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DETERMINATION OF LATERAL SOIL PRESSURE AGAINST THE GRAVITY TYPE QUAY WALL REGARDING STRUCTURE KINEMATICS

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INTRODUCTION

Influence of structure displacements and deformations on the loads produced by lateral soil pressure is considered as rather important issue and it is reflected in the corresponding European cods and regulations[1, 2]. The mentioned problem has been intensively studied, particularly, in Odessa National Maritime University at the Department "Sea and River Ports, Waterways and their Technical Operation" during the last fifty years. It is worth mentioning researches produced by Prof., D.Sc. F. Shikhiev, Prof., D.Sc. P. Yakovlev, Prof., D.Sc. V. Kovtun, Prof., Ph.D. V. Bugaev, Prof., D.Sc. M. Doubrovsky, Ass. Prof., Ph.D. R. Lubenov, Ass. Prof., Ph.D. M. Vargin, Ass. Prof., Ph.D. N. Khonelia and some others. The main idea of these studies stipulates taking into account the influence of structure kinematics on loads induced by soil pressure. Some researches were based either on experimental testing "retaining wall – soil media" system, mainly in the sandbox with rigid or flexible wall models (F. Shikhiev, R. Lubenov, P. Yakovlev, V. Bugaev) or on theoretical investigations and numerical modeling (F. Shikhiev, V. Kovtun, M. Doubrovsky, N. Khonelia).

Some of the obtained experimental results gave very interesting and useful information, but they were not generalized and did not bring to the well-grounded theory. Some of the proposed theoretical approaches were not supported by reliable numerical algorithms and applied non-standard geotechnical parameters. That's why we have tried to apply in one model both influence of structure kinematics confirmed by known experimental studies and the use of conventional soil parameters that are available from the standard on-site or laboratory tests.

Regarding volume provided for this paper, we present the part related to the interaction of the rigid retaining wall (i.e. gravity type quay wall or another high wall) with non-cohesive soil. For this system development of both flat and non-flat slip surfaces is analyzed.

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[1] CEN – European Committee for Standartization (1993). Eurocode 7, Part 1. Geotechnical Design, General Rules.

[2] Doubrovsky, M. P. (1994) Determination of soil pressure against a retaining wall with allowance for the structure's kinematics. *Soil Mechanics and Foundation Engineering*, Vol. 31, No. 2, Plenum Publishing Corporation, USA, pp. 46–51.

CASE OF PLANE SLIP SURFACES

Proposed kinematics model of the interaction between the components of the "retaining-wall/soil medium" system[3, 4, 5] is based only on two premises. They reduce to the following in examining a retaining wall with an arbitrarily inclined contact face (at angle α_0 to the vertical) and bottom surface of the backfill soil (at angle β to the horizontal) when a distributed load with an intensity q acts on the latter (Fig. 1) under conditions of the plane problem:

1. The character of the stress state at an arbitrary point on the contact between the lateral surface of the structure and soil is determined by the ratio of the horizontal displacement u (z) of the structure cross-section to the embedment depth z of this section concerning the point of intersection between the surface of fee ground and the retaining wall. The soil will then be in the sublimiting state when $u(z)/z < \alpha$, and in the limiting stress state for the inverse ratio. Considering that the condition promoting the formation of the soil limiting stress state over the entire contact face of the structure (with a height H) and along its section (in die segment with a height z) are analogous, we can assume that $\alpha = 0.001–0.0015$ and $\alpha = 0.01–0.03$ during the development of an active and passive pressure, respectively.

The premise in question is distinguished from the condition normally considered whereby a limiting state characterized by the ratio of the so-called critical displacement of the wall to its height $u_{cr}/H = \alpha$ sets in; this requires appropriate confirmation. Under natural conditions, the process whereby the soil goes over from a state of rest for a stationary wall to the limiting stress state occurs not jumpwise, but smoothly in the displacement interval $[0; u_{cr}]$. For current displacements, $u < u_{cr}$ (for example, with progressive displacements or displacement combined with rotation about the bottom of the wall), consequently, the part of the soil in contact with the wall in its upper part adjacent to the ground surface) is in the limiting, and part (in the underlying region of the wall) in the sublimiting stress state. When it becomes equal to zero (when $u = u_{cr}$), therefore, the ratio u/H characterizes not the current, but only the final state, which for some structure, may not set in under small displacements.

[3] Doubrovsky, M., M. B. Poizner. (2008) Foundations of onshore and offshore constructions on Ukrainian Black Sea and Azov Sea coasts. Proceedings of 11th Baltic Sea Geotechnical Conference *"Geotechnics in Maritime Engineering"*. Gdansk, Vol. 2, pp. 935–940.

[4] Gabi, S., Doubrovsky M. and Belakrouf A. New Development of Port Structures Design and Construction. *Journal of Shipping and Ocean Engineering*, Vol. 1, Number 3, 2011, David Publishing Company, USA. pp. 150–157.

[5] M. Doubrovsky, M. Poizner. (2016) Innovative development of coastal, port and marine engineering. Saarbrucken: Lambert Academic Publishing.

