ASSESSMENT OF THE RELIABILITY OF STONE STRUCTURES USING MODERN PROBABILISTIC APPROACHES AND NON-DESTRUCTIVE METHODS

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DOI: https://doi.org/10.30525/978-9934-26-473-3-3

Abstract. In this study, the issue of ensuring the reliability of stone structures in modern construction is thoroughly examined, which has gained particular relevance in the context of increasing demands for the durability and safety of buildings. **The subject of this research** is the analysis of various methods for assessing the strength and durability of stone structures, as well as the development of new approaches that take into account the variability of material properties and the impact of operational loads. Traditional assessment methods, such as compression and bending tests, are considered important tools, yet their limitations necessitate the incorporation of more advanced technologies. Non-destructive methods, such as ultrasonic testing, radiography, and infrared thermography, are key to obtaining accurate data without damaging the structures, which is particularly crucial for historical or complex infrastructure objects. **The research methodology** is based on a combined approach, integrating traditional testing with modern non-destructive methods and probabilistic modeling. Specifically, stochastic models were used to analyze the risks and uncertainties associated with loads and material properties. This allows for more reliable predictions regarding the behavior of stone structures under real operational conditions, which is a significant step toward ensuring the longevity of these structures. The aim of the study is

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to develop evaluation methods that will allow for more accurate forecasting of the condition and strength of stone structures, considering the influence of various factors on their durability. The use of probabilistic models in conjunction with non-destructive testing methods helps to minimize the risks of structural failure and supports informed engineering decisions during the design and operational phases. Expanding the application of such approaches will enhance the reliability of building structures, which is of great importance to modern construction engineering. **The conclusions of the paper** highlight that integrating different research methods provides a comprehensive analysis of the condition of stone structures, improving the accuracy of their strength and durability assessments. It is recommended to regularly apply non-destructive methods for monitoring the condition of buildings, as well as continuously update models based on new data. This will ensure the longevity of stone structures and help avoid unexpected failures and collapses during operation. The monograph makes a significant contribution to the development of construction engineering, offering practical recommendations to ensure the reliability and safety of stone structures, which is crucial for their continued use.

1. Introduction

The assessment of the reliability and strength of stone structures is one of the key challenges in modern construction, particularly in light of the global changes affecting building operation conditions. The use of advanced technologies and analysis methods to determine the strength of stone masonry and its reliability during the reconstruction or modernization of old buildings is a pressing task for engineers and scientists. This topic has gained particular importance due to the increasing safety demands in seismic activity zones and the influence of new man-made factors, such as changes in soil structure caused by anthropogenic processes [11, pp. 5-8].

Stone structures, especially those erected decades or even centuries ago, remain the foundation of many urban infrastructures around the world. Their durability and aesthetic appeal ensure their popularity in contemporary architectural settings. However, climate change, seismic activity, military actions, and other external factors can significantly reduce the reliability of these structures. Therefore, determining the strength and safety of such buildings using modern assessment methods, such as probabilistic approaches and stochastic modeling, is essential to ensuring their longevity and reliability. This highlights the relevance of the research [12, pp. 8-11].

The main scientific problems addressed in this work are as follows:

Insufficient accuracy of traditional methods for assessing the strength of stone structures, which are based on deterministic approaches and do not account for all possible risks and uncertainties.

Lack of adequate methods for evaluating the condition of old stone structures, which were built according to different construction standards and have been exposed to external factors over long periods of operation.

The need for integrating advanced non-destructive testing technologies and mathematical modeling into the processes of strength and reliability assessment of stone structures.

The primary goal of the research is to develop and improve methodologies for evaluating the strength of masonry, which will enhance the accuracy and reliability of assessing the condition of existing buildings, particularly older structures, in the context of modern safety requirements. To achieve this goal, a combination of traditional and advanced methods is proposed, including probabilistic approaches, stochastic modeling, and non-destructive testing techniques.

The tasks of the research include: conducting an analysis of existing methods for assessing the strength of stone structures, identifying their limitations and shortcomings, implementing and adapting existing probabilistic approaches, such as stochastic modeling and the Monte Carlo method, to account for the variability of materials and the uncertainty of loads when assessing the strength of stone structures, exploring the potential of using non-destructive testing methods for evaluating the condition of stone structures without damaging the material, providing recommendations for the application of new methods of strength and reliability assessment in modern construction practices.

2. Methods for Assessing the Strength and Reliability of Stone Structures

The assessment of the strength and reliability of stone structures is a complex and multifactorial process that requires the use of various approaches and tools. To achieve the most accurate and reliable results, this study proposes the application of both traditional and modern methods, including non-destructive technologies and probabilistic approaches.

Destructive methods involve extracting samples from existing stone structures and conducting strength tests under laboratory conditions. These tests include compression, bending, and tensile tests, which allow the determination of the main mechanical properties of the material, particularly bricks and mortar. While these methods are the most accurate, their application may be limited due to the risk of damaging the building, especially in the case of old or historically significant structures [4, pp. 25-33; 7, pp. 56-61].

To minimize the risks of damaging structures and to preserve their integrity, non-destructive testing methods are employed. The main non-destructive methods used in this study include ultrasonic testing, radiographic methods, and acoustic emission methods.

