg.$$

The SM problems lead to the new theories of extra dimensions: Randall-Sundrum (RS) model, Arkani-Hamed-Dimopoulos-Dvali (ADD) model, and TeV^{-1} model. In the framework of these models, the production cross sections for KK particles at various energies at the LHC were calculated. The graviton production processes through the gluon-gluon, quark-gluon, and quark-quark fusion processes were studied and production cross sections multiplied by branching fractions were calculated for the massive graviton, G, at 13 TeV, 14 TeV, 100 TeV, [10]. We considered three processes of graviton decay, $G \to gg$, $G \to ll$, and $G \to hh$, for further $\sigma \times Br$ calculations within RS scenario for graviton production process gg → G at 13 TeV, 14 TeV, and 100 TeV at the center of mass energies. In Figure 5, our calculations performed with the help of Pythia8.2 are presented, with parameters $k/M = 1$ and 0.1.

4. Black hole radiation and entropy

The internal structure of black holes (BH) can be studied within the framework of string theory. In some cases, it is even possible to describe the microstructure of BH. It is easiest to understand the system of BH living

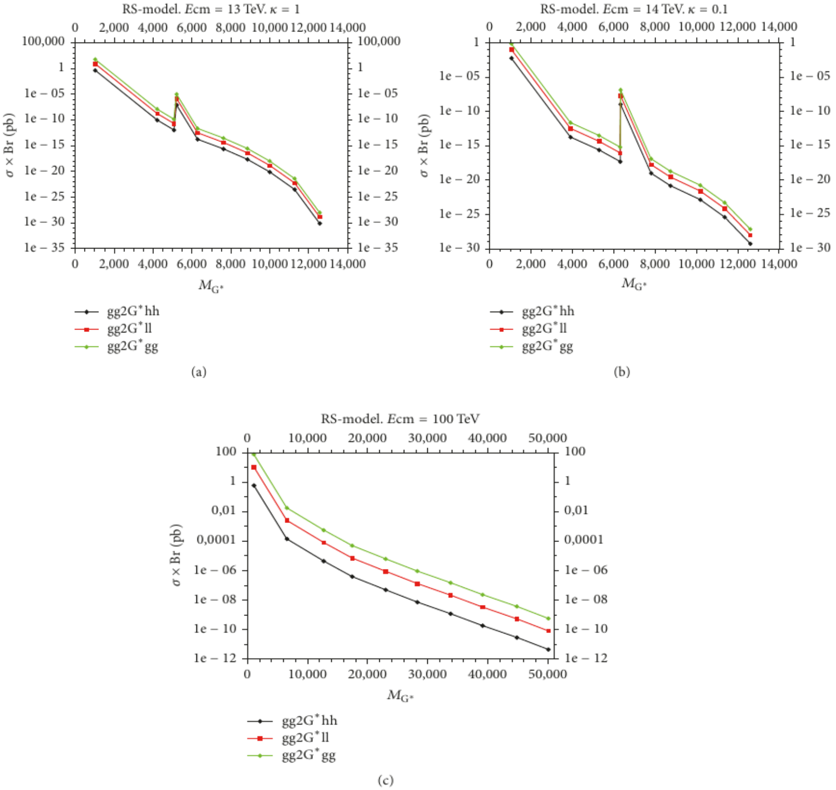


Figure 5. $\sigma \times Br$ for graviton production and decay as the function of graviton mass, MG, at (a) 13 TeV, $k/M = 1$; (b) 14 TeV, $k/M = 0.1$; and (c) 100 TeV, $k/M = 0.1$

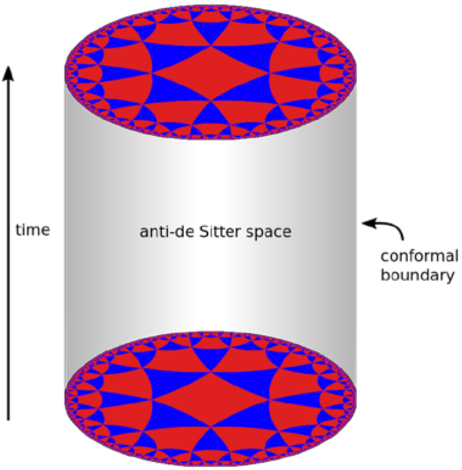


Figure 6. Cylindrical space-time

in a space-time continuum of constant negative curvature. Such space-time continua are the simplest generalization of ordinary rectified space. The curvature of the straightened space is zero, and its two-dimensional analogue is a plane. A two-dimensional analogue of space with positive curvature is the surface of a sphere. Space-time continuums with negative curvature essentially have a closed boundary at infinity. In 1997, Maldacena ventured to suggest that all gravitational physical interactions in such a space can be described through the theory of interactions of ordinary particles located on its boundary [11]. The three-dimensional anti-de Sitter space is like a stack of hyperbolic disks, each of which represents the state of the Universe at a certain moment in time, Figure 6.