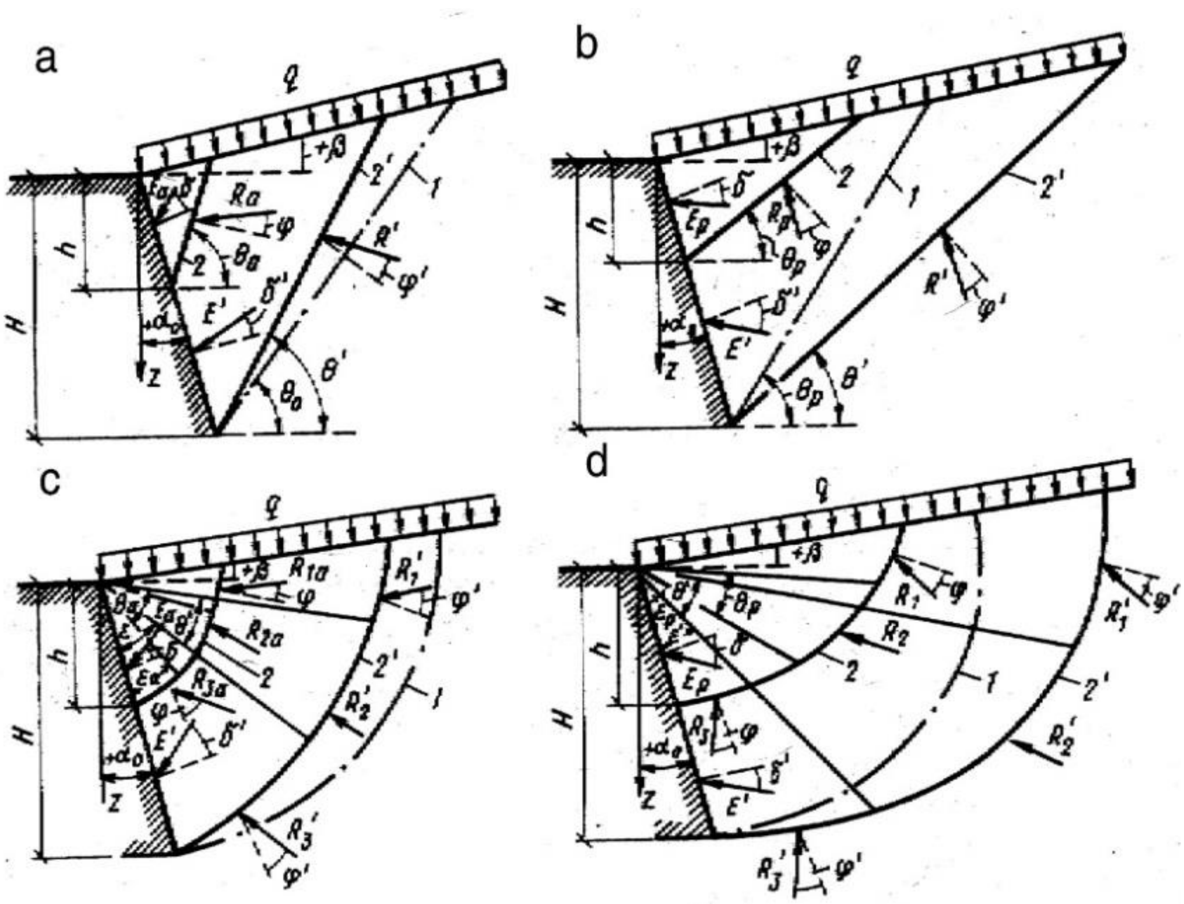

Fig. 1. Statement of the mixed problem of determination of lateral soil pressure against retaining wall with flat (a, b) and non-flat (c, d) slip surfaces. a, b) as the wall moves away from the soil, and thrust pressure is formed; c, d) as the wall moves toward soil and support pressure is developed; 1) conditional slip surface corresponding to a pressure of soil at rest; 2 and 2') slip surfaces bounding, respectively, limiting and submitting regions of the soil stress state.

Let us examine the following imaginary experiment to illustrate the logical nature of the premise that we have adopted. Let there be N similar retaining walls with heights $H_1 > H_2 > \ldots > H_N$ for the faces in contact with the soil, which are displaced with respect to the backfill soil, for example, forward at the same distance u. In that case, let the height HN of the N-th wall be such that $u/H_N \geq \alpha$, i.e., the soil over the entire height of the contact face of the N-th retaining wall goes over to the limiting stress state, and the height of the remaining walls such that $u/H_1 < \alpha$; $u/H_2 < \alpha$; ...; u/H_N–1 < α, i.e., for the remaining N – 1 wall, not all of the soil interacting with their contact faces goes over to the limiting state. If thereafter, the soil within the bounds of the entire height of the N-th wall goes over to the limiting stress state as the wall is displaced by a distance u, it is logical to assume that for the same displacement and other walls, the soil within the limits of the same height H_N (referenced from the surface of the backfill) will be within the limiting stress state, and the soil in the lower part of the contact faces of the walls in the sections with a height $H_1 - H_N$, $H_2 – H_N$, ..., $h_{N-1} – H_N$ will be in the sublimiting stress state. This may be caused, among other things, by the fact that the extent to which deformations develop in the overlying (spilling over immediately beyond the retreating or bulging advancing contact face of the retaining wall), and not the underlying soil is decisive for the formation of the soil stress state at a depth HN along the contact face of the i-th retaining wall.

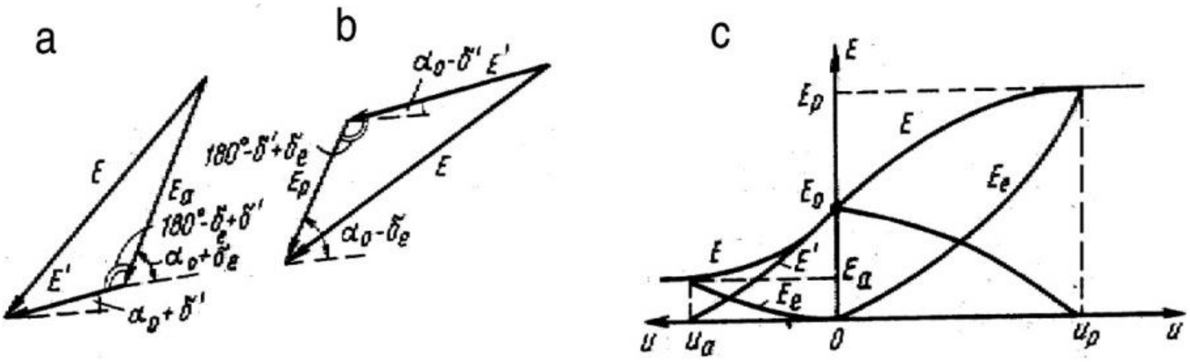

Fig. 2. Determination of resulting E of lateral soil pressure using its limiting Ee and sublimiting E' components; a, b) addition of force vectors, respectively, under thrust and support pressure, c) relationship between lateral soil pressure and displacement of structure (E_0 is lateral soil pressure in the at-rest state).

Conversion from a criterion based on the development of a limiting stress state at an arbitrary point on the contact face of a retaining wall in the form of the ratio u/H (resulting in jumpwise development of the limiting state simultaneously at an all points of the contact face) to a criterion in the form of the ratio u(z)/z (ensuring a gradual conversion of the soil to the limiting stress state as the displacements of the structure increase and making it possible to account for the existence and transformation of the zone of the soil limiting stress state) is also motivated by these discussions.

The boundary between the zones of the soil limiting and sublimiting stress states (or height h of the zone of contact between the soil, which is in the limiting state, and the structure) can be found from the condition u(h)/h = a, for the use of which the form of the u(z) function, which can be determined by the pattern of the structure deformations, should be given (for example, this function is linear for rigid structures, and u = const and h = u/α; for advancing displacements).

2. The angles of deviation of the resultant reactive pressure of the soil mass behind the thrust (or support) prism from the normal to the boundary of this prism and the resultant lateral pressure of the soil from the normal to die contact face of the structure are assumed equal to the angles of the soil internal friction φ and the contact friction $\delta = m\varphi(0 \leq m \leq 1)$ for the zone of the limiting

stress state with height h, and equal, respectively, to φ' and δ' for the zone of the sublimiting stress state with height H – h; in that case

$$\varphi' = \varphi_0 + n(\varphi - \varphi_0); \; \delta' = \delta_0 + n(\delta - \delta_0), \tag{1}$$

where n is a parameter dependent on the relationship between the dimensions of the zones of the limiting and sublimiting stress states of the soil ($0 \le n \le 1$) and defined by the ratio $n = V_e/V$ (V_e and V are, respectively, the volumes of the soil prism in the limiting stress state, and all of the soil that interacts with the contact face of the structure, as defined from geometric considerations in conformity with the assumed shape of the slip surface), φ_0 is the conditional angle of internal friction of the soil under pressure in the at-rest state (maybe determined from familiar recommendations, for example[6]), and δ_0 is the conditional angle of contact friction under pressure in the at-rest state.

The premises adopted are sufficient for the determination of both zones (limiting and sublimiting) of the stress state in the mass of soil interacting with the structure and for determination of the lateral pressure, which can be found from the side of these zones against the structure. The kinematics model adopted makes it possible to implement any of the premises traditionally employed in soil mechanics for the shape of the slip surface, and the computational relationships for determination of the thrust and support pressure are determined by universal equations that are distinguished only by sign in the indicated cases (in further calculations, the upper and lower signs, respectively) before the values of the angles φ and δ, or φ' and δ' (this principle of analogy is brought to light by P. Yakovlev[7]).

The resultant E of the lateral soil pressure against the structure can be defined for each current strain state of the structure as the algebraic sum of its two components: the limiting component E_e, which acts over a section with height h, and the sublimiting component E' which acts over a section with height H – h (Fig. 2a and b) in conformity with the expression

$$\vec{E} = \vec{E}_e + \vec{E}' = \left[E_e^2 + E'^2 + 2E_e E' \cos(\delta_e - \delta')\right]^{1/2} \tag{2}$$

The indicated components of the lateral pressure are found by successive examination of the equilibrium conditions of the limiting and sublimitmg soil prisms, the geometry of which is determined by the shape assumed for the slip surface.