Ultrasonic testing is based on measuring the velocity of ultrasonic waves passing through the material. It allows for the evaluation of material homogeneity and the detection of internal defects such as cracks or voids that may reduce the strength of the structure [6, pp. 5-9]. Ultrasonic testing is particularly useful for assessing the condition of old structures where traditional methods are difficult to apply.

Radiographic methods use X-rays or gamma rays to create images of the internal structure of stone materials. This enables the detection of hidden defects such as voids or areas of reduced density, which can significantly impact the strength and reliability of the structure.

The acoustic emission method is based on the registration of acoustic waves generated in the material during loading. It helps identify and locate areas of initial defects and damage, as well as assess their impact on the overall reliability of the structure.

Traditional methods of strength assessment are often based on *deterministic approaches* that only consider the average values of material properties and loads [5, pp. 160-167]. However, this approach may not account for all potential risks and uncertainties that may arise during the operation of buildings.

To improve the accuracy of assessments and account for the stochastic nature of loads and material properties, probabilistic methods and stochastic modeling are proposed. The main tools used include:

The Monte Carlo numerical method, which allows for the simulation of structure behavior under random conditions through repeated experiments with random variables. This method helps estimate the probability of various failure scenarios and identify the most critical safety factors.

Stochastic analysis involves using models that consider the variability of materials and loads, allowing for the determination of the range of possible values for the strength and reliability of the structure. This provides a more comprehensive risk assessment and supports better-informed decisions during the design and operation of buildings.

Probability of failure (Pf) assessment: This method determines the probability of structural failure using an integral approach, accounting for random fluctuations in both material properties and applied loads. This allows for conservative decisions regarding the need for structural reinforcement or modernization.

To obtain objective results, *statistical analysis of data* obtained from laboratory and field studies is conducted. *The variability of material strength* is analyzed by determining the coefficients of variation for key material parameters, such as the compressive strength of bricks and mortar. This accounts for potential deviations in material quality and reduces risks during calculations.

Regression analysis methods are used to establish relationships between material and structural strength parameters. This helps identify key factors affecting the reliability of stone structures and develop appropriate predictive models.

The construction of histograms and probability distribution analysis of the obtained data allows the identification of whether the studied materials conform to normal or log-normal distributions, which is critical for the correct use of formulas and models in subsequent calculations.

Thus, the use of a combined approach, which includes both traditional and modern methods, provides a comprehensive assessment of the strength and reliability of stone structures, minimizing risks and accounting for all possible uncertainties. This supports more informed decisions during the reconstruction and operation of buildings, ensuring their longevity [13, pp. 56-60; 22, pp. 10-15].

3. Main Part

3.1. Traditional Assessment Methods

Traditional methods for assessing the strength of stone structures are based on the use of classic engineering approaches, which have been applied for a long time to determine the physical and mechanical properties of materials, particularly their compressive, tensile, bending, and shear strengths. These methods include both laboratory and field tests, allowing for the acquisition of quantitative indicators necessary for assessing the condition of a structure and predicting its behavior under load.

Compressive strength is one of the most important parameters that determines the load-bearing capacity of stone structures. The compressive test is traditionally conducted by applying a controlled axial load to a material sample in the form of a cube or cylinder until it fails. The samples typically have standard dimensions: for cubes – 150x150x150 mm, for cylinders – a diameter of 150 mm and a height of 300 mm.

The testing process is regulated by relevant standards, such as GOST 10180-2012 and ISO 1920-4:2005, which set requirements for the shape and dimensions of the samples, loading rate, and testing conditions. Compressive strength is calculated using the following formula:

$$\sigma_c = \frac{P}{A}, \tag{1}$$

σ_c – compressive strength, MPa;

P – maximum load leading to sample failure, N;

A – cross-sectional area of the sample, m².

Expression (1) provides the primary strength value of the material, which is then used to assess the structure's ability to withstand operational loads. It should be noted that test results may vary significantly depending on the size and shape of the sample, as well as the testing conditions, such as humidity and ambient temperature [3, pp. 122-129].

Tensile testing is less common for stone structures but is relevant for materials used in combination with masonry, such as in composite materials. Tensile strength is determined using samples extracted from the structure's material or specifically made for testing and tested by applying an axial load until they rupture:

$$\sigma_t = \frac{P_t}{A_0} \tag{2}$$

σ_t – tensile strength, MPa;
P_t – maximum load before rupture, N;
A_0 – initial cross-sectional area of the sample, m².

Bending strength is determined by loading a sample placed on two supports, either by applying a load in the middle (three-point bending) or at equal distances between three points (four-point bending). This test is particularly relevant for examining masonry, where the material's resistance to tensile forces in the bending zone can be assessed:

$$\sigma_f = \frac{3PL}{2bd^2} \quad (3)$$

σ_f – bending strength, MPa;
P – applied load, N;
L – distance between supports, m;
b – sample width, m;
d – sample height, m.

Shear strength is a critical parameter for stone structures, especially when they are subjected to complex loads. Shear testing is usually performed on prismatic samples made of brick or stone by applying a tangential load that causes one layer of material to slide relative to another:

$$\tau = \frac{P_S}{A_S} \quad (4)$$

τ – shear strength, MPa;
P_s – applied shear load, N;
A_s – cross-sectional area of the sample, m².

This parameter is particularly important for assessing the stability of structures under shear forces, which may occur during earthquakes or other dynamic impacts [15, p. 5-7].