Such a space is called anti-de-Sitter, AdS space, and AdS black hole is a solution of general relativity. In general, the space of extra dimensions is AdS, and the ten-dimensional metric is defined as follows

$$ds^2 = -(k^2 r^2 + 1 - \frac{C}{r^{d-2}})\, dt^2 + \frac{1}{k^2 r^2 + 1 - \frac{C}{r^{d-2}}}\, dr^2 + r^2 d\Omega^2.$$

According to the AdS/CFT correspondence, the AdS black hole is a binary state at the conformal boundary, which would correspond to the deconfinement phase of the quark-gluon plasma in the context of QCD. Later, this hypothesis was developed in detail by S. Gubser, I. Klebanov, A. Polyakov, E. Witten and many other scientists. In a rectified space, we have, by the law of conservation of energy, real particles. That is, in a vacuum, particles actually annihilate each other before they even have time to be born. In 1974, Stephen Hawking proved that this is not the case near the horizon. There is a non-zero probability of the birth of a pair of particles that are immediately detected on different sides of an infinitely thin horizon, and the law of conservation of energy is not violated, since a particle outside the horizon has, from the point of view of an outside observer, positive energy, and a particle inside the horizon has negative energy. The mass of isolated BH becomes smaller, its temperature becomes higher, and it "evaporates" faster until it evaporates completely. In fact, if we compress just a few kilograms of matter to the density of BH, such BH will evaporate and a huge amount of energy will be released. The thermodynamic state of the BH within the framework of this model is described by the temperature of the particles in its boundary layer, Figure 7.

That is, as we can see from Figure 7, the properties of the BH (a) are determined by the boundary particles located on branes 1 and 2 (b), and the transformation of the BH into another state is realized with the help of closed and open strings located on the boundary of this space.

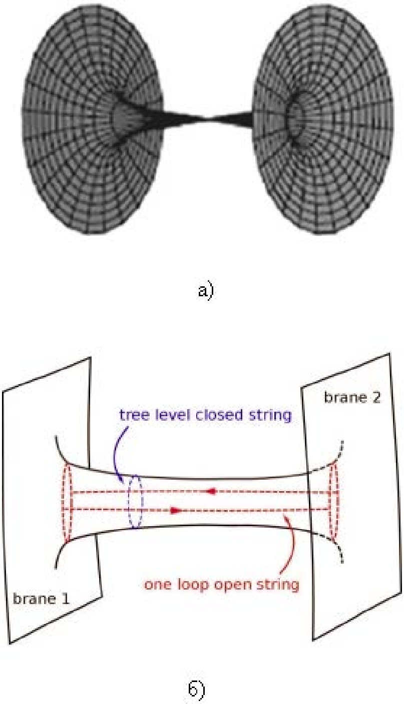


Figure 7. Schematic representation of BH formation and decay (a) through two D-branes and superstrings between them (b)

The presence of thermal radiation in the BH immediately creates a puzzle as to the cause of the increase in the entropy of the BH. It is not entirely clear what the entropy of the BH "consists of", since there are no obvious components that could contribute to the infinite increase of entropy by their chaotic movement inside the BH. Since a "black" hole is a bottomless pit in space-time, it is necessary to find some fundamental constituent elements into which the geometry of space-time can be decomposed. It is also extremely interesting that the entropy of a BH is proportional to its area (the square of the radius), and not to the volume (the cube of the radius). In the early 1990s, t Hooft and Susskind suggested that, in a theory combining quantum mechanics and gravity, the number of elementary components needed to comprehensively describe a system is proportional to the area of the surrounding surface in which it is enclosed. From a theoretical point of view, this leads to a radical change in ideas about the world, since it turns out to be possible to describe a closed space-time domain exclusively by the behavior of components located on its outer boundary.