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[6] M. Doubrovsky, M. Poizner. (2016) Innovative development of coastal, port and marine engineering. Saarbrucken: Lambert Academic Publishing.

[7] Doubrovsky, M. P. (1997) Determination of lateral soil pressure against retaining walls with allowance for nonplanar slip surfaces and kinematic factors. *Soil Mechanics and Foundation Engineering*, Vol. 31, No. 2, Plenum Publishing Corporation, USA. Pp. 15–21.

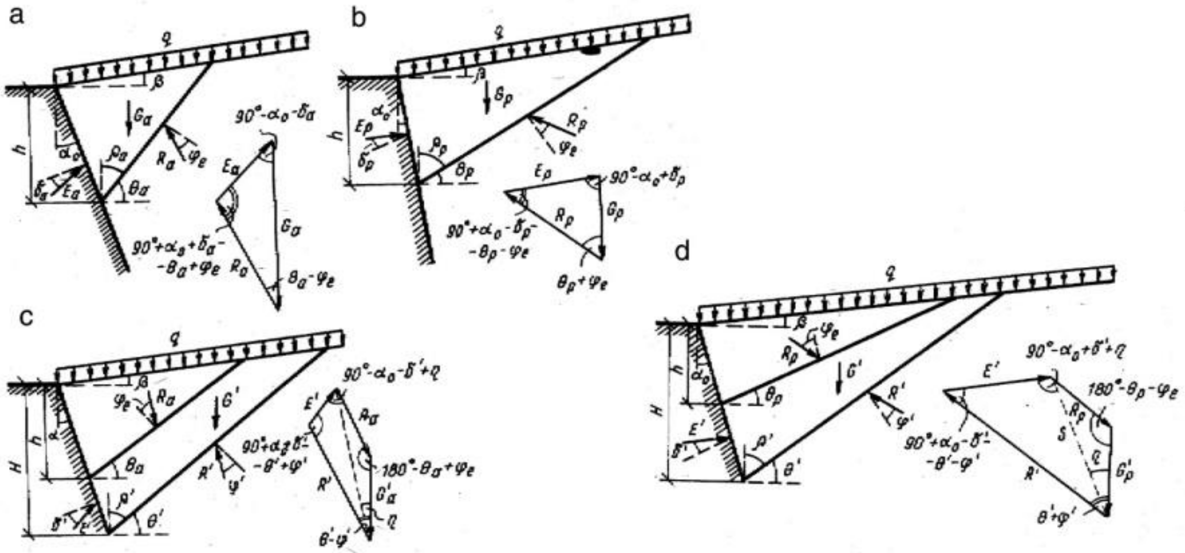

**Fig. 3. Calculation of soil pressure against the retain wall with allowance for kinematics of structure. a, b) computational schemes and force polygons for limiting prism under thrust and support pressures, respectively;
c, d) the same for subimiting prism.**

For place slip surfaces, therefore, we obtain

$$E_e = G_e \sin(\theta \mp \varphi_e)/\cos(\alpha_0 \pm \delta_e \pm \varphi_e - \theta_e),$$

from the equilibrium condition of the limiting prism (Fig. 3a and b), where

$$G_e = G_{\gamma e} + G_{qe};$$

$$G_{\gamma e} = 0.5\gamma h^2(\tan\alpha_0 + \tan\rho_e)(1 + \tan\alpha_0 tg\beta)/(1 - \tan\rho_e tg\beta);$$

$$G_{qe} = qh(\tan\alpha_0 + \tan\rho_e)/(1 - \tan\rho_e \tan\beta);$$

$$\tan\theta_e = (\cos\alpha_0 \pm k_0 \sin\varphi_e)/(k_0 \cos\varphi_e - \sin\alpha_0); \quad (3)$$

$$k_0 = \{[\pm\sin(\varphi_e + \delta_e)\cos(\alpha_0 - \beta)]/[\cos(\alpha_0 \pm \delta_e)\sin(\pm\varphi_e - \beta)]\}^{1/2}; \quad (4)$$

$$\rho_e = 0.5\pi - \theta_e;$$

$$R_e = G_e \cos(\alpha_0 \pm \delta_e)/\cos(\alpha_0 \pm \delta_e \pm \varphi_e - \theta_e),$$

Here, γ is the specific gravity of the soil, and the subscript “e” indicates that the parameters being determined to apply to the extremal (i.e. limiting – active or passive) stress state.

Proceeding then to the analysis of the equilibrium conditions of the sublimiting prism (Fig. 3c and d), we obtain

$$S = \left[R_e^2 + G'^2 + 2R_e G' \cos(\theta_e \mp \varphi_e)\right]^{1/2};$$

$$\eta = \arcsin\left[R_e \sin(\theta_e \mp \varphi_e)/S\right];$$

$$G' = G'_\gamma + G'_q; \;\; \rho' = 0.5\pi - \theta';$$

$$G'_\gamma = 0.5\gamma(1 + \tan\alpha_0 \tan\beta)\left[H^2 \frac{\tan\alpha_0 + \tan\rho'}{1 - \tan\beta \tan\rho'} - h^2 \frac{\tan\alpha_0 + \tan\rho_e}{1 - \tan\rho_e \tan\beta}\right];$$

$$G'_q = q\left[H \frac{\tan\alpha_0 + \tan\rho'}{1 - \tan\beta \tan\rho'} - h \frac{\tan\alpha_0 + \tan\rho_e}{1 - \tan\rho_e \tan\beta}\right],$$

and the relationships for determination of the angle θ’ are similar to Eqs. (3) and (4), if the angles φ and δ in them are replaced by φ’ and δ’. For calculation of the parameter n, we obtain

$$n = \frac{h^2}{H^2} \frac{\cos(\theta_e - \alpha_0)\sin(\theta' - \beta)}{\cos(\theta' - \alpha_0)\sin(\theta_e - \beta)},$$

from geometric considerations (see Fig. 2); hence, it is possible to express a function of the form

$$y(n) = h^2 \cos(\theta_e - \alpha_0)\sin(\theta' - \beta) - nH^2 \cos(\theta' - \alpha_0)\sin(\theta_e - \beta).$$

From the latter, it is also possible to find the desired parameter n by the iteration method (given n and calculating the angles φ’ and δ’ from relationships (1)); this is possible when the condition y(n) = 0 is satisfied.

Determining the height h of the zone of the limiting stress state of the soil and the value of the parameter n corresponding to it for each increment of the structure displacement, it is possible to obtain a computational “E–u” curve similar to that presented in Fig. 2c within the range from the active (E_a) to passive (E_p) pressure. It is apparent from the latter, among other things, that the limiting component increases from zero (when there are no displacements) to E_a or E_p (depending on the direction of the displacements of the contact face of the structure) in the displacement interval under consideration [0; u_{cr}], while the sublimiting component decreases from the at-rest pressure to zero, respectively.

The computational algorithm has been developed for the calculation and plotting of the “E–u”-type curve.

The special investigation has indicated a substantially lower effect of the accuracy with which the parameter α is assigned (within the above-adopted intervals of its values) on the computed value of the soil lateral pressure as compared with the accuracy with which the physics-mechanical characteristics of the soil, as determined by traditional means, are assigned.

