(10^{-8} – 10^{-4} s), its stimulation by pulsed light irradiation can be the basis for new technologies for controlling the size of nanocrystals in the manufacture of film nc-Si.

References:

1. Beard, M. C., Luther, J. M., & Nozik, A. J. (2014). The promise and challenge of nanostructured solar cells. *Nature Nanotechnology,* no. 9, pp. 951–954.

2. Gavrylyuk, O. O., Semchuk, O. Yu., Steblova, O. V., Evtukh, A. A., & Fedorenko, L. L. (2014). Study of the distrubution of temperature profiles in nonstoichiometric SiO_x films at laser annealing. *Ukrainian journal of physics*, vol. 59, no. 7, pp. 712–718.

DEVELOPMENT OF MATHEMATICAL LOGIC AS A POWERFUL APPARATUS OF MODERN PHYSICS

Tetiana Obikod [1]

DOI: https://doi.org/10.30525/978-9934-26-002-5-51

The history of logic is rich and contradictory. However, one cannot fail to see the benefits of its development for science as a whole, which gave rise to a new, first formal, and then mathematical logic. The latter had such a strong influence on the development of modern mathematics and physics that we cannot ignore this topic. The history of the development of logic is usually associated with Greece. In spite of the fact, that no one mathematical work on the rules of reasoning has reached us, however, even then the axioms / Archimedes / began to be used. Plato said that «those who study geometry and arithmetic... act in order to arrive at what they had in mind to consider by common agreement» [1]. The transition to axioms seemed to the Greek mathematicians a solid foundation of science [2]. Euclid's «Beginnings» (III century BC) became an example of mathematical theory, an integral axiomatic system of mathematics. In the first book of the «Elements» Euclid gives 14 axioms of geometry and arithmetic, then numerous theorems are logically derived from them. Each theorem is deduced either from axioms or from other proved theorems, and according to the laws of Aristotle's logic / IV century BC / the new theorem is also true. The theory of real numbers of Eudoxus, set out by Euclid in the fifth book of the Elements, was used in Europe until the 17th century.

[1] Institute for Nuclear Research NAS of Ukraine, Ukraine

In the 12th and 13th centuries, after all the works of Aristotle were canonized, a medieval logic appeared, known under the name logica modernorum. It received its final design in the late 13th – mid-14th centuries. In the works of William Sherwood, Walter Burley, William Ockham, Jean Buridan and Albert of Saxony, [3]. In these writings a «universe of speech» appears and the idea of using language to express thoughts about facts and to express thoughts about language itself.

At the beginning of the 17th century, formal logic is developed. Representatives of this direction are, in particular, Viet and Descartes, [4], who reworked mathematical symbolism and made it close to modern one. «I wash, therefore I am», is the first and surest of all cognitions. A formalized language is considered as a combination of signs, in which only their concatenation or logical operations matters. Logical operations with concepts, as a result of which the content or volume of concepts changes, as well as the formation of new concepts, are carried out using: denial; restrictions; generalizations. Changes in the scope of concepts include: logical addition; logical multiplication; logical subtraction. These operations can be written mathematically using set theory.

Following R. Descartes, T. Hobbes and the Logicians of Port-Royal, Leibniz considered it possible to create a «universal symbolism», an artificial language free from the ambiguities inherent in natural languages, which would accurately and unambiguously express thoughts, [5]. Analyzing the categories of Aristotle, Leibniz came to the idea of isolating the simplest initial concepts – the «alphabet of human thoughts», which, when combined according to certain rules, should give all other concepts. Leibniz believed that simultaneously with such an analysis of concepts, it was possible to create a universal algorithm that would allow proving all known truths and thereby to compile an «evidential encyclopedia».

This idea later played an important role in mathematics and logic thanks to the works of G. Cantor and K. Gödel, [6]. Cantor developed the theory of infinite sets (see Set theory), established the existence of nonequivalent infinite sets, and formulated (1878) the general concept of the cardinality of a set.

In the XX century, it was possible to construct axiomatic set theories, free from the contradictions discovered earlier. Three main approaches were distinguished, called logicism, intuitionism and formalism.

Bertrand Russell expounded the ideas of logicism in his joint three-volume monograph Principia Mathematica (1910–1913), [7] with Alfred Whitehead, which made a significant contribution to the development of mathematical logic. Logicism asserts that mathematics and logic are a single whole, that is, the concepts and laws of logic are sufficient not only for the derivation of theorems, but also for the definition of mathematical objects.

The supporters of intuitionism, whose representatives are considered to be L. Kronecker, A. Poincaré, L. Brouwer, [8], on the contrary, put intuition as a source of truth above logic. According to Brower and other intuitionists, mathematics is completely the creation of human thought and does not depend on the external world. The practice of human action is useful for the development of new mathematical ideas, but is not necessary for their emergence.

The representative of «formalism» in the first half of the 20th century was Hilbert, [8]. Inspired by the success of his Foundations of Geometry, Hilbert wanted to build all mathematics (and, in the future, physics) on a single logical basis. He believed that for the disciplines lying in the foundations of mathematics, such as set theory and arithmetic, one can find a system of axioms from which any theorem of a given theory can be derived by purely syntactic transformations.

