FORMATION OF WORKING SURFACES AND RESEARCH OF QUALITATIVE INDICATORS OF NON-EVOLVENT GEARS (REVIEW AND PROSPECTS OF DEVELOPMENT)

Yuriy Gutsalenko[1]
Tetyana Tretyak[2]

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Abstract. From the standpoint of the development of possibilities of application in theory and practice, the works of Prof. B. A. Perepelitsa from Kharkov Polytechnic Institute and his disciples to develop an applied methodology of multiparameter mappings in relation to the profiling and functioning of complex curvilinear objects and transmission mechanisms in mechanical engineering, mainly with examples of gears, are presented. The work substantiates the relevance of the study of gears with a complex non-involute profile of the side surfaces of the teeth, which in some applications have advantages over involute gears and are devoid of some of their drawbacks associated with quality indicators. A technique for obtaining mating surfaces of the teeth of non-invasive gears as envelopes of the specified surfaces of the teeth of tools is described. A scheme for forming pairs of non-involute gears, from which a gearing can be composed, is proposed. At the same time, diamond-abrasive tools are considered as shaping the working gear profile in its cutting according to the copying scheme and finishing according to the rolling honing scheme. In the first case, the profile of a special shaped tool on a high-strength metal bond is supported by a master electrode according to the scheme of the anodic connection of the tool into the electric circuit of dressing, similar to diamond spark grinding. In the second case, the use of gear wheels-hones on elastic ligaments is shown. It is shown that to obtain the mating surfaces of the

[1] Senior Staff Scientist, Senior Lecturer at M.F. Semko
Integrated Engineering Technology Department,
Institution of Education and Science in Mechanical Engineering and Transport,
National Technical University «Kharkiv Polytechnic Institute», Ukraine
[2] Senior Lecturer at M.F. Semko Integrated Engineering Technology Department,
Institution of Education and Science in Mechanical Engineering and Transport,
National Technical University «Kharkiv Polytechnic Institute», Ukraine

teeth of two non-involute gears, two tool rails can be used with the profiles of the side surfaces of the teeth opposite to each other. As a nonlinear profile of the tooth lateral surface of the tool rail, some part of one of the simulated flat kinematic curves is considered. A description of the program developed in accordance with the described method is given, which allows you to calculate the geometric characteristics of the shaped profiles of the gear pair wheels, visualize the shaping process, and also determine the quality indicators of the gearing. Thus, the prerequisites were created for choosing from the resulting geometric modeling of the curve field of such tooth profiles of the tools, which would provide the most rational combination of the tooth profiles of the gears processed by them and the required quality parameters of the gear teeth. The results of the study of the pressure ratio between the teeth of a gear and the overlap ratio of gears when choosing the shape of the tooth profiles are presented. A series of numerical experiments for gearing, formed by pairs of tool rails with different profiles of the side surfaces of the teeth   straight, convex and concave, as well as convex-concave   were performed. It is shown that non-involute gearing can have large reduced radii of curvature (and consequently smaller pressure coefficients) at the points of tangency of the profiles compared to involute gearing with a slight increase or decrease in the gearing overlap ratio. The most preferable is the variant of the rails with convex and concave tooth profiles, which provides the best values of both quality indicators of the engagement.

1. Brief excurse to beginnings of development

Historically, Ukraine has been traditionally noticeable in the global market for the development of the theory and practice of complex shaping, especially in the current challenges of the aerospace, nuclear power engineering, defense industry, including in terms of compact gears and CVTs for armored military and advanced civil equipment. So, in the world chronological list of works of scientists who made the most notable contribution to the theory of gears according to the modern version [1] (S. P. Radzevich, 2018), the first seven, followed by the representation of Italy (L. Da Vinci, 1493), Russia (L. Euler, 1754), England (R. Willis, 1841), France (T. Olivier, 1842) and the USA (F. Reuleaux, 1861, and J. W. Gibbs, 1863) closes H. I. Gofman's master degree work on mechanics [2]. It developed the analytical form of arbitrary gears and the basics of the modern analytical theory of gearing, created engineering methods for calculating and designing gearing, developed new types of gearing, the theory of kinematic pairs and chains, investigated the structure and derived the general equations of mechanisms, first classified them based on objective features which are determined by structure levels and movements.