CASE OF NON-PLANE SLIP SURFACES

A kinematics computational model[8, 9, 10, 11] of the "structure/soil-medium" system makes it possible to determine the loads due to thrust and resisting pressures over the entire interval of generalized displacements (from the pressure of the soil in repose to the active and passive pressures, respectively). In this section, a developed approach will be evolved into the more general (and to a greater degree, corresponding to reality) case of the realization of non-planar slip surfaces. Let it be understood that this method is based on an analysis of the interaction between zones of the limiting and sublimiting stress state of soil medium for an arbitrary deformed state of the system under investigation. Working relationships of the theory of limiting stress state, which utilize the angles of deviation of the resultant reactive pressure of the soil mass behind the thrust (or resisting) prism from the normal to the boundary of this prism (an analogy of the angle of internal friction φ) and the resultant lateral pressure of the soil from the normal to the contact face of the structure (angle of contact friction δ) are valid in the first of these regions. The working formulas for the zone of the sublimiting stress state operate with intermediate φ' and δ' values of these angles, which are determined as a function of the generalized displacement realized for a retaining wall in the interval from the initial values corresponding to the soil pressure in a state of repose (φ_0 and δ_0) to the maximum values (φ_c and δ_c) is (1).

Retaining the adopted system of notations, we will hereinafter affix the subscript "e" to the parameters characterizing the extremal (limiting) stress state, and a prime to the parameters relating to the sublimiting state.

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8 Doubrovsky M. P., Meltsov G. I., Pereiras R. Papova et al. (2015) Some innovative structural and technological solutions for near-shore and offshore development. *Proceedings of the 16th European Conference on Soil Mechanics and Geotechnical Engineering*. Edinburg, pp. 1273–1278.

9 Doubrovsky M., Poizner M. (2016) Innovative development of coastal, port and marine engineering. Saarbrucken: Lambert Academic Publishing.

10 Doubrovsky M., Samorodov A., Muliar D. et al. (2017) Innovative design and technological solutions and test method for pile supports with increased bearing capacity. *Proceedings of the 19th International Conference on Soil Mechanics and Geotechnical Engineering*, Seoul, pp. 2735–2738.

11 Doubrovsky M., Poizner M. (2016) Innovative development of coastal, port and marine engineering. Saarbrucken: Lambert Academic Publishing.

Steep retaining walls for which the angle relationship $\theta_1 \geq \alpha_0$ (Fig. 4), where

$$\theta_1' = 0.5\left(\arccos\frac{\sin\beta}{\pm\sin\varphi'} \mp \varphi' + \beta\right) \tag{5}$$

is valid in conformity with the theory of limiting stress state, are addressed below.

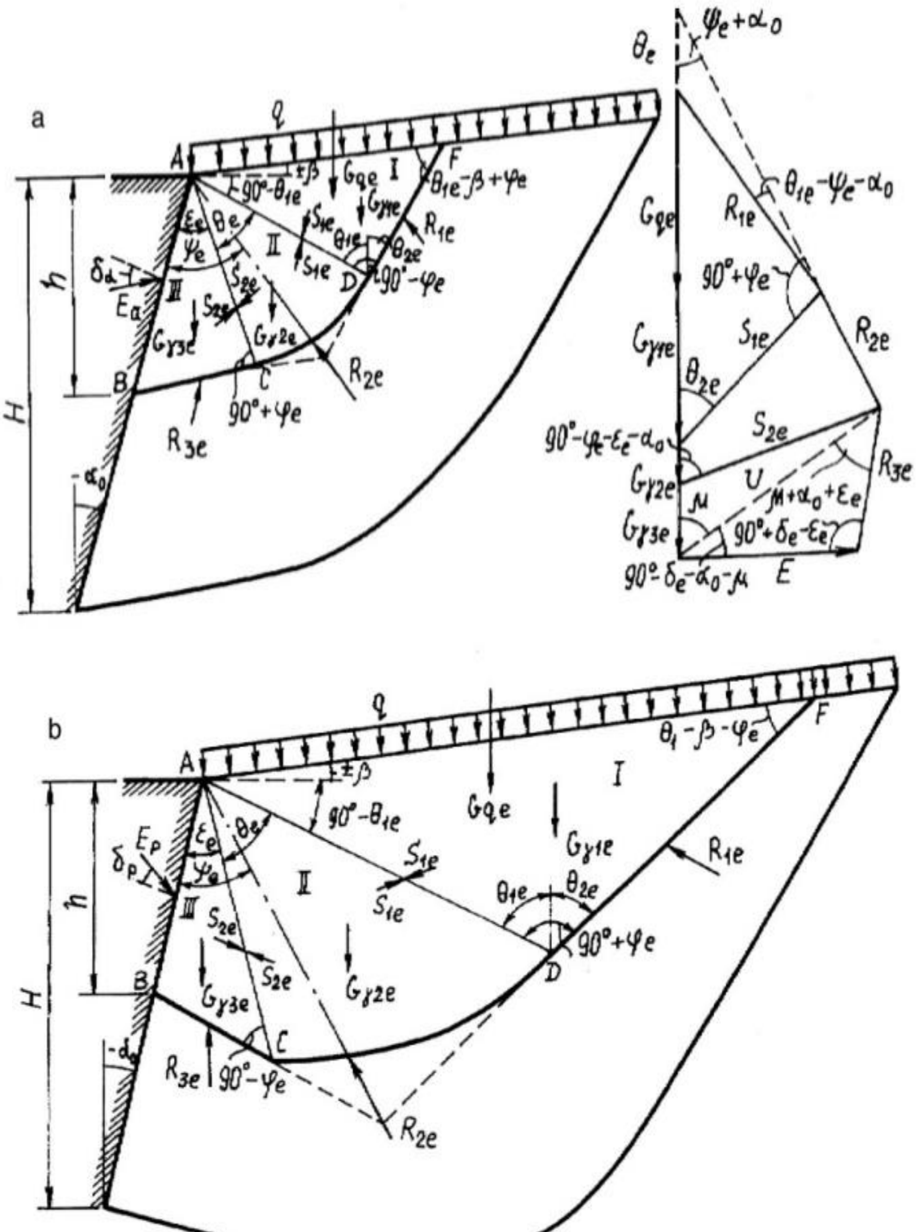

Fig. 4. Working diagrams for determination of limiting component of lateral soil pressure against retaining wall with nonplanar slip surfaces for an arbitrary generalized structure displacement: a) thrust pressure and corresponding force polygon; b) resisting pressure

Consequently, the stress state of the soil medium interacting with the structure is uniform, and a Prandtl transition zone is located between the zones of minimum and maximum stress state, while the stress components are continuous functions of the coordinates.

The general approach we used earlier for determination of the resultant lateral soil pressure for plans slip surfaces as the vector sum of its limiting Ee and sublimiting E' components remains in effect. Determination of the sublimiting pressure is, however, a more complex problem. The conditions of equilibrium of the upper (limiting) and lower (sublimiting) zones of the soil stress state can be subsequently examined for its solution[12].

Considering the relationships of the technical theory of limiting stress state, let us define the lateral pressure against a retaining wall, which can be transmitted through the region of the limiting stress state (see Fig. 4) as[13]

$$E_e = U_e \sin(\mu + \alpha_0 + \varepsilon_e) / \cos(\varepsilon_e \mp \delta_c), \qquad (2.6)$$

where:

$$U_e = \left(Q_e + G_{\gamma c} - G_{qe}\right) \sin \frac{\psi_c + \alpha_0}{\sin(\psi_c + \alpha_0 + \mu)};$$

$$\mu = arcctg\left[\left(Q_e + G_{\gamma c} + G_{qe}\right) tg(\eta_e + \alpha_0) + G_{\gamma 3c} ctg(\psi_c + \alpha_0)\right] /$$

$$/ (Q_e + G_{qc} - G_{\gamma 3c});$$

$$G_{\gamma c} = G_{\gamma 1c} + G_{\gamma 2c} + G_{\gamma 3c};$$

$$Q_e = \left[\left(G_{\gamma 1c} + G_{qe}\right) \sin\theta_{2c} \sin(\theta_{1c} - \psi_c - \alpha_0)\right] / \left[\cos\varphi_e \times \sin(\psi_c + \alpha_0)\right];$$

$$\psi_c = [\eta_e - arctg(\exp(\mp\theta_c tg\varphi_e)) - \cos\theta_c] / \sin\theta_c;$$

$$\eta_e = \varepsilon_e \pm \varphi_e;$$

$$\varepsilon_e = 0{,}5[0{,}5\pi \pm \delta_c \mp \varphi_e - \arcsin((\sin\delta_c)/(\sin\varphi_e))];$$

$$\theta_c = \theta_{1c} - \alpha_0 - \varepsilon_e;$$

$$\theta_{2c} = 0{,}5\pi \mp \varphi_e - \theta_{1c};$$

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12 Doubrovsky M., Poizner M. (2016) Innovative development of coastal, port and marine engineering. Saarbrucken: Lambert Academic Publishing.