The entropy of the BH is equal to the total entropy of the particles (strings) located on its boundary, which are the "elementary quanta" of the space-time AdS geometry. Then its analytical value is represented by the area of the corresponding five-dimensional surface

$$S = \frac{A_{10}}{4G_N^{10}} = \frac{A_5}{4G_N^5}.$$

Calculations by Maldacena led to the following formula for the entropy of black holes [11]

$$S_e = \sqrt{\pi\left(2\boldsymbol{N_B} + \boldsymbol{N_F}\right)\boldsymbol{EL}/6} = 2\,\pi\sqrt{\boldsymbol{Q}_1\boldsymbol{Q}_5\boldsymbol{N}},$$

$$E_L = N/R_9,\ L = 2\pi\, R_9.$$

It is necessary to pay attention to the fact that it is proportional to the moment P=N/R for Q5 and Q1 branes, where N means the level of energy excitation of a soliton object of the D-brane type. Since, according to the second law of thermodynamics, entropy increases, this indicates an increase in the excitation level of the soliton state, which, when decaying in the low-energy approximation into other particles in the deconfinement phase of the quark-gluon plasma in the context of QCD, generates the release of a large amount of energy. BH can be imagined as two-phase structures double to each other, or it is a plasma of gluons in the deconfinement phase ↔ or it is a "big black hole ~ high T-phase QG" [12].

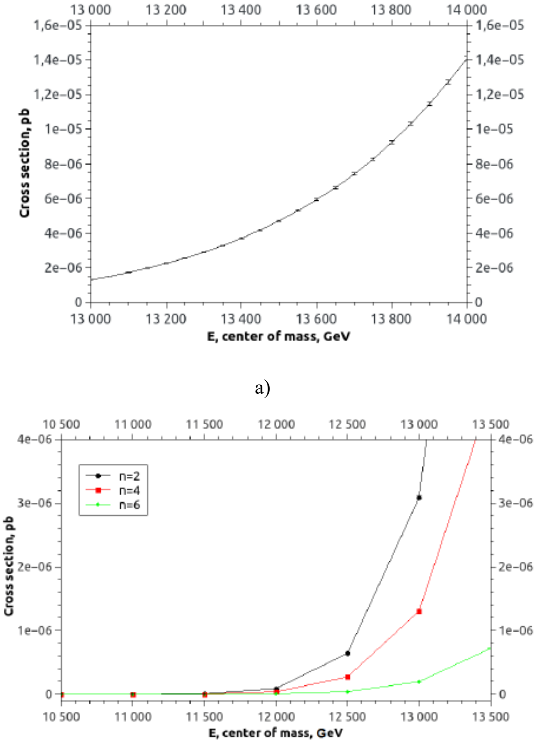
5. Experimental searches for exotic particles at the LHC

The concept of the existence of BH smaller than the mass of the star was proposed in 1971 by Stephen Hawking [13]. Some hypotheses involving additional measurements suggest that micro-black holes may be formed at energies in the TeV range that are available at the LHC [14] For example, microscopic BHs can be born at the LHC collider, since the mass of a microscopic BH M=10 TeV and its size R=10^{-17} cm satisfy the hierarchy formula [15]

$$M_P = M \cdot e^M \pi R.$$

Using the BlackMax program [16] we calculated the cross section of the formation of a non-rotating micro BH for the ADD model (n = 4, M = 10 TeV) and rotating black holes in the energy range of 13-14 TeV [17], Figure 8.

Figure 9 shows experimental searches for exotics at the LHC, [18].



a)

b)

Figure 8. Cross-section of the formation of non-rotating micro BH (a) and rotating micro BH (b) as a function of energy