The same prestigious list of works by 60 scientists includes Kharkov researcher A. I. Pavlov [3] and Hungarian researcher I. Dudas [4], who collaborates with the Kharkov scientific school in the applied development of the mathematical apparatus of multi-parameter mappings of space for gear wheels and gears [5]. The works [6-8] and some others in the list [1] also represent a number of ideologically close to development presented here on the mathematical apparatus and approaches used in it. In our experience, [9–14] also should be considered in this series of works.

At the same time, some well-known developments to enhance the adaptive capabilities of the mathematical apparatus of multiparameter mappings of affine space, for which the improvement of the theory and practice of gears is not an end in itself, but one of the directions of applied implementations, i. e. subordinate theme, are missing in the selected series of particularly significant works on the theory of gears [1]. Most likely, this is what explains the absence. Development of the Kharkov scientific school of Prof. B. A. Perepelitsa and his disciples at the Kharkov Polytechnic Institute (NTU «KhPI») [15–21] refers precisely to such works of a wide range of applications, including with respect to the description, improvement and synthesis of new gears and gearings. The history of this development is especially rich in fruitful cooperation with the Kiev Polytechnic Institute (P. R. Rodin, academic fundamentalist of the theory of instrumental production for complex cutting, and others at the NTUU “Igor Sikorsky KPI”) [22; 23] and the Institute for Superhard Materials (A. V. Krivosheya, world class coryphaeus in the theory and practice of tooth processing, now the leading specialist in Ukraine in this object-oriented area of applied development of the general theory of multiparameter mappings, and others at the V. Bakul ISM of the NAS of Ukraine) [24–27]. The significant results of this collaboration by the key participants mentioned above are jointly summarized in [28].

2. Introduction to general theory and methodology

The fundamentals of the new direction in the theory of shaping and designing of cutting tools and objects to be processed by them, based on the formalizing and generalizing capabilities of the mathematical apparatus of multiparameter mappings of affine space, were developed to the beginning of the 80-s of last century [15–17].

It was established that a general mathematical model of shaping during cutting can be a regulated multiparameter mapping. The general character of the model is that the workpieces with their edges are images, and the shaping movements of any complexity are one-parameter mappings. Consequently, to build one or another model of shaping it means to synthesize the corresponding mapping, that is, to completely determine its components. Initially, this requires completely, but in general terms, to determine the structure of the model, parameters and operators, the nature of the pre-image, operators, and functional connections between the parameters. Then it is necessary to assign or calculate the specific numerical content of all components of the model in accordance with the initial data. In the synthesis of the model, it is necessary to take into account the known regulatory conditions of contact and intersection, as well as the conditions for the completeness of shaping.

There are formulated the theoretical principles of modeling, which reflect different degrees of algorithmic generality: the principles of directional synthesis, coincidence, replacement, and inversion of mappings; principle of regulated intersection of images, etc. Based on these principles and the general model, systems of typical and specific models can be formed, each of which corresponds to a method or group of methods of real shaping, which potentially allows us to predict and develop new processing methods.

Multiparameter mappings are used as a general methodological basis for geometric design and shaping. In both processes, the same theoretical principles of modeling and the general mathematical apparatus operate. This commonality allowed to develop an automated structural modeling system, which covers a wide range of seemingly different in nature modeling and control problems in the design and shaping.

The basis of the universality of the system are a multiparameter mapping and a structural method based on the unification of parameters, operators and functional relationships. It is enough to specify a mapping in order to fully define the corresponding part or tool as a geometric shape.

By specifying the structural models of the workpiece, the machining tool and the shaping, as well as the corresponding numerical parameter arrays, this information can be converted into a form convenient for technological realization, for example, for computer-controlled machines and systems equipped with microprocessors and mini-computers. At the same time, it is advisable to introduce unified operators into the structure of the forming surface at the design stage of the part, providing for the use of the same operators during processing. In other words, if necessary, it is possible to foresee and use an algorithmic generality between the design of the part and its shaping.

A technique has been developed for applying of mappings for the mathematical description and construction of parts and tools as limited multi-parameter images. For example, by successive one-parameter mappings with the help of two operators (rotation and parallel transfer), it is possible to synthesize a gear from a starting point as a multi-parameter area of space. Gear equations in vector, matrix and coordinate form can be obtained by substituting operators into the general equation of their mapping. The combination of these equations and parameter intervals fully describes the gear wheel as a set of its points (inside and on the surface). In the particular case when the operators of rotation and parallel translations are mapping, and the independent parameter is time, the one-parameter mapping represents motion. Consequently, the general mapping equation in this case is an equation of arbitrarily complex motion.