13 Doubrovsky M., Poizner M. (2016) Innovative development of coastal, port and marine engineering. Saarbrucken: Lambert Academic Publishing.

Here and below, the double signs before certain angles make it possible to use the working formulas to determine both the thrust (upper sign) and resisting (lower signs) pressures.

Relationships for determination of the weights of the three zones of soil existing in the limiting stress state assume the form

$$G_{\gamma 1e} = 0{,}5\gamma h^2 \Phi_1(\varphi_e); \tag{7}$$

$$G_{\gamma 2e} = 0{,}5\gamma h^2 \Phi_2(\varphi_e); \tag{8}$$

$$G_{\gamma 3e} = 0{,}5\gamma h^2 \Phi_3(\varphi_e); \tag{9}$$

where

$$\Phi_1(\varphi_e) = \frac{\cos(\theta_{1c} - \beta)\cos^2(\eta_e)\exp(\mp 20_c tg\varphi_e)}{\sin(\theta_{1c} - \beta \pm \varphi_c)\cos\varphi_c \cos^2\alpha_0};$$

$$\Phi_2(\varphi_e) = \frac{\mp\cos^2(\eta_e)[\exp(\mp 20_e tg\varphi_e) - 1]}{\sin 2\varphi_e \cos^2\alpha_0};$$

$$\Phi_3(\varphi_e) = \frac{\sin\varepsilon_e \cos\eta_e}{\cos\varphi_e \cos^2\alpha_0};$$

On examining the force polygon (see Fig. la), let us determine the reactions on the side of the underlying sublimiting zone

$$R_{1e} = (G_{1\gamma e} + G_{qe})\sin\theta_{2e}/\cos\theta_e;$$

$$R_{2e} = \left(G_{\gamma 1e} + G_{qe} + G_{\gamma 2e} + Q_e\right)\cos\frac{\pm\varphi_e + \varepsilon_e + \alpha_0}{\cos(\mp\varphi_e - \varepsilon_e + \psi_e)} -$$

$$-R_{1e}\sin\theta_{1e}/\sin(\phi_e + \alpha_0),$$

where

$$G_{qe} = qh\Phi_s(\varphi_e);$$

$$\Phi_s = \frac{cos\beta cos\eta_e \exp(\mp\theta_c tg\varphi_e)}{\sin(\theta_{1e} - \beta \pm \varphi_e)cos\alpha_0};$$

$$R_{3e} = U_e\cos(\pm\delta_e + \alpha_0 + \mu)/\cos(\varepsilon_e \mp \delta_e).$$

For further analysis applicable to the zone of the sublimiting stress state (Fig. 2.5), let us introduce an assumption consistent with physical notions.

Although the corresponding common boundaries of zones I and I’, II and II’, and in and III’ of the regions of the limiting and sublimiting states of the soil do not completely coincide in the working diagram (as a result of differences in the angles ε_e and ε’, θ_e and θ’, θ_{1e} and θ'_1), no jumpwise boundary between zones of the limiting and sublimiting stress states is formed in the real soil medium, such that it is possible to assume that reactive forces R_{1e}, R_{2e}, and R_{3e} act entirely on the appropriate boundaries of zones I and I’, II and II’, and in and III’. From geometric considerations dictated by the character of the working diagram under consideration, the following expressions for the weights of the three zones under consideration can be attained after trigonometric transformations:

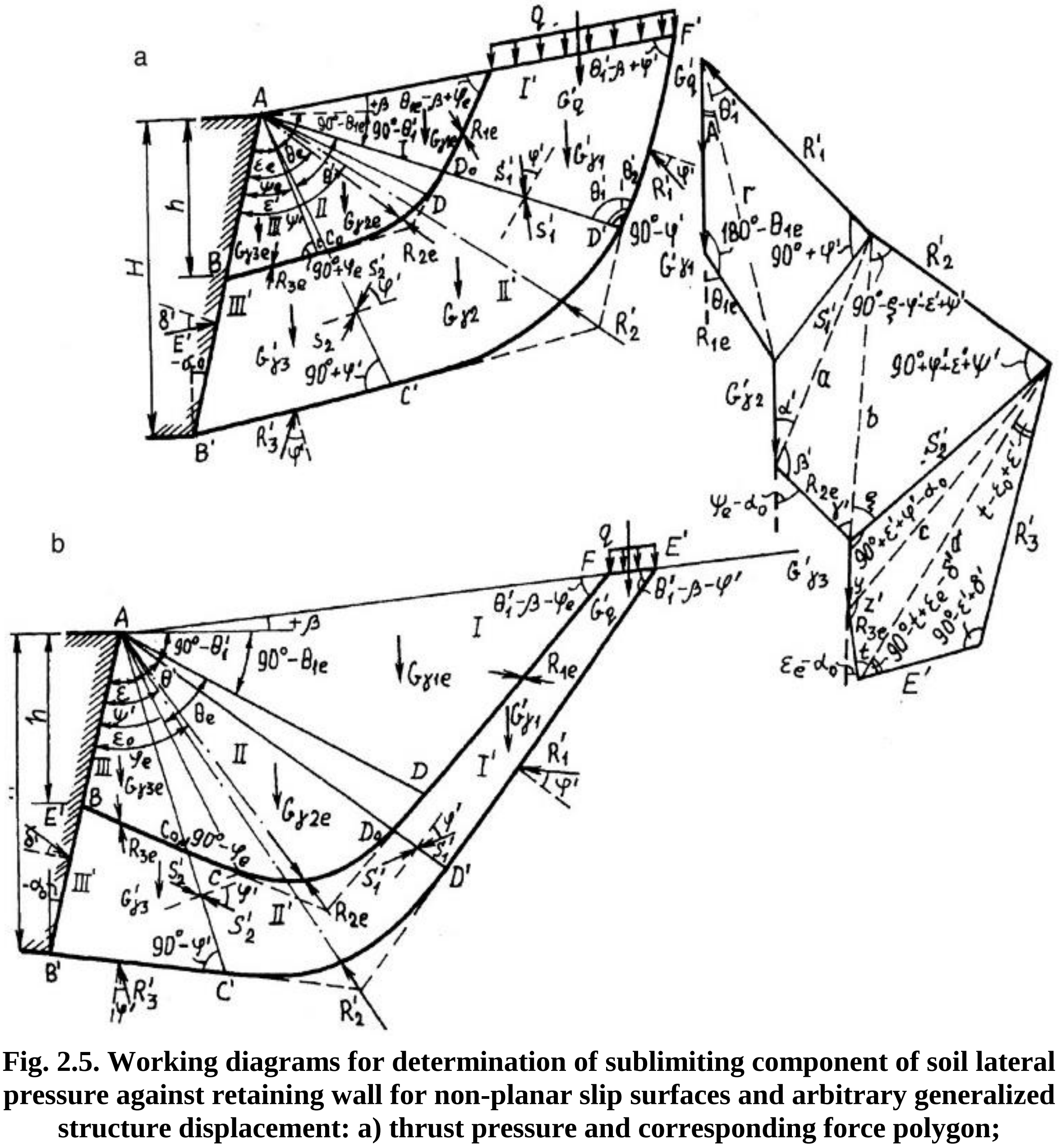

Fig. 2.5. Working diagrams for determination of sublimiting component of soil lateral pressure against retaining wall for non-planar slip surfaces and arbitrary generalized structure displacement: a) thrust pressure and corresponding force polygon; b) resisting pressure.