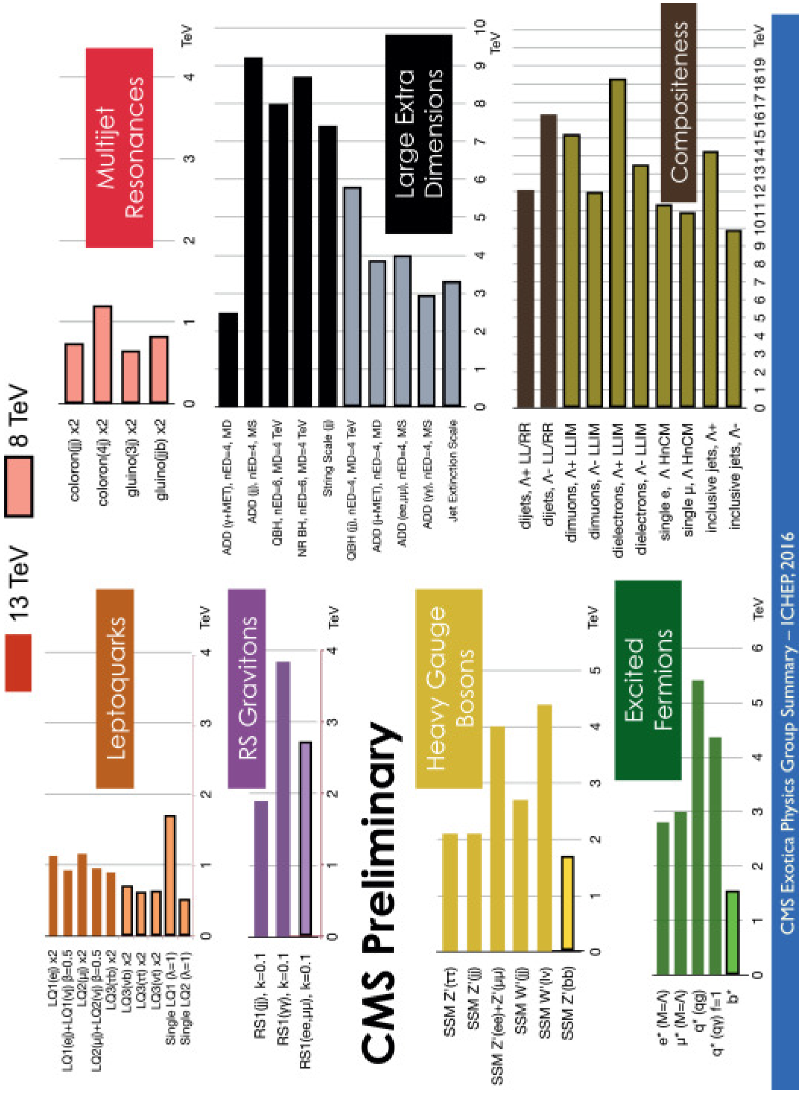


Figure 9. Searches for exotics on the CMS detector

Among the data presented are experiments on the search for the space of extra dimensions, Randall-Sandrum gravitons, heavy gauge bosons, multi-jet resonances, microscopic BH in the form of restrictions on the masses of the corresponding particles.

6. Conclusions

We have shown the possibility of using symmetry groups to consider the Theory of Grand Unification. We presented nuclear theories constructed in accordance with various symmetry groups, as well as the matter content of models with extended gauge groups. String theory provides a large number of the observed features of the Universe. There are signs that string theory includes many of the qualitative features of the SM such as gauge groups and matter content. These are ubiquitous features of D-brane realizations of gauge theories. Both gauge coupling unification and the matter content of the SM hint at the presence of a unified gauge group structure at high energies. Indeed, there is a natural sequence of E-group embeddings which give the SM gauge group and matter structure in an elegant manner.

Modern high energy physics is connected with experimental searches of new physics beyond the SM. Calculations of the production cross sections of rotating, non-rotating ADD microscopic BH using computer simulations and information on the searches for exotics at the LHC at 8 and 13 TeV were presented. The studying of the properties of the new particles predicted by the theories of extra dimensions stimulated us to perform calculations at different parameters and energies within the RS model. Our calculations of $\sigma \times Br$ of gravitons production show that the resonance peak shifts from 5 TeV to 7 TeV with increasing of energy at the colliders from 13 TeV to 14 TeV as well as the absence of peak at energy of 100 TeV at the center of mass energies.

In the framework of the space of extra dimensions, formulas of the energy and entropy of BH were presented and the proportionality of the energy to the number of degrees of freedom of the strings located between the branes, N, which has the meaning of the level of energy excitation of a soliton object of the D-brane type, was shown. Since the entropy increases and this corresponds to an increase in the excitation level of the soliton state, the decay of BH would be accompanied by an energy release greater than the explosion of a hydrogen bomb.

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