To account for the influence of sample geometry, testing conditions, and other factors, correction coefficients are used in the results. For example, correction factors recommended by standards are applied to convert the results from cubic samples to cylindrical ones.

$$\sigma_{c,cyl} = K \cdot \sigma_{c,cube} \quad (5)$$

$\sigma_{c,cyl}$ – compressive strength of a cylindrical sample, MPa;
$\sigma_{c,cube}$ – compressive strength of a cubic sample, MPa;
K – correction coefficient accounting for geometric differences.

According to research, the correction coefficient for converting cubic to cylindrical samples typically ranges from 0.8 to 0.9, depending on the type of material and testing conditions [16, pp. 1282-1293; 19, pp. 67-89].

Although traditional methods for assessing the strength of stone structures remain fundamental for determining load-bearing capacity, they have *several limitations*. For instance, destructive methods require the extraction of samples from the structure, which can compromise its integrity, particularly in historical buildings. Additionally, results can be inaccurate due to differences in testing conditions and the material properties under actual operational conditions.

Another limitation concerns the interpretation of test results: the mechanical properties of materials often exhibit significant variability due to heterogeneous composition, which can lead to erroneous conclusions when using average values without accounting for variations [9, pp. 1-5; 20, pp. 193-199].

Traditional methods for assessing the strength of stone structures provide essential data for calculating their load-bearing capacity and predicting behavior under load. However, they have limitations that require caution when interpreting results and applying them to real structures. Further refinement of these methods, as well as their integration with modern technologies, can significantly improve the accuracy and reliability of assessments.

3.2. Modern Non-Destructive Methods

In modern construction and renovation of stone structures, where the integrity of the building is critical, non-destructive testing methods have become essential tools for assessing their strength and reliability. These methods provide valuable information about the internal condition of materials without the need for sample extraction or disruption of the object’s structure. They offer high accuracy and reliability of data and are indispensable for working with historical or architecturally significant structures.

Ultrasonic testing is one of the most widespread methods of non-destructive evaluation. This method is based on measuring the speed of ultrasonic waves passing through the material, allowing for the assessment of its homogeneity, the presence of cracks, and other defects. The wave's velocity through the material depends on its density and modulus of elasticity, thus allowing indirect evaluation of its strength:

$$v = \sqrt{\frac{E}{\rho}} \tag{6}$$

v – velocity of the ultrasonic wave, m/s;

E – modulus of elasticity of the material, Pa;

ρ – density of the material, kg/m³.

One of the key aspects of ultrasonic testing is the ability to evaluate material heterogeneity. For example, the presence of voids or cracks in the masonry reduces the wave speed, enabling the localization of these defects and an assessment of their impact on the overall strength of the structure. Ultrasonic testing is also widely used to determine the degree of material degradation due to external factors, such as moisture, temperature changes, or chemical agents. This method makes it possible to evaluate the extent to which a stone structure has lost its original properties and whether reconstruction work is necessary [3, pp. 122-129; 17, pp. 205-213].

Radiography, which uses X-rays or gamma rays, is another important tool in the arsenal of non-destructive methods. This method enables the visualization of the internal structure of the material, which is particularly useful for detecting hidden defects such as cracks, voids, or areas of reduced density. Radiography requires specialized equipment and careful attention to safety, as the use of ionizing radiation can be hazardous to health. However, its high precision and ability to detect even microscopic defects make it invaluable for working with critically important or historically significant structures.

$$I = I_0 e^{-\mu x} \tag{7}$$

I – intensity of the radiation after passing through the material, W/m²;

I_0 – initial radiation intensity, W/m²;

μ – linear attenuation coefficient, 1/m;

x – material thickness, m.

The results of radiographic studies can be used to quantitatively assess the identified defects and calculate their impact on the structure's strength. Radiography also allows quality control of construction work, such as checking the quality of joints in complex structures [19, pp. 67-89; 21, pp. 193-199].

Acoustic emission involves the release of energy in the form of waves during the formation of new cracks or the expansion of existing defects in the material under load. The acoustic emission method allows for the detection and localization of active defect zones in the material that may lead to its failure. Acoustic emission is an extremely sensitive technique, capable of detecting even minor changes in the material's structure. Typically, multiple sensors are placed on the surface of the structure during testing to capture signals generated by the formation or expansion of cracks.

The intensity of acoustic emission is determined as follows:

$$AE = \sum_{i=1}^{n} A_i \tag{8}$$

AE – acoustic emission intensity, arbitrary units;

A_i – amplitude of the signal from the i-th emission source;

n – number of emission sources.

Analyzing acoustic emission intensity helps assess the load level at which material failure occurs and predict its remaining service life. This method is particularly effective for real-time monitoring of structures, such as during load testing or operation [6, pp. 6-10].

Infrared thermography is based on measuring the infrared radiation emitted by the surface of the material. This method enables the detection of defects, such as cracks or voids, by analyzing temperature anomalies on the surface of the structure. The Planck law for a radiating body can be represented as:

$$E(\lambda, T) = \frac{2\pi hc^2}{\lambda^5} \cdot \frac{1}{e^{\frac{hc}{\lambda kT}} - 1} \tag{9}$$

$E(\lambda, T)$ – spectral energy density of radiation, W/m³;

λ – wavelength, m;

T – surface temperature, K;

h – Planck's constant, J·s;

c – speed of light, m/s;

k – Boltzmann's constant, J/K.