Schematically generalized kinematic scheme of shaping for gear links of various classes, types and types proposed by prof. B.A. Perepelitsa can be represented in Figure 1. The movements of the initial shape forming link in its own coordinate system are indicated in the first coordinate system, and movements of the first coordinate system with the moving initial shape forming link are indicated in the second coordinate system, etc.

According to the generalized kinematic scheme of shaping (Figure 1), its generalized unified mathematical model, using 4th order matrices, can be represented as [21, p. 48]:

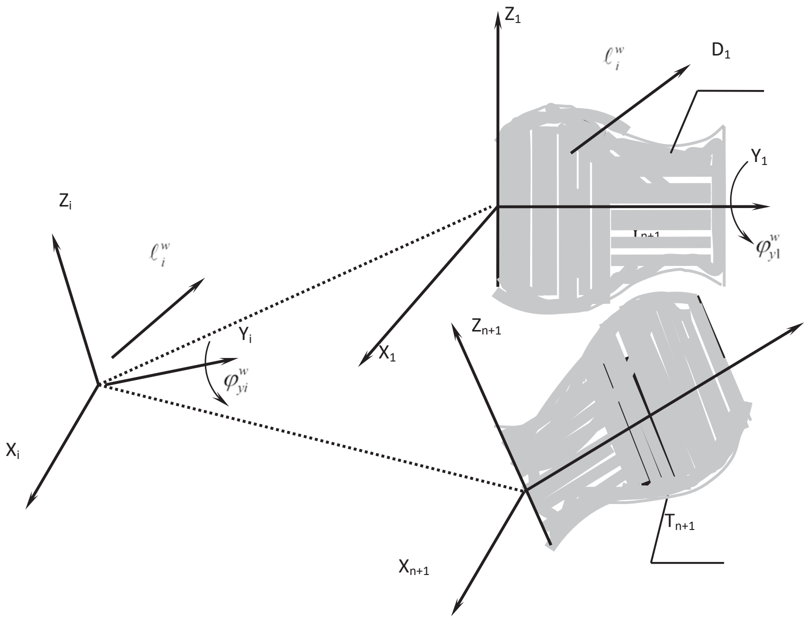


Figure 1. The generalized kinematic scheme of shaping for gear links [21, p. 47]:
T_1 and Dn+1 – forming and formable gear links, respectively

$$\begin{cases} m_{r\partial(n+1)T/D} = m_{\nu_n^w c_n^w} m_{\varphi_n^w \ell_n^w} \ldots \\ m_{\nu_i^w c_i^w} m_{\varphi_i^w \ell_i^w} \ldots m_{\nu_1^w c_1^w} m_{\varphi_1^w \ell_1^w} m_{r_{u1}} \\ \varphi_{1A}^u < \varphi_1^u < \varphi_{1A}^u \\ \varphi_{1A}^v < \varphi_1^v < \varphi_{1B}^v \\ \varphi_{1A}^w < \varphi_1^w < \varphi_{1B}^w \\ \left.\begin{aligned} \varphi_i &= f_{\varphi_i}\left(\varphi_1^w\right) \\ \ell_i &= f_{\ell_i}\left(\varphi_1^w\right) \end{aligned}\right\} \text{ coupling equations} \end{cases} \quad . \qquad (1)$$

In system (1), the last two equations are the equations of communication; φ_1^u, φ_1^v – independent parameters of the surface of the gear rims of the forming gear link; $\varphi_{1A}^{u}, \varphi_{1B}^{u}$ and $\varphi_{1A}^{v}, \varphi_{1B}^{v}$ – respectively, the initial and final values of the independent parameters of the surface of the gear rims of the forming gear link; φ_{1}^{w} – independent parameter of the movement of the forming gear link; $\varphi_{1A}^{w}, \varphi_{1B}^{w}$ – the initial and final value of the independent parameter of the movement of the forming gear link; $m_{r_{u1}}$ – matrix equation of the forming surface or line of the forming gear link in the first coordinate system; $m_{\varphi_i^w \ell_i^w}$ – motion matrix of the forming gear link in the coordinate system i; $m_{\nu_i^w c_i^w}$ – position matrix, i. e. matrix of coordinate transformations in the transition from the coordinate system i to the coordinate system $i + 1$; $m_{r_{\partial}(n+1)\,T/D}$ – the matrix equation of motion of the forming link (T_1), specified in the 1st coordinate system relative to the fixed formable gear link (D), written in the coordinate system $n + 1$, i. e. in the coordinate system of the taken form gear link.