$$G'_{\gamma_1} = 0{,}5 \cdot \gamma \cdot (H^2 \cdot \Phi'_1(\varphi') - h^2 \cdot \Phi_2(\varphi')) \quad (10)$$

$$G'_{\gamma_2} = 0{,}5 \cdot \gamma \cdot (H^2 \cdot \Phi'_3(\varphi') - h^2 \cdot \Phi_4(\varphi')) \quad (11)$$

$$G'_{\gamma_3} = 0{,}5 \cdot \gamma \cdot (H^2 \cdot \Phi'_5(\varphi') - h^2 \cdot \Phi_4(\varphi_e)) \quad (12)$$

where

$$\Phi'_1(\varphi') = \frac{\cos(\theta'_1 - \beta)\cos^2(\eta')\exp(\mp 2\theta' tg\varphi')}{\sin(\theta'_1 - \beta \pm \varphi')\cos\varphi' \cos^2\alpha_0};$$

$$\Phi'_2(\varphi') = \frac{\cos(\theta_{1e} - \beta)\cos\eta_0 \cos\eta_e \exp[\mp(\theta' + \theta_e)tg\varphi_e]}{\sin(\theta_{1e} - \beta \pm \varphi_e)\cos\varphi_e \cos^2\alpha_0};$$

$$\Phi'_3(\varphi') = \frac{\cos^2(\eta')\exp(\mp 2\theta' tg\varphi') - 1}{\mp\sin 2\varphi' \cos\varphi' \cos^2\alpha_0};$$

$$\Phi'_4(\varphi') = \{\frac{\pm\cos^2(\eta_e)[\exp(\mp 2\theta_e tg\varphi_e) - 1]}{\sin 2\varphi_e} +$$

$$+\sin\varepsilon_e \cos\eta_e - \sin\varepsilon' \cos\eta_0 - \frac{\cos(\theta_{1e} - \beta)}{\sin(\theta_{1e} - \beta \pm \varphi_e)} \times$$

$$\times\cos\eta_0 \cos\eta_e \exp[\mp(\theta' + \theta_e)tg\varphi_e] -$$

$$-\cos^2\eta_e \exp(\mp 2\theta_e tg\varphi_e)\} / (\cos\varphi_e \cos^2\alpha_0);$$

$$\Phi'_5(\varphi') = \frac{\sin\varepsilon' \cos\eta'}{\cos\varphi' \cos^2\alpha_0}; \quad \Phi_4(\varphi_e) = \frac{\sin\varepsilon' \cos\eta_0}{\cos\varphi_e \cos^2\alpha_0};$$

$$\eta' = \varepsilon' \pm \varphi'; \quad \eta_0 = \varepsilon' \pm \varphi_e; \quad \theta' = \theta_1' - \alpha_0 - \varepsilon';$$

$$\theta_1' = 0{,}5\ [\arccos(\sin\beta / \pm\sin\varphi') \mp \varphi' + \beta];$$

$$\varepsilon' = 0{,}5\ (0{,}5\pi \pm \delta' \mp \varphi' - \arcsin(\sin\delta' / \sin\varphi')).$$

The resultant surface load is

$$G_q' = q\cos\beta\,[H\Phi'_q(\varphi') - h\Phi_{qe}(\varphi_e)] / \cos\alpha_0, \quad (13)$$

where

$$\Phi'_q(\varphi') = \cos\eta' \exp(\mp\theta' tg\varphi') / \sin(\theta_1' - \beta \pm \varphi');$$

$$\Phi'_{qe}(\varphi_e) = \cos\eta_e \exp(\mp\theta_e tg\varphi_e) / \sin(\theta_{1e} - \beta \pm \varphi_e).$$

Having determined the weight of all three zones of the region of the soil sublimiting stress state, let us construct the force polygon for this region (see Fig. 2a) from which we can obtain formulas for calculation of the sublimiting component of the lateral soil pressure:

$$E' = d\sin(t - \varepsilon + \varepsilon') / \cos(\varepsilon' \mp \delta'), \qquad (14)$$

where

$$t = \arcsin[c\sin(z'/d)];\; z' = \pi - y' - \varepsilon_e - \alpha_0;$$
$$d = [c^2 + R_{3e}^2 - 2cR_{3e}\cos z']^{0,5};$$
$$y' = \arcsin[S_2' \cos(\pm\varphi_e + \varepsilon' + \alpha_0)/c];$$
$$c = [G_{\gamma 3}'^2 + S_2'^2 + 2G_{\gamma 3}' S_2' \cos(\pm\varphi_e + \varepsilon' + \alpha_0)]^{0,5};$$
$$S_2' = b\cos(\xi + \varepsilon' \pm \varphi' - \psi')/\cos(\psi' \mp \varphi' - \varepsilon');$$

$$\xi = 0,5\pi - \gamma' + \varphi_e \mp \varphi' - \varepsilon';\; \gamma' = \arcsin[a\sin(\beta'/b)];$$
$$b = (a^2 + R_{2e}^2 - 2aR_{2e}\cos\beta')^{0,5};\; \beta' = \pi - \alpha' \mp \varphi_e - \alpha_0;$$
$$\alpha' = \arcsin[S_1' \cos(\theta_1' \pm \varphi')/a];$$
$$a = [G_{\gamma 2}'^2 + S_1'^2 + 2G_{\gamma 2}' S_1' \sin(\theta_1' \pm \varphi')]^{0,5};$$
$$S_1' = (g + G_q' + G_{\gamma 1}')\sin\theta_1'/\cos\varphi';$$
$$g = R_{1e}\sin(\theta_{1e} - \theta_1')/\sin\theta_1';$$
$$\psi' = \eta' + \operatorname{arctg}[\exp(\mp\theta'\operatorname{tg}\varphi') - \cos\theta']/\sin\theta'.$$

For use of the formulas cited, the angles φ’ and δ’, which correspond to some deformed stale of the structure, should be predetermined using relationships (1), i.e. the coefficient n should be found for the case under considerations. It follows from geometric considerations (see Fig. 5) that

$$n(h) = (V_{\gamma 1e} + V_{\gamma 2e} + V_{\gamma 3e})/(V_{\gamma 1e} + V_{\gamma 2e} + V_{\gamma 3e} + V'_{\gamma 1} + V'_{\gamma 2} + V'_{\gamma 3}). \qquad (15)$$

Using relationships (7)-(12) for the weights of the zones of the limiting and sublimiting stress states of the soil and proceeding to the volumes corresponding to them, it is possible to write

$$n(h) = [\Phi_1(\varphi_e) + \Phi_2(\varphi_e) + \Phi_3(\varphi_e)]/(\Phi_1(\varphi_e) + \Phi_2(\varphi_e) + \Phi_3(\varphi_e) - \Phi_2'(\varphi') - \Phi_4'(\varphi') - \Phi_4(\varphi_e') + \left(\frac{H}{h}\right)^2 [\Phi_1'(\varphi') + \Phi_3'(\varphi') + \Phi_5'(\varphi_e')]), \qquad (16)$$

where the functions $\Phi'_i(\varphi')$ are also expressed in terms of the parameter n through equations for determination of the angles θ', θ_1', η', E', φ' and δ'.