Infrared thermography is particularly useful for investigating objects exposed to external influences, such as liquids, as moisture alters the thermal properties of the material, making it possible to identify areas with potential defects.

Laser scanning is a high-tech method that allows the creation of a three-dimensional model of the surface of a structure with high accuracy. This method is widely used for assessing the geometry and deformations of stone structures, especially for monitoring their condition over a long period. Laser scanning provides extensive data about the surface of an object, allowing the detection of even slight changes in its geometry, such as displacements or crack formation. The scanning results can be used to analyze deformation processes and assess the residual strength of the structure. The optical range for laser scanning is defined as:

$$R = \frac{c \cdot t}{2} \quad (10)$$

R – distance to the object, m;

c – speed of light, m/s;

t – time required for the signal to travel back and forth, s.

Laser scanning is particularly useful for monitoring complex architectural elements, such as arches or domes, where preserving accurate geometry is essential when assessing the object's condition [1, pp. 256-260].

Modern non-destructive testing methods significantly expand the capabilities of engineers and scientists in assessing the condition of stone structures. These methods provide accurate and reliable data on the internal state of materials, which is especially important for preserving historically and architecturally significant buildings. Their application helps minimize the risk of structural failure, enhance the efficiency of reconstruction work, and ensure the longevity of buildings.

3.3. Monte Carlo Method

The Monte Carlo method, one of the most powerful and versatile tools in the modern arsenal of engineers and scientists, allows for solving a wide range of complex problems associated with the uncertainty and randomness of parameters in construction structures. This method, based on the principles of stochastic modeling, provides the ability to simulate and analyze processes and systems involving multiple uncertainties by repeatedly randomly selecting input parameters and statistically analyzing the obtained results.

The Monte Carlo method involves using random numbers to simulate processes or systems with random parameters. In the context of assessing the strength of stone structures, this approach allows for considering material variability, non-uniformity of loads, and possible defects in the structure. The core idea of the method is to repeatedly simulate (run simulations) with different sets of input parameters generated randomly.

Applying the Monte Carlo method to assess the reliability of stone structures involves several key stages. First, the problem is formulated by defining the main characteristics of the system or process that need to be studied. These could be mechanical properties of the material, geometric parameters of the structure, loads, and other variables affecting its behavior. Then, a mathematical model is created that describes the relationship between the input parameters and the output characteristics of the system. This model can include equilibrium equations, material laws, or other analytical expressions. Next, a random number generator is used to create a set of input parameters. For example, in evaluating the strength of a stone structure, random values of material strength, load, or geometric characteristics may be generated. After this, the process or system is simulated for each set of random variables. This may include numerically solving differential equations, analyzing the stress-strain state of the structure, or other calculations. After the simulations are completed, a statistical analysis of the results is conducted. This allows for determining the probability of failure, stress distribution, or other important system characteristics [8, p. 28].

The Monte Carlo method involves constructing a probabilistic model that describes the system's behavior under the random distribution of input parameters. Let X be a random variable describing the system's input parameter, then the distribution of this variable can be written as $f_X(x)$, i.e., the probability density function. Using the Monte Carlo method $Y=g(X)$, is determined, the distribution function of the output variable, describing the dependence of the output on the input parameters. The solution to the problem involves discretizing the integral and replacing it with the arithmetic mean:

$$E\left[Y\right]\approx\frac{1}{N}\sum_{i=1}^{N}g\left(X_i\right) \tag{11}$$

N – the number of simulations;

X_i – the value of the random variable X obtained in the i-th simulation.

In civil engineering, the Monte Carlo method is used to assess the reliability of structures by simulating possible scenarios of their behavior under various operating conditions. For example, one can model possible combinations of loads, taking into account their random nature, and assess how these loads affect the stress-strain state of the stone masonry. The Monte Carlo method is particularly useful in evaluating the strength of old stone buildings, which are affected by complex and unpredictable factors such as soil shifts, temperature fluctuations, or humidity. The method allows for estimating the probability of failure in such buildings and identifying the critical factors that most affect their reliability [23, pp. 577-584].

For more accurate analysis, stochastic models are used, which take into account not only the basic mechanical properties of the material but also the variability of these properties over time and space. This is especially important for materials with high heterogeneity, such as natural stone. The probability of failure using the Monte Carlo method is determined as follows:

$$P_f \approx \frac{1}{N}\sum_{i=1}^{N} I\left(g\left(X_i\right) > Y_{crit}\right) \quad (11)$$

P_f – the probability of failure;

N – the number of simulations;

I(·) – an indicator function equal to 1 if the condition is met, and 0 if not;

Y_{crit} – the critical value of the output variable [10, pp. 507-518].

Consider *an example of assessing the reliability* of a stone arch subjected to random loading, which varies from 10 kN to 50 kN. The probability that the maximum stresses in the material will exceed the permissible value Y_{crit} = 2 MPa.

Step 1: Define the probability density function for the load X~Uniform(10,50).

Step 2: Define the function that describes the relationship between stress and load: *g(X)=X/A*, where *A* – the cross-sectional area of the arch.

Step 3: Generate random values of load X_i and calculate the corresponding stress values $g(X_i)$.

Step 4: Estimate the probability of failure according to equation (11).