The development of structural mathematical modeling in relation to the processes of shaping the cutting of parts of complex shape leads to the development of matrix formalization of geometric objects of processing, that allow, firstly, on a uniform algorithmic basis, to consider both the geometric design of surfaces, and their shaping, and with a significant reduction in the amount of processed information; secondly, not only to model, but also to investigate the geometric properties of structures, to predict and develop on the basis of this new surfaces, tools and kinematic schemes of shaping; thirdly, to provide the necessary prerequisites for processing complex-shaped parts without a drawing, using structural mathematical models of type (1) as an intermediate storage medium. Based on the previous national experience of the calculation and methodological support for the processing of complex non-linear surfaces on machine tools with numerical control [29], a consistent development of a common method for calculating the trajectory and coordinates of instantaneous points of contact, applied to any shaped instrumental surface and based on special improvements and applied system development of the mathematical apparatus of multiparameter mappings of affine space, is carried out [18–20], Figure 2. The addition of the existing methods of analytical description of discretely defined surfaces by the developed method of virtual modeling made it possible to expand the range of types and profiles of tool surfaces used on CNC machines.

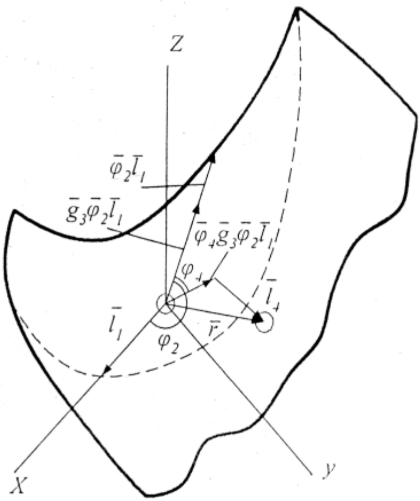


Figure 2. 3D model of the turbine blade surface in the structure with mapping operators (fragment) [30, p. 70]

3. Model and tools for shaping of gears

The relevance of improving the mathematical apparatus of analysis and synthesis of gearing, especially from the last quarter of the last century, respectively, and the development response, is fueled by the globalization of competition in the automotive industry, the demands of the aerospace industry and the military-industrial complex to create gears and variators of increased compactness and (or) supporting capacity, in particular two-parameter [31–35] (Figure 3), with the spherical design of the teeth carrying elements [36; 37], etc.

The main difficulties of design and practical implementation of gear-shaping technologies in consideration contacting in gearing reducer solids according Figure 3 are tied with shape-generation complexity of bevel gear with equidistant teeth lines. We [34, p. 69] are investigated three models of its shaping based on copying and revolving shaper tools. Omitting here consideration of the first two models respectively supported on the use of pencil and disk shaping cutters (milling cutters) we will consider third scheme of shaping (Figure 4) remembering that permanent meshing of

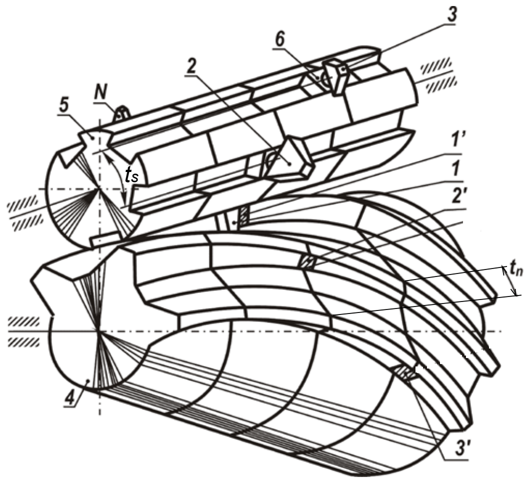


Figure 3. General schematics of double-link variator on the base of bevel gear with equidistant tooth lengthwise curves: *1, 2, 3, … N* – movable tooth of spur gear; *1', 2', 3'* – applicable contact zones; *4* – bevel gear with constant normal pitch; *5* – spur compound gear; *6* – sliding elements jointed with tooth [21, p. 221]

kinematics by rise of movement number is taken placed when conversion from the first to the third shaping method [34, p. 69].