Subsequent investigations in this area led to the refinement of the concepts of «axiomatic system» and «proof», a restructuring on this basis of mathematical logic, and to the construction of formal axiomatic theories of sets, which are now recognized as the foundation of all mathematics.

The most prominent representatives of mathematical logic are Nicola Bourbaki – a group of French mathematicians /1935/, who defined mathematics as «the science of relations between objects, about which nothing is known, except for some of the properties or axioms that describe them. The ultimate idealization of the objects of mathematics made it possible to establish connections between the objects under consideration up to the construction of a hierarchy between them with the selection of elementary objects from which all the rest are built, [9].

As a result, the following mathematical theories were developed: number theory, mathematical analysis, groups, topology, lattices, rings, modules, fields, vector spaces, manifolds with all possible additional structures such as metric, connection, curvature and category theory.

In the 1960s, W. Loveer, [10], proposed a theory describing the concept of a category autonomously, without reference to set theory. Informally, a category in mathematics is understood as a set of objects with a system of transformations (morphisms) of one object into another. In the language of set theory, the concept of an object is interpreted as a set with an additional structure, and a morphism as a relation (usually a mapping) that preserves such a structure.

The Bourbaki school has found a particularly striking application not only in modern high-energy physics /HEP/, but also in astrophysics in the aspect of the theory of the Everything and evolution of the Universe. Here, instead of the classical vacuum, we consider a quantum vacuum with a complex topological structure, characterized by topological invariants, [11]. To characterize such a complex structure in physics, the concept of a manifold such as Calabi-Yau or orbifolds is introduced. These manifolds are, in turn, the bases of fibered spaces with a fiber – a structure group or C* algebra. They are characterized by K-groups, and their calculation gives information about the topological invariant – an analogue of energy in nuclear theory. The consideration of orbifolds in the HEP led to the category of McKay quivers, which are obtained after the blowing up of the orbifold singularities. This approach allows one to consider quivers as D-branes, and the morphisms between them as strings, whose vibrational modes give sets of elementary particles. The emergence of string and D-brane theory is associated with multidimensional spaces, which are searched for at the LHC as part of the search for microscopic black holes or Kaluza-Klein gauge boson partners. From the other hand the vibrational modes of strings are associated with superparticles and the theory of supersymmetry, which explains the problem of hierarchy and the problem of dark matter.

The theory of the evolution of the Universe operates with the concept of the problem of the initial singularity and the insufficiency of the description of the Universe by the classical general theory of relativity. According to one of the most popular hypotheses, associated with Stephen Hawking, Lawrence Krauss and Roger Penrose, the Universe could have arisen from nothing, i.e. from the «area» where there is no matter, through «quantum fluctuations». Here a special role is played by the mathematical apparatus, according to which space can be represented as a complex singular manifold. Within the framework of toric geometry, it is divided into polyhedra, and fluctuations between different vacuum states are described in the language of nesting or embedding of polyhedral, [11]. To each polyhedron is assigned its own Euler characteristic, which has the meaning of a triple in our world. Finding a variety with such a number is the task of modern cosmologists.

Summing up, it should be emphasized that the development of logic from natural to formal and then mathematical logic gave a powerful impetus to the development of modern mathematical directions in science, such as algebraic geometry, algebraic topology and category theory. A striking demonstration of the ideas of the deductive approach and logical positivism was the application of modern mathematical constructions to the modern HEP physics and the theory of the evolution of the Universe.

References:

1. Platon, La Republique, trad. E. Chambry, vol. Paris / Les Belles Lettres /, 1932–1949.
2. «Beginnings» of Hippocrates of Chios, 450 BC.

3. Ashworth, E.J., 'Terminist logic', in The Cambridge History of Medieval Philosophy, ed. Pasnau, R. two volumes, Cambridge: Cambridge University Press, 2010, pp. 146–158.

4. The Philosophical Writings of Descartes, v. I, II, transl. J. Cottingham, R. Stoothoff, & D. Murdoch, and v. III, transl. J. Cottingham, R. Stoothoff, D. Murdoch, & A. Kenny (Cambridge: Cambridge University Press, 1984, 1985, 1991).

5. Lenzen, W., 2004, «Leibniz's Logic», in Handbook of the History of Logic by D. M. Gabbay/J. Woods (eds.), volume 3: The Rise of Modern Logic: From Leibniz to Frege, Amsterdam et al.: Elsevier-North-Holland, pp. 1–83.

6. Zermelo, Ernst (1908). «Untersuchungen über die Grundlagen der Mengenlehre I». Mathematische Annalen. 65(2): 261–281.

7. Russell, Bertrand. The principles of mathematics, Cambridge: Cambridge University Press, 1923.

8. Ewald, William B., ed. (1996). From Kant to Hilbert: A Source Book in the Foundations of Mathematics. Oxford, UK: Oxford University Press.

9. URL: http://www.bourbaki.ens.fr/Ouvrages.html

10. F. William Lawvere. The Category of Categories as a Foundation for Mathematics (англ.) // Proceedings of the Conference on Categorical Algebra. – Springer, Berlin, Heidelberg, 1966. P. 1–20.

11. Tetiana Obikhod. High Energy Physics in the Context of D-BRANE Theory and Searches for New Physics at the LHC. Theoretical and practical aspects of the development of the European Research Area: monograph /edited by authors. 4th ed. Riga, Latvia: «Baltija Publishing», 2020. 354 p.