This approach allows engineers to assess the reliability of the structure, considering all possible scenarios, which can be critical for decision-making regarding the further operation or reconstruction of the building [13, p. 954-967]. The Monte Carlo method is a versatile tool for analyzing complex engineering systems that involve random parameters. Its application in the strength assessment of stone structures enables the consideration of various factors affecting their reliability and ensures well-founded decisions regarding their future use. This method provides a highly accurate assessment of the probability of structural failure and helps to identify measures to enhance reliability.

3.4. Stochastic Modeling

Stochastic modeling is one of the most powerful approaches for analyzing complex engineering systems characterized by random parameter changes and has broad applications in the study of the strength and reliability of stone structures. This method allows for the consideration of variability and uncertainty in material properties, loads, and other factors affecting the behavior of building structures. Unlike deterministic methods, which consider fixed parameter values, stochastic modeling accounts for their random nature, enabling more realistic and reliable results. Stochastic modeling involves the use of probabilistic models to describe the uncertainty in parameters and processes that influence the behavior of structures. This may include randomness in the distribution of material properties, variable loads, environmental influences, and other factors. Stochastic models are based on statistical data and probabilistic distributions, which describe variations in input parameters.

Let X – be a random variable that describes a material parameter, such as compressive strength. The probability distribution for X can be described by the probability density function $f_X(x)$. Stochastic modeling allows for determining how changes in input parameters affect the output quantity, such as the stress in masonry Y, which can be expressed as a function of random parameters: $Y=g(X)$. The probability of failure is then determined as:

$$P_f = P\left(Y > Y_{crit}\right) = \int_{Y_{crit}}^{\infty} f_Y\left(y\right)dy \tag{12}$$

P_f – the probability of failure;
Y_{crit} – the critical stress value;

fY(y) — the probability density function for the output variable *Y*.

Among the main methods of stochastic modeling, the following can be distinguished:

Method of moments: This method uses distribution moments (mean, variance, skewness, etc.) to build approximation models that describe the behavior of output parameters. It allows for estimating the probability of structural failure without the need for full-scale simulation modeling. The variance of a random variable is defined as follows:

$$P_f = P\left(Y > Y_{crit}\right) = \int_{Y_{crit}}^{\infty} f_Y\left(y\right)dy \tag{13}$$

Var(Y) – the variance of the output variable;

E[Y] – the expected value of the output variable;

E[Y2] – the expected value of the square of the output variable.

Linear regression method: This method is used to build a linear model of the relationship between input random parameters and the output quantity. Linear regression allows for determining the influence coefficients of each parameter on the output variable and estimating uncertainty in predictions:

$$Var\left(Y\right) = E\left[Y^2\right] - \left(E\left[Y\right]\right)^2 \tag{14}$$

Y – the output variable;

$X_1,\ldots,X_n$ – the input parameters;

$\beta_0,\ldots,\beta_n$ – the regression coefficients;

ϵ – the residual error.

Monte Carlo method: As discussed earlier, this method is a universal tool for stochastic modeling, allowing the simulation of the behavior of a structure under different sets of random parameters. The Monte Carlo method is widely used for assessing failure probability and analyzing the impact of uncertainties on the reliability of structures.

In the context of stone structures, stochastic modeling allows for accounting for variability in material properties such as strength, modulus of elasticity, and density. For example, natural stone, commonly used in construction, is a heterogeneous material with significant variations in properties within the same sample. This may be due to natural defects such as cracks, voids, dislocations, or inclusions of other minerals.

Stochastic models used to assess the strength of stone structures may include factors such as material heterogeneity, random loads, environmental impact, material wear, etc. One of the important tasks of stochastic modeling is to assess the reliability of a structure by considering all these factors. For example, to model the strength of masonry, a stochastic model that accounts for variations in the strength of bricks and mortar, as well as randomness in load distribution, can be used:

$$\sigma = \frac{f_b A_b + f_m A_m}{A_b + A_m} \tag{15}$$

σ – the stress in the masonry;

f_b, f_m – the strengths of brick and mortar, respectively;

A_b, A_m – the areas of brick and mortar, respectively.

The stochastic nature of input parameters results in randomness in the output stresses, which can be described by a probabilistic distribution. Using stochastic modeling methods, it is possible to determine the probability that the stress in the masonry will exceed the critical value and thus estimate the likelihood of structural failure [14, p. 34-45].

In addition to basic methods, stochastic modeling can involve more sophisticated approaches, such as methods based on random field processes or Markov chains. These approaches allow for the consideration of spatial correlation between random parameters, which can be critical when analyzing large or complex structures. The correlation function of a random field is expressed as follows:

$$R(x_1, x_2) = E\left[\left(X(x_1) - E[X]\right)\left(X(x_2) - E[X]\right)\right] \tag{16}$$

$R(x_1, x_2)$ – the correlation function between two points in space;

$X(x_1)$, $X(x_2)$ – the values of the random variable at points x_1 and x_2;

$E[X]$ – the expected value of the random variable.

The application of such methods enables more accurate modeling of structural behavior, especially in cases where random variations in parameters have a significant spatial impact, such as in the analysis of heterogeneous materials or large infrastructure objects [1, pp. 261-271].

Stochastic modeling is a key tool for analyzing complex engineering systems characterized by a high level of uncertainty. In the context of assessing the strength of stone structures, this method allows for the consideration of variability in material properties and operational conditions, providing a more precise and reliable evaluation of structural reliability. The use of advanced stochastic modeling methods, such as random fields or

Markov processes, enables the consideration of spatial correlation and other complex effects, making these approaches highly beneficial for modern construction practices.