According to the third scheme of shaping a gear production is realized by revolving than with indexing movement such as by the first two schemes. In generalized structural formulation:

$$\overline{r} = \overline{\varphi}\overline{u}\overline{\beta}\overline{R}_1 + \overline{\varphi}_1\overline{\psi}\overline{R}_2 + \overline{\varphi}_1\overline{\psi}\overline{u}\overline{R}_2 \ . \qquad (1)$$

Accordingly (1), tool with generatrix $\overline{R}_1$ makes additional turn $\overline{\beta}$ ($\overline{\beta}_{\partial on}$, Figure 4) and at the same time translational movement $\overline{u}$ along pitch element $\overline{R}_2$ which commits rotary $\overline{\psi}$ and reverse $\overline{\varphi}_1$ motions about proper axis.

Shaping by the first two methods does not reproduce the kinematics of real gearing in full, necessity of divider application is leading to accumulation of error on every next tooth. The first method (with point contact of tool surface) can be considered as scheme of preprocessing. The second one (with more accurate line shape-generating contact) can be used for making of

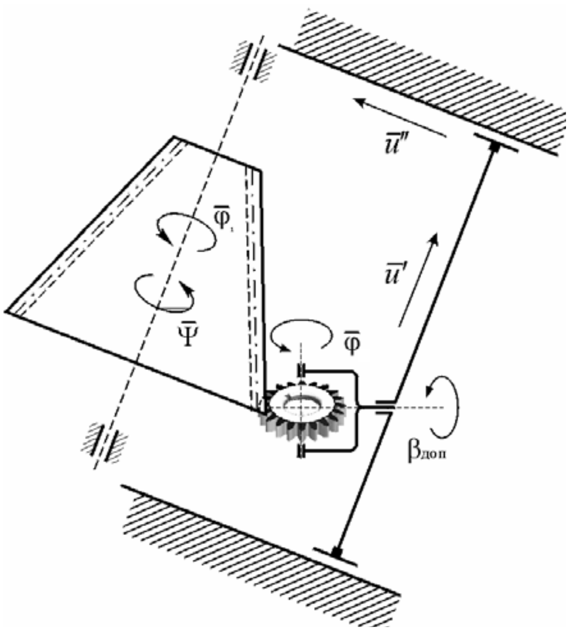


Figure 4. Kinematic scheme of model with initial position of revolving tool (shaper cutter) relative to feedwork [34, p. 69]

gears with mean quality indexes. Examined here the third method excludes all listed shortcomings, but needs in high-value equipment. Realization of this processing technique by revolving method allows to obtain theoretically accurate tooth surface profiles and make bevel gears of examined tooth double-link variable-speed mechanisms with high quality metrics.

At the same time, diamond-abrasive tools are considered as shaping the working gear profile in its cutting according to the copying scheme and finishing according to the rolling honing scheme. In the first case, the profile of a special shaped tool on a high-strength metal bond is supported by a master electrode according to the scheme of the anodic connection of the tool into the electric circuit of dressing, similar to diamond spark grinding [39, p. 137]. In the second case, the use of gear wheels-hones on elastic ligaments is shown [40].

In fulfilled engineering of edge gear cutting tools for shaping of noninvolute gears with constant normal pitch it is provided for exclusion of distorted profiling after tool regrinds. There are proposed constructive approaches; procedure for estimate of influence of bevel gear's with constant normal pitch shaping parameters on quality metrics of two-parameter toothed gearing [38, p. 196]; generalized [21, p. 159–176] and particular calculation algorithms, which may be used in dataware of respective CAD/CAM systems of maintenance for tooling backup. Among developed tools there are assembled shaping cutters with prismatic and round cutters [21, c. 213–218]. Compensatory possibilities of proposed assembled shaping cutters are ensured by repositioning of shaped cutting edges after their regrindings by linear displacement of prismatic shaper cutters and angular displacement round ones respectively.

