SECTION 4. GEOLOGY

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THE APPLICATION OF ADVANCED MATHEMATICAL METHODS IN PETROLEUM GEOLOGY TO ENHANCE THE ACCURACY OF HYDROCARBON RESERVOIR PREDICTION

ЗАСТОСУВАННЯ СУЧАСНИХ МАТЕМАТИЧНИХ МЕТОДІВ У НАФТОГАЗОВІЙ ГЕОЛОГІЇ ДЛЯ ПІДВИЩЕННЯ ТОЧНОСТІ ПРОГНОЗУ ПОКЛАДІВ ВУГЛЕВОДНІ

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In contemporary petroleum geology, methods of mathematical statistics form the foundation for solving a wide range of problems. To study the structure of random variable distributions and their comparison, methods of analysis of variance *(ANOVA)* and factor analysis are applied. To identify statistical relationships between two or more random variables, correlation analysis is used. The detection of homogeneous regions in multidimensional random variable space constitutes the essence of cluster analysis. For classification tasks, discriminant analysis and numerous non-parametric (heuristic) methods are employed. The latter, known as pattern recognition methods, are based on various similarity measures between objects in a multidimensional space. Pattern recognition techniques are widely applied both in classification and clustering of geological objects [4, 7, 9].

Below is a brief overview of the most commonly used and most effective mathematical methods in solving geological problems.

Analysis of Variance (ANOVA). This method is based on sample data to identify and assess variability of geological features related to various sources. The essence of variance analysis lies in comparing quantitative measures of variability – variances across several sample populations [3].

It is used to analyze observational results affected by multiple simultaneous factors, to identify the most significant among them, and to evaluate their influence. A key question in variance analysis is whether a given sample represents observations of a single normally distributed variable, or a mixture of observations of several normally distributed variables differing only in their mean values [6].

Practical significance of variance analysis lies in its ability to identify the most influential factors from a broader group of potential variables. In petroleum geology, ANOVA helps to:

1. Determine the degree of isotropy of an oil-bearing reservoir by permeability;
2. Test hypotheses on the impact of lithological, structural, and other factors on hydrocarbon localization;
3. Assess the influence of landscape conditions on the manifestation of petroleum prospecting criteria;
4. Evaluate the effect of measurement methods on reservoir parameter values for possible parameter unification;
5. Address the role of hypergenic processes in the reliability of geochemical anomalies in direct hydrocarbon prospecting;
6. Identify zonal elements of various petroleum-bearing objects;
7. Establish major factors influencing oil, gas, and condensate recovery coefficients [2].

Factor Analysis. Application of factor analysis in geology is based on determining eigenvalues and eigenvectors of either the correlation matrix or the similarity matrix of objects [8,9].

Thus, factor analysis orthogonalizes the original feature space, producing a new system of independent coordinates where the correlation matrix is diagonal, and diagonal elements represent the variances of new features. From these, a smaller subset of features explaining most of the variability can be selected. Consequently, factor analysis serves as both a dimensionality reduction technique and a tool for classification and clustering tasks [3].

Correlation Analysis. This method establishes the presence, strength, and form of statistical relationships between random variables. Among statistical techniques, correlation analysis is the most widely applied in geology. Correlation provides a convenient model for investigating complex geological properties and is an essential component of any multivariate statistical procedure [5, 9].

In petroleum geology, correlation analysis is often applied to study the relationships between reservoir properties, technological indicators of hydrocarbon extraction, the development of prospecting criteria, and the selection of rational combinations of geological research methods. It is also useful for constructing hypothetical models describing dependencies between geological processes and external factors [5].

Classification and Clustering Methods. Classification determines an object's membership in a predefined class within a given feature space, while clustering identifies the classes themselves, grouping objects with similar characteristics.

Both parametric (based on known probability distributions) and non-parametric statistical methods are applied to classification and clustering, generally known as pattern recognition methods. The essence of pattern recognition is to compare geological features of an investigated object (a local petroleum prospect) with those of reference objects (known hydrocarbon fields and barren structures). Based on similarity, the object is assigned to the most probable class [5,8,9].

Discriminant Analysis. This method examines differences between two or more groups of objects across several variables simultaneously, and classifies new objects based on these differences. Discriminant analysis is highly useful in forecasting within natural and technical sciences, particularly in petroleum geology, where it can serve as a basis for hydrocarbon prospectivity prediction.

Among non-parametric classification methods, well-known and effective approaches include the potential functions method, the hypersphere inclusion method, and the angle-based classification method. In the potential functions approach, similarity between objects is measured using elementary (often power-law) functions of Euclidean distance in feature space. If Euclidean distance itself is used, this corresponds to the hypersphere method. In angle-based classification, correlation (angular distance) between objects plays a central role, rather than their absolute or normalized feature values [1,3].

Mathematical Methods of Spatial Analysis in Geological Research. Traditionally, spatial variations of geological features in petroleum geology are visualized using maps, cross-sections, profiles, charts, and diagrams. Such visual documents typically highlight both the direction of feature change and the location of anomalous zones. Of particular importance is identifying regional trends of geological parameters.

For example, a regional increase in petroleum reservoir productivity may reflect the position of hydrocarbon generation zones, while changes in granulometric composition may indicate source areas of denudation. In geologically simple or poorly studied regions, such regional trends are relatively easy to establish from graphical materials. However, in more complex conditions with mosaic local anomalies, identifying regional tendencies is challenging and often influenced by subjective geological interpretations.

The concepts of "regional" and "local" are themselves subjective and strongly dependent on the scale of the study area [4, 5, 9]. The availability of data also significantly affects the established trends: local patterns cannot be reliably inferred when their scale is comparable to sampling areas. Such dependencies can be quantified for regular sampling networks but are difficult to establish for irregular datasets, which are far less suitable for mathematical processing [2].

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