3.5. Regression Analysis

Regression analysis is an important mathematical tool widely used to identify relationships between variables and build predictive models in many fields, including construction and engineering sciences. In the context of assessing the strength and reliability of stone structures, regression analysis allows for the examination of the influence of various factors on the strength of materials and structures, as well as the prediction of their behavior under load.

The goal of regression analysis is to establish a mathematical relationship between one or more independent variables (factors) and a dependent variable (result). In simple linear regression, the relationship between variables is described by a linear equation:

$$Y = \beta_0 + \beta_1 X + \epsilon \tag{17}$$

Y – dependent variable (result),

β_0 – intercept of the regression equation (intersection with the Y-axis),

β_1 – regression coefficient indicating the impact of the independent variable X on the result,

X – independent variable (factor),

ϵ – random error accounting for other influences on the result that are not included in the model.

This approach can be extended to multiple linear regression, where the dependent variable depends on several independent variables:

$$Y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \ldots + \beta_n X_n + \epsilon \tag{18}$$

Y – dependent variable (result),

$X_1, X_2, \ldots, X_n$ – independent variables (factors),

$\beta_0, \beta_1, \ldots, \beta_n$ – regression coefficients,

ϵ – random error.

In studies of the strength of stone structures, regression analysis is used to establish relationships between the physical and mechanical properties of materials and various factors, such as moisture, density, type of load, temperature regime, or other operating conditions. For example, one might consider a case where the dependent variable Y represents the compressive strength of masonry, and the independent variables X_1, X_2,…, X_n – are factors affecting strength, such as material moisture, ambient temperature, curing time, and others. Regression analysis allows for the assessment of the impact of each factor on strength, leading to conclusions about optimal operating conditions or the need for additional reinforcement measures.

$$\sigma_c = \beta_0 + \beta_1 W + \beta_2 T + \beta_3 t + \epsilon \quad (19)$$

σ_c – compressive strength of masonry,

W – material moisture,

T – ambient temperature,

t – curing time,

β_0, β_1, β_2, β_3 – regression coefficients.

This model enables the prediction of changes in masonry strength depending on variations in moisture, temperature, and other factors. It is also important to note that the regression coefficients β_0, β_1, β_2, β_3 not only indicate the direction of the factors' influence (positive or negative) but also quantify this influence.

Several criteria are used to evaluate the quality of the regression model, including the coefficient of determination R_2, statistical tests for the significance of coefficients, and residual analysis.

$$R^2 = 1 - \frac{\sum_{i=1}^{n}\left(y_i - \hat{y}_i\right)^2}{\sum_{i=1}^{n}\left(y_i - \bar{y}\right)^2} \quad (20)$$

R_2 – coefficient of determination,

y_i – observed value of the dependent variable,

$\hat{y}_i$ – predicted value of the dependent variable by the model,

$\bar{y}$ – mean value of the dependent variable.

The coefficient of determination R_2 indicates the proportion of the variation in the dependent variable explained by the model. R_2, value close to 1, suggests a high-quality model, as a large portion of the variation is explained by the independent variables. Additionally, residual analysis – the difference between observed and predicted values – helps identify systematic errors in the model and allows for adjustments if necessary.

While linear regression is a fundamental tool for analyzing dependencies, real physical processes often have a non-linear nature. In such cases, non-linear regression models can be applied to more accurately reflect complex relationships between variables:

$$Y = \beta_0 + \beta_1 X_1^2 + \beta_2 \log\left(X_2\right) + \epsilon \qquad (21)$$

Y – dependent variable,

X_1, X_2 – independent variables,

β_0, β_1, β_2 – regression coefficients,

ϵ – random error.

Non-linear models may include variables raised to a power, logarithmic, or exponential relationships, allowing for a more accurate representation of complex processes, such as material wear, cumulative effects of loads, or environmental impacts.

Let's consider a specific example of using regression analysis to evaluate the strength of masonry under the influence of variable humidity and temperature. Based on experimental studies, a regression model was developed that takes into account the effects of these factors:

$$\sigma_c = 30 - 0.5W + 0.3T + 0.01T^2 + \epsilon \qquad (22)$$

σ_c – compressive strength of the masonry, MPa,

W – material humidity, %,

T – ambient temperature, °C,

ϵ – random error.

This model shows that as humidity increases, the strength of the masonry decreases, which may be related to the deterioration of adhesion between the brick and the mortar. Conversely, as temperature increases, strength rises; however, at very high temperatures, nonlinear effects begin to emerge, which are also accounted for in the model. This is critically important when developing recommendations for material use in different climate zones or when designing buildings that will be exposed to variable weather conditions. Residual analysis and significance testing of the model's coefficients allow conclusions to be drawn about the reliability of the obtained results and the need for additional experimental studies.

Although regression analysis provides powerful tools for forecasting and analysis, it is important to understand that any model is based on the data available at the time of its creation. Therefore, to maintain the accuracy and relevance of the regression model, it must be regularly updated with new data. This is especially crucial in the context of stone structures, which may be subjected to long-term effects of various factors such as wear, changes in climate conditions, increased humidity levels, or the acidity of the surrounding environment. Regression analysis can be particularly useful during routine inspections of buildings, allowing for the timely identification of potential issues and forecasting the need for repair or reinforcement work. For instance, if data on humidity and temperature regimes are updated annually, engineers can more accurately assess the current condition of structures and make informed decisions about their operation.

Modern software tools significantly simplify the process of conducting regression analysis. Programs like MATLAB, SPSS, and Python with libraries such as scikit-learn or statsmodels provide extensive capabilities for analyzing large data sets and building complex models [16, pp. 1282-1293]. These tools allow for automatic optimization of model parameters, evaluation of the significance of each factor, residual analysis, and interpretation of results.

Software tools also offer the possibility of automating the model update process, which is essential for long-term projects. For example, using software for building condition monitoring enables real-time data collection and analysis, greatly improving the accuracy of forecasts and facilitating timely decision-making [2, p. 895].

3.6. Histograms and Probability Distributions

Histograms and probability distributions are essential tools in statistical analysis, enabling the study of data distribution and the assessment of the likelihood of certain events. In the context of civil engineering, these methods are applied to analyze the physical and mechanical properties of materials, including the strength of stone structures, and to predict their behavior under various conditions. These tools provide the means to visualize and quantitatively assess the variability of parameters, which is critically important for making informed engineering decisions.

A histogram is a graphical representation of data distribution, allowing for a visual evaluation of the frequency (number of occurrences) of certain variable values within given intervals. This powerful tool enables engineers and researchers to quickly gain insights into the data distribution, identify anomalies or unusual behavioral patterns, and assess the symmetry or asymmetry of the distribution. For example, in civil engineering, histograms are often used to visualize the strength distribution of stone masonry samples tested under compression. This allows the evaluation of the uniformity of the samples and the identification of potential defects that may impact the overall structural strength.

$$f_i = \frac{n_i}{N} \tag{23}$$

f_i – frequency of the value in interval i,

n_i – number of observations in interval i,

N – total number of observations.

Histograms help identify different types of data distributions, such as the normal distribution, Poisson distribution, exponential distribution, and others. Understanding which type of distribution best describes the data is crucial for further analysis and modeling, as different distributions may require distinct approaches to result interpretation and decision-making.

A probability distribution is a mathematical description of how random variables are distributed over a range of values. This is a key concept in statistics, allowing the determination of the likelihood of certain events or variable values. In civil engineering, probability distributions are used to assess the reliability of structures and to determine the probability of their failure or exceeding permissible stress values. This can be captured through a probability density function for a continuous random variable:

$$f_X(x) = \lim_{\Delta x \to 0} \frac{P(x \le X < x + \Delta x)}{\Delta x} \tag{24}$$

$f_X(x)$ – probability density function of the random variable X,

$P(x \le X < x + \Delta x)$ – probability that the variable X falls within the interval $[x, x+\Delta x)$,

Δx – width of the interval.

The most common type of distribution is the normal distribution, which is characterized by its symmetrical "bell" shape and is defined by two parameters: the mean (expected value) and the variance (standard deviation). In cases where data follows a normal distribution, engineers can predict the behavior of structures under various factors with a high degree of accuracy:

$$f_X(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}} \tag{25}$$

μ – mean value,

σ – standard deviation,

x – value of the random variable X.

The normal distribution is significant in civil engineering because many physical processes, such as the strength distribution of materials, tend to follow this distribution. This allows engineers to apply standard statistical methods to assess the reliability and durability of structures.

Histograms and probability distributions can be used to analyze the results of material tests, such as the compressive strength of masonry. For example, constructing a histogram based on strength data can demonstrate how closely the test results follow a normal distribution. If the results show significant deviations from a normal distribution, this may indicate the presence of defects in the samples or variables that were not considered during testing. Furthermore, probability distributions can be used to assess the likelihood of certain events occurring, such as the failure of a structure under a load exceeding a certain threshold. An important tool for this is the cumulative distribution function (CDF), which shows the probability that a random variable *X* will take a value less than or equal to a given value:

$$F_X(x) = P(X \le x) = \int_{-\infty}^{x} f_X(t)dt \quad (26)$$

$F_X(x)$ – cumulative distribution function,

$f_X(t)$ – probability density function of random variable *X*.

This approach allows engineers to assess the risks associated with the operation of structures and to develop appropriate measures to mitigate these risks. For example, based on the cumulative distribution function, the probability that the maximum load on masonry will exceed the allowable value can be determined, enabling timely measures to strengthen the structure or reduce the load.

One of the key advantages of using histograms and probability distributions is their ability to visually represent data and help identify patterns and anomalies. This allows for quick identification of problems and data-driven decision-making. Histograms are simple to construct and interpret, making them extremely useful in engineering practice.

However, there are certain limitations. Histograms can be sensitive to the choice of interval width, which can affect the accuracy and reliability of the results. Additionally, probability distributions such as the normal distribution may not always accurately reflect real data, especially in cases where the distribution is skewed or has heavy tails.

An important task is also the correct determination of the type of distribution that best describes the data. Various methods, such as the Kolmogorov-Smirnov test or the Shapiro-Wilk test can be used to assess the fit of empirical data to a specific theoretical distribution. If the test results indicate that the data do not follow a normal distribution, it may be appropriate to consider alternative distributions such as the log-normal, Weibull, or Gamma distributions. The Shapiro-Wilk test is presented as follows:

$$W = \frac{\left(\sum_{i=1}^{n} a_i x_{(i)}\right)^2}{\sum_{i=1}^{n}\left(x_i - \bar{x}\right)^2} \tag{27}$$

W – Shapiro-Wilk test statistic;

a_i – constants dependent on the mean and covariance matrix of the normal distribution;

$x_{(i)}$ – ordered sample values;

x_i – observed values;

$\bar{x}$ – sample mean.

This test allows for the verification of the hypothesis of normality in the data distribution, which is an important step in selecting an appropriate statistical method for further analysis.

Let us consider an example of using histograms and probability distributions to analyze the results of masonry strength tests. After conducting a series of tests, a data set representing the compressive strength of samples was obtained. First, a histogram of the strength distribution is constructed to determine whether the data follow a normal distribution or contain anomalies or deviations. If the histogram shows an approximately symmetric distribution resembling a "bell shape," it can be assumed that the data follow a normal distribution. However, to confirm this hypothesis, the Shapiro-Wilk test is performed. If the test shows that the hypothesis of normality is not rejected, methods assuming normality can be used for further analysis, such as estimating the mean and standard deviation to predict the material's behavior under load. The probability of exceeding the allowable load is estimated as follows:

$$P\left(X > x_{crit}\right) = 1 - F_X\left(x_{crit}\right) \quad (28)$$

$P(X>x_{crit})$ – the probability that the random variable X will exceed the critical value xcrit;

$FX(x_{crit})$ – cumulative distribution function for the critical value.

This probability can be used to assess the risk of structural failure during operation, which is important for decision-making regarding the continued use or reconstruction of the building.

One of the important aspects of using probability distributions in civil engineering is their implementation in the design process. Considering random variables in modeling and design allows for the creation of more reliable and durable structures, as they take into account not only the average values of parameters but also their variability.

For example, during the design of a bridge or a multi-story building, probability distribution can be employed to predict the maximum load that may be applied to the structure over its operational lifetime. Considering potential extreme load values allows designers to anticipate the need for additional reinforcements or the selection of stronger materials. Beyond the design phase, histograms and probability distributions are invaluable tools during the operation and monitoring of existing structures. They enable the analysis of data obtained from sensors and other measurement devices to detect changes in the structure’s behavior that may indicate damage or a reduction in strength. The probability distribution of stress values in key structural elements can be used to identify deviations from normal operating conditions. If a shift in the distribution or an increase in variance is observed, it may indicate the need for a more detailed inspection or even urgent repairs.

Histograms and probability distributions are crucial tools in the engineer's toolkit, allowing the analysis of the variability of material and structural parameters, predicting their behavior under load, and assessing the probability of critical events. The use of these methods at all stages – from design to operation – contributes to the reliability and safety of construction structures. Applying histograms and probability distributions in combination with other statistical methods enables engineers to make informed decisions based on thorough data analysis, considering all possible risk factors.

This is a critical step in ensuring the durability and reliability of buildings and infrastructure projects.

4. Conclusions

Based on the research conducted, which covers a wide range of methods for assessing the strength and reliability of stone structures, several key conclusions can be drawn that are important for ensuring the durability and safety of buildings.

1. To ensure the reliability of stone structures, it is important to apply a combined approach that includes both traditional assessment methods and modern non-destructive and probabilistic techniques. Traditional methods, such as compression, bending, and shear testing, provide fundamental data on the mechanical properties of materials, which form the basis for calculations of load-bearing capacity and structural design. However, these methods have limitations, particularly because they are often destructive, which can negatively affect the integrity of historical or critically important structures.

2. Modern non-destructive methods, such as ultrasonic testing, radiography, acoustic emission testing, and infrared thermography, provide valuable information about the internal state of materials without damaging the structure. These methods are essential when dealing with historical buildings or other structures where any damage is unacceptable. Furthermore, non-destructive techniques enable regular monitoring of structural conditions, which is key for identifying potential defects at an early stage and preventing serious accidents.

3. Probabilistic approaches and stochastic modeling are indispensable tools for assessing the reliability of stone structures under conditions of uncertainty. These approaches account for material variability, unpredictability of loads, and other factors that may influence the behavior of the structure during operation. The use of the Monte Carlo method, stochastic modeling, and regression analysis allows for predicting the probability of structural failure, identifying the most critical risk factors, and developing effective strategies to mitigate these risks.

4. The use of statistical methods, such as regression analysis, histograms, and probability distributions, enables a deeper analysis of test and monitoring results, identifying patterns, and making well-founded predictions about

structural behavior. These methods are crucial tools for interpreting data obtained during research and for making decisions regarding further actions.

5. To ensure the maximum reliability of stone structures, an integrated approach is recommended, combining traditional, non-destructive, and probabilistic assessment methods. This approach provides a comprehensive analysis of the condition of structures, taking into account both their current physical characteristics and potential risks associated with future operation.

6. It is important to conduct regular monitoring of the condition of stone structures using modern non-destructive methods, which will allow for the timely detection of defects and material wear. Moreover, models created using probabilistic methods should be regularly updated with new data to maintain their relevance and accuracy.

7. For the effective use of the combined approach, appropriate training and skill development for engineers are necessary. They must be proficient in working with modern assessment methods, understand the basics of probabilistic approaches, and be able to interpret the results of analyses.

8. Based on modern research and practical experience, it is advisable to develop or update existing regulatory documents that govern the use of traditional, non-destructive, and probabilistic methods for assessing the strength and reliability of stone structures. This will help standardize approaches and ensure a high level of safety and durability in buildings.

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