4. Geometric analysis of shaping by rolling gear cutting tools

Based on the previously developed algorithm for calculating the profile of envelope surfaces [41], a method for geometric analysis of the process of surfaces shaping by rolling gear cutting tools has been developed. In this case, a new structural approach to find the formable and tool surfaces as envelopes [42], which does not require the derivation of specific analytical equations, is used [43].

Using computer graphics, the sequential shaping of the space between the teeth of the gear wheel is studied. A plane geometric task is being solved. The workpiece and the shaping tooth of the tool (for example, tool rack) can be considered as geometric figures, i.e. bounded subsets of plane points. As noted earlier, the boundaries of the tooth of the tool can be not only straight line segments, but also various other types of curves.

In motion relative to the workpiece, the tool tooth as a geometric figure sweeps on the plane a region representing a set of trajectories of points. The boundaries of this area. which is swept, are either the trajectories of single points of the tooth, or the envelopes of certain curves that bound the tooth [44].

Examples of computer graphics for shaping of gear wheel with the module $m = 5$ and the number of teeth $z = 20$ are shown in Figure 5 and Figure 6 (more see [45]).

In Figure 5, a, b the initial profile of the tool rack with a concave profile of the tooth flanks and the successive shaping of the space between the teeth of wheel are shown. In the figures, the workpiece points, in which, at the current time, the condition of touching the surfaces is fulfilled, are highlighted. The combination of these points represents the envelopes of certain curves that limit the tool tooth. They are marked in Figure 5, c. Figure 5, d images the boundaries of the desired space between the wheel teeth.

Figure 6 represent the space shaping between the teeth of gear when cutting by tool rack with a convex profile of the tooth flanks.

The proposed method allows to analyze the process of shaping the surface of the gear wheel, detect the appearance of undercuts, and also determine the range of gear wheels which can be obtained using a specific rolling tool.

5. Method of obtaining of corrupted surfaces

When cutting gears by the running in method, both the tool rail and the tool gear can act as a tool [46; 47]. The surfaces of the teeth of two involute gears, which are envelopes of the surfaces of the teeth of the same tool, are mating. To obtain the mating surfaces of the teeth of two non-involute gears, two different tools are required. So, for example, if the tools are two tool rails, then the profiles of the side surfaces of the teeth of the rails should be inverse to each other, i.e. the tooth profile of the lath 1 should coincide with the groove profile of the lath 2. Accordingly, the gears 1 and 2, the side surfaces of the teeth of which are the envelopes of the side surfaces of the teeth of the tool racks 1 and 2, can participate in the gearing. Based on this, a technique has been developed for obtaining the mating surfaces of the teeth of non-involute gears as envelopes of the specified surfaces of the tool teeth.

At the first stage of forming a pair of gears, tool racks 1 and 2 with nonlinear profiles of the side surfaces of the teeth opposite to each other can be considered as tools. With these rails, the process of manufacturing two non-involute gears with given numbers of teeth is modeled. These wheels can be further considered as instrumental, with the help of which the process of cutting other gears with different numbers of teeth is simulated in the next stage of shaping. This sequence is repeated until the gears that make up the gearing are molded. The formation sequences of wheel 1 and wheel 2 may contain a different number of stages, but this number for both wheels must be either odd or even.

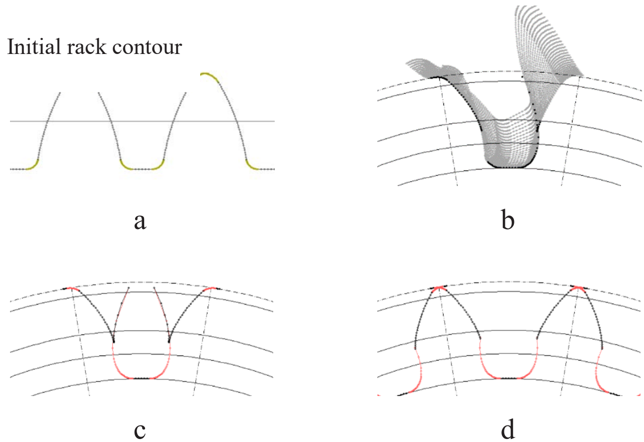


Figure 5. The shaping of the space between the teeth of the gear wheel with *m* = 5 and *z* = 20 by tool rack with a concave profile of the tooth flanks

