PHYSICAL AND MATHEMATICAL SCIENCES

MODELING OF CONTACT INTERACTION OF ARTICULAR CARTILAGES CONSIDERING SYNOVIAL FLUID

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DOI: https://doi.org/10.30525/978-9934-26-472-6-5

Introduction. Synovial fluid is an important aspect of joint function. Its amount plays an important role during physical activity, as it performs the function of intra-articular lubrication, preventing friction of articular cartilage and their wear; participates in maintaining the normal ratio of articular surfaces, increases their mobility; provides nutrition to articular cartilage; serves as an additional shock absorber. At low load on the joint, it is absorbed into the pores of the cartilage tissue, with increasing pressure, it is squeezed out of the pores and provides lubrication. Without adequate lubrication and shock absorption, cartilage can wear out faster, leading to the development of diseases. The balance between the production of synovial fluid and its absorption is also important. If this balance is disturbed, the risk of increased pressure in the joint increases, which can lead to degenerative changes. The synovial fluid, along with the elastic structure of the cartilage, reduces the risk of damage and prevents premature aging of the joints. That is why articular cartilage slides almost frictionlessly during significant physical activity.

Problem Formulation. Consider the contact interaction of articular cartilages under plane strain conditions. The surface of one cartilage is smooth, and the other has irregularities described by a periodic function. The cartilages are in contact under the action of uniform compressive stresses set at infinity P^{∞}. Due to the irregularity of one of the cartilages, their contact is incomplete and gaps appear between the cartilages with a certain height $g(x)$, and width 2γ. We assume that these gaps are partially filled with synovial fluid, i.e., the volume of the fluid V_{sl0} and the volume of one section of the joint cavity in the initial state in the absence of pressure V_0 are related by the dependence:

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$V_{sl0} = \delta V_0$, where δ is the coefficient of volumetric filling of the joint cavity with fluid ($0 < \delta < 1$).

We consider two stages of loading:

1) at the initial stage, the volume of the gap between the cartilage is larger than the volume of synovial fluid, which does not resist the convergence of the cartilage surfaces;

2) the second stage begins when a critical stress value is reached, at which the volume of the gap between the cartilages is equal to the initial volume of synovial fluid, in which a pressure p arises that resists cartilage convergence.

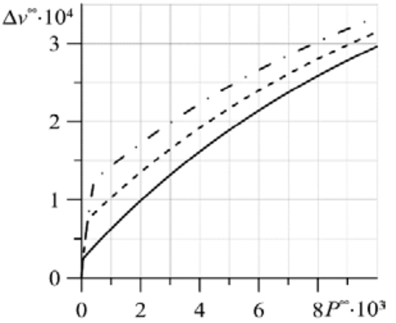
The volume of synovial fluid V_{sl} and the initial volume of fluid (in the absence of pressure in it) are related by the equation of state of a compressible barotropic fluid $V_{sl} = V_{sl0} e^{-p/B}$, where B is the modulus of volumetric elasticity of the fluid.

Solution of the formulated problem. Using the method of complex potentials and the method of intercontact gap functions [1, p. 3384], the formulated problem was reduced to a singular integral equation (SIE) with a Hilbert kernel, which, after replacing the variables, was transformed into an SIE [2, p. 603] with a Cauchy kernel with respect to the derivative of the height of the gaps between the cartilage. According to the SIE theory, we searched for a bounded solution to the equation that exists when an additional condition is imposed on its right-hand side.

At the initial stage of loading, when the external stresses are less than the critical stresses, we find analytically the height of the gaps between the cartilage cartilages $g(x)$. At the second stage of loading, when the external stresses are greater than the critical ones, the transcendental equation for determining the unknown half-width of the cartilage gaps γ is obtained from the existence of a bounded solution. We also analytically found the pressure of the synovial fluid, which resists the convergence of cartilages and the closure of the gap between them.

Considering that the surface of one of the cartilages has irregularities, the integral effect of which is manifested in the fact that at large distances from the contact surface in the direction of the applied compressive stresses at infinity, an additional approach of the cartilage surfaces Δv^{∞} occurs [3, p. 49], which was also found analytically.

Numerical results. Calculated for different coefficients of volumetric filling of the joint cavity with synovial fluid: solid curve – $\delta = 0{,}95$; dashed curve – $\delta = 0{,}85$; dashed-dotted curve – $\delta = 0{,}75$. The figure shows the contact approach of the cartilages Δv^{∞}, which increases with the external compressive stress. At the initial stage of loading, Δv^{∞} increases rapidly because the synovial fluid does not resist, and at the second stage, this rate slows down. The greater the coefficient of volumetric filling of the joint cavity with synovial fluid, the smaller the contact approach of the cartilages.



Conclusions. The contact of articular cartilages, the surface of one of which is smooth and the other has irregularities, has been studied. Due to the irregularity of one boundary, the cartilage contact is incomplete and gaps appear between them, which are partially filled with synovial fluid. With some assumptions, the formulated contact problem is reduced to a singular integral equation with a Hilbert kernel with respect to the derivative of the height of the cartilage gaps and a system of two transcendental equations for the width of the cartilage gaps and the pressure of the synovial fluid. The solution of the singular integral equation is constructed in a closed form and, on this basis, the transformation of the cartilage gaps and their contact convergence with increasing compressive stress applied to them are analyzed. It was found that this behavior is qualitatively different for two stages of loading. At the first stage, as long as the volume of each cartilage gap remains greater than the initial volume of synovial fluid, the geometric parameters of the cartilage gap decrease sharply with increasing compressive load. This decrease slows down significantly at the second stage, after the compressive load exceeds the critical value at which the volume of the gap between the cartilages becomes equal to the volume of synovial fluid. It has been established that the less synovial fluid there is at the initial moment of cartilage contact, the greater their contact approach.

References:

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