Having obtained a function of the form

$$x(n)=h^2\{(n-1)[\Phi_1(\varphi_e)+\Phi_2(\varphi_e)+\Phi_3(\varphi_e)]- \\ -n[\Phi'_2(\varphi')+\Phi'_4(\varphi')+\Phi_4(\varphi_e)]\}+ \\ +nH^2[\Phi'_1(\varphi')+\Phi'_3(\varphi')+\Phi'_5(\varphi')], \quad (17)$$

from (16), it is possible to satisfy the condition x(n) = 0 by the method of iterations (given the value of n, determining the angles φ' and δ', calculating the function $\Phi'_i(\varphi')$, and substituting them to the expression (17)).

Implementation of the kinematics methods that we have developed is simplest and most effective when determining loads taken up by operational thrust-bearing structures for which regular observations, including, among other things, geodesic measurements of displacements (settlements, shifts, tilts) and deformations (this applies primarily to retaining walls of such critical structures as quay walls, locks, docks and shore-protection constructions) are conducted in fulfilling the established operating regime.

Knowing specific parameters of the current deformed state of the structure, i.e., its generalized displacements, from observations and geodesic measurements, it is also possible, using the "E–u" curve, to estimate the current parameters of the loads due to the lateral pressure of the soil, which acts on the structure. Having defined the load values more precisely, it is easy to correct the calculation of the stress-strain state of the structure, linking it to a specific operational situation.

The problem of implementing kinematics methods in designing the entities of new construction when generalized displacements of the structure are not known a priori is more complex. In that case, the following algorithm may be recommended for the solution of the problem under consideration:

1) proceeding from operating conditions assigned in design for the structure, to determine the most probable pattern and direction of generalized displacements for the structure;

2) to assign incremental increase for the generalized displacement and determine (including graphically) the "E–u" relationship;

3) to calculate the external force effect T on the structure, which will be transmitted onto the soil through the contact face of the retaining wall (for the resisting pressure), or the force Q by which the structure resists displacements, as dictated by the thrust pressure of the soil; and,

4) laying off the T or Q values (depending on the direction of the possible generalized displacement of the structure) against the pressure axis of the E = E(u) diagram, to find on the U axis the corresponding value of the generalized pressure for which the perception of force T by the resisting

pressure, or force Q, which is not exceeded by the thrust pressure, is ensured, and estimate this displacement from positions that satisfy the requirements of the structure reliability.

All procedures for calculating the “structure/soil-medium” system which are discussed in this section have been represented by an algorithm; appropriate computational programs have been developed and mathematical modeling performed for these procedures. Numerical modeling was performed for considered problems; during these calculations, we investigated the effect of such factors as the direction, pattern, and magnitude of the generalized displacements of the structure; the topology of the “structure/soil-medium” system; the shape of the slip surface; and the accuracy of the assignment of the initial data.

Summarizing the results of the analysis of the numerical modeling and their comparison with experimental data[14, 15], it should be pointed out that employed kinematics method makes it possible to account accurately qualitatively and quantitatively the influence of generalized structure displacements on the condition and pattern of its interaction with the soil medium (for both the resisting and thrust soil zones).

DETERMINATION OF SOIL LATERAL PRESSURE LOADS ON A RETAINING QUAY WALL TAKING INTO CONSIDERATION ITS DISPLACEMENTS AND DEFORMATIONS

As mentioned above, the suggested method allows calculating the “lateral pressure (E) versus generalized displacement (u)” dependence within the entire range from the soil pressure at rest to the limit (active or passive) state achieved at the certain critical value of the generalized displacement. As far as we know, specific parameters of the current strain condition in the course of observations and geodetic measurements, it is possible to evaluate the characteristics of current soil lateral pressure loads acting upon the structure with the aid of the “E-u” dependence. Having specified load values, it is easy to correct the calculation of the stress-strain condition of the facility in its relation to a particular practical case.

When designing new facilities and in case if the generalized displacements of the facility are not known a priori, the following algorithm for a solution of the discussed problem may be recommended:

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[14] Doubrovsky M., Samorodov A., Muliar D. et al. (2017) Innovative design and technological solutions and test method for pile supports with increased bearing capacity. *Proceedings of the 19th International Conference on Soil Mechanics and Geotechnical Engineering*, Seoul, pp. 2735–2738.

[15] Doubrovsky M., Poizner M. (2016) Innovative development of coastal, port and marine engineering. Saarbrucken: Lambert Academic Publishing.

– based on the operational conditions of the facility that are assigned in the project, the most probable pattern and direction of the generalized displacements are determined;

– the increment of the generalized displacement is pre-set and the dependence "E-u" is calculated and determined (also, graphically);

– the external force T acting on the facility via the contact surface of the retaining wall and transmitted to soil (in case of passive soil pressure) or the resistance Q to the displacement of the facility caused by the lateral soil pressure (in case of active soil pressure) is determined;

– by plotting the force value T or Q (depending on the direction of possible generalized displacement of the facility) on the pressure axis of the plot E = E(u) to find on the u-axis the respective value of the generalized displacement (which in case of passive pressure withstands the force T or in case of active pressure does not exceed the force Q); the displacement "u" is evaluated in order to meet the requirements of the operational reliability of the structure.

The calculation method based on this algorithm has been developed and implemented in design for the systems "retaining structure – soil medium" in general and for some port structures in particular[16].

Several methods are developed to plot the diagrams of lateral soil pressure on the contact surface of a rigid retaining wall both for the limit (initial and final) and for the intermediate strain states of the system "structure – soil medium". They are based upon a possibility to define the resultant force of the lateral soil pressure as well as its limit and sublimit components.

The first method preserves the conventional linearity of the pressure diagrams for both zones of the stress state of soil and within the limits of the height $h < z < H$ for the sublimit zone. However, the latter case is considered for several variants of diagrams that reflect actual conditions of the interaction of the structure with soil medium including those that result in a partial non-linearity of the diagram.

The second method preserves linearity for the limit stress state zone only (i.e. for the active or passive pressure zone), while for the sublimit stress state zone the diagram is obtained as curvilinear.

According to the third method, the curvilinear diagram based on a parabolic approximation of soil lateral pressure intensity is plotted along with the entire height of the contact surface of the structure.

Let us consider the last (most general) method.

As the tests prove (for example, works[17, 18] and others), the soil lateral pressure diagrams are close to the parabolic (convex) curve. In most cases, the

[16] Doubrovsky M., Poizner M. (2016) Innovative development of coastal, port and marine engineering. Saarbrucken: Lambert Academic Publishing.

centre of pressure (the point where the resultant force of soil lateral pressure is applied) is located higher than the centre of gravity of conventional rectilinear (Coulomb) diagrams (from mentioned experiments, at about (0.40-0.45) H distance from the bottom of the wall, where H – wall height). In case if there is a surface load, the distance may be bigger – up to (0.45-0.53) H. In particular instances (when the structure obtains certain deformations) the diagram becomes concave one; its centre of gravity is located lower than in the Coulomb diagrams.

Generally, the equation describing pressure diagram ordinates versus depth is:

$$s(z) = az^2 + bz + c, \tag{18}$$

where: a, b, c – unknown coefficients;

z – the ordinate plotted vertically down from the point where soil surface crosses the contact surface of the wall (0<z<H).

To find the unknown coefficients in equation (18), we shall consider the following prerequisites and respective boundary conditions:

1) At any (even very small) displacement of the wall which means the respective displacement of its top end (it is not practically possible to fix it absolutely rigidly in an actually erected structure) the limit stress state appears at the point z= 0, i.e.:

$$s(z = 0) = c = ql, \tag{19}$$

where: q – surface load intensity;

1 – coefficient of soil lateral pressure (active or passive, depending on the direction of structure displacement).

2 – The pressure diagram area is numerically equal to the lateral pressure force related to the unit of the structure length, i.e.:

$$\int_0^H s(z)dz = E, \tag{20}$$

The value of E when the kinematics method is used, maybe determined for an intermediate strain state of the structure as the vector sum of the limit and sublimit components. In the extreme particular case when the limit soil pressure is achieved across the entire length of the contact surface E value may be found by means of one of the known methods without resort to plotting conventional rectilinear diagrams.

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[17] Gabi S., Doubrovsky M. and A. Belakrouf. New Development of Port Structures Design and Construction. *Journal of Shipping and Ocean Engineering*, Vol. 1, No 3, 2011, David Publishing Company, USA. pp. 150–157.

[18] Doubrovsky M., Poizner M. (2016) Innovative development of coastal, port and marine engineering. Saarbrucken: Lambert Academic Publishing.

(3) Taking into consideration mentioned experimental data, we may assume that the pressure centre location is known as well as its ordinate z_o in the adopted system of coordinates, then:

$$\int_0^H z\, s(z)dz \,/\, \int_0^H s(z)dz \ = z_0 \tag{21}$$

After integrating expressions (20) and (21) with due account of function (18) and condition (19), we obtain the following system of three equations containing three desired unknown coefficients a, b, c:

$$\begin{cases} c = ql \\ at_1 + bt_2 + ct_3 = E \\ \dfrac{at_4 + bt_1 + ct_2}{at_1 + bt_2 + ct_3} = z_0 \end{cases} \tag{22}$$

$$\begin{cases} a = (1/H[2A(3\nu - 2) + B]); \\ b = -A(4\nu - 3) - B; \\ c = ql = BH/6 \end{cases} \tag{23}$$

where: $A = 6E/H^2$; $B = 6ql/H$; $\nu = z_0/H$.

The obtained general formulae enable to come to the classic triangular diagram of pressures for the particular case v=2/3, i.e. when force E is applied at a distance of H/3 from the bottom of the wall, as in this case, when q = Q, the parameter “a” is always equal to zero irrespective of the value of E, and equation (18) is simplified to s (z) = bz.

Assessing the efficiency of kinematic calculation methods applied to retaining walls of the reviewed type which is revealed by comparison of the experimental and calculated data, considered methods and means have been used when analysing stress strain state of the gravity type berthing structures located in a number of the Black Sea ports and ship-repair yards.

For example, curvilinear (parabolic) diagrams of active soil pressure have been used for the berth’s designs in two Black Sea ports[19]. In the first case, a trapezoidal section concrete blockwork and cyclopean concrete superstructure (Fig. 6a), and in the second case the blockwork with the top cantilever block (Fig. 6d) designed by R&D Institute “Soyuzmorniiproekt” have been used. Shown in mentioned figures for comparison, the diagrams of contact pressures in stone bedding and the foundation soil were determined both by the conventional linear active pressure diagrams (Fig. 6 b, e) and by the parabolic

[19] Timashev S. A. Reliability of large mechanical systems. Moskva : Nauka, 1982. 184 p.

diagrams (Fig. 6 c, f). Both cases may be characterized by a considerably more unfavourable distribution of contact pressures in the case when parabolic diagrams are used. It is caused, evidently, by a higher location of the point where the resultant of the active pressure is applied. The latter consideration has resulted, in particular, in an increase of the maximum compressing pressures by 10-15 % as well as in the appearance of negative pressures near the rear edge of the wall or in a transformation of the trapezoidal diagram to the triangular one.

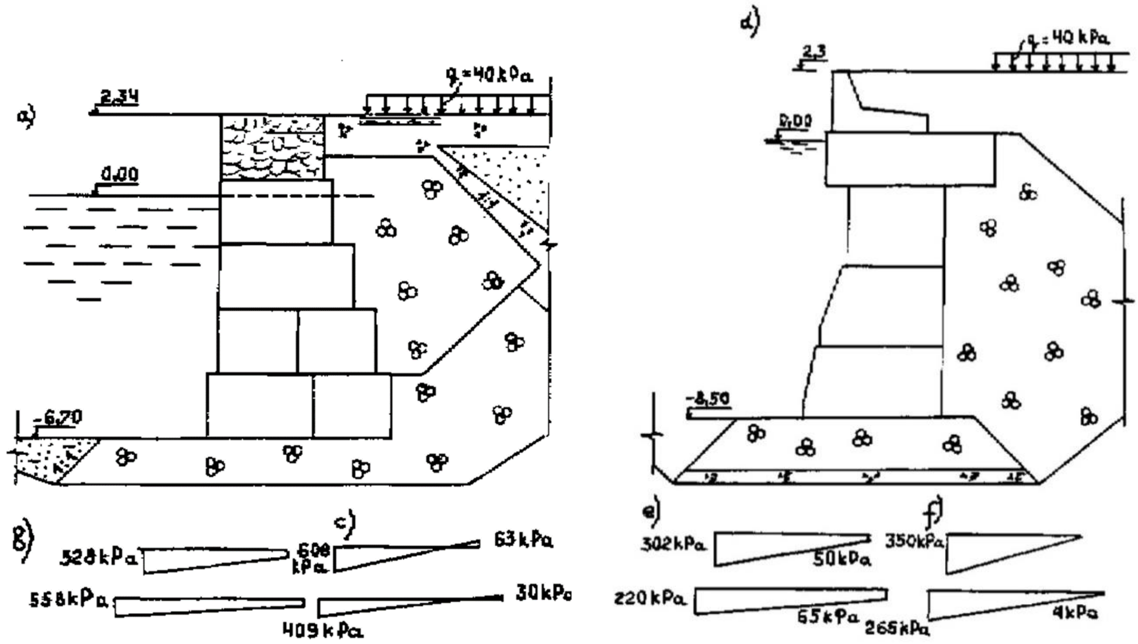

Fig. 2.6. Gravity type quay walls and contact pressure distribution diagrams
a – quay wall of trapezoidal section; b and e – contact pressures in stone bedding and in the foundation soil determined by use of the conventional linear active pressure diagrams; c, f – contact pressures in stone bedding and in the foundation soil determined by use of the proposed parabolic active pressure diagrams; d – quay wall with the top cantilever block

CONCLUSIONS

The applied kinematics model describing the interaction of the elements in "retaining quay wall structure – soil medium" system allows calculating the dependence of "lateral pressure – generalized displacement" type within the entire possible range of displacements and deformations for the considered structures that are alike by quality and close by quantity to the test data. The test-based diagrams of active and passive soil pressure acting on the contact surface of the structure make it possible to reflect, with a sufficient accuracy, the actual distribution of the lateral pressure intensity and to take into consideration the realized kinematics factors.

The calculation model applied in this work for the actual structures allowed to explain the phenomena observed in situ and to develop recommendations (both as to changing the loads exerted upon the berths and to structures reinforcement and reconstruction) on their further optimum and reliable operation.

SUMMARY

The article considers a new kinematics model describing the interaction of the elements of “retaining quay wall structure – soil medium” system. Both planar and non-planar slip surfaces for the studied system have been applied. The model allows calculating the dependence of “lateral pressure – generalized displacement” type within the entire possible range of displacements and deformations for the considered structures that are alike by quality and close by quantity to the test data. The test-based diagrams of active and passive soil pressure acting on the contact surface of the structure make it possible to reflect, with a sufficient accuracy, the actual distribution of the lateral pressure intensity and to take into consideration the realized kinematics factors. The calculation model applied in the paper for the actual structures allowed us to explain the phenomena observed in situ and to develop recommendations (both as to changing the loads exerted upon the berths and to structure reinforcement and reconstruction) on their further optimum and reliable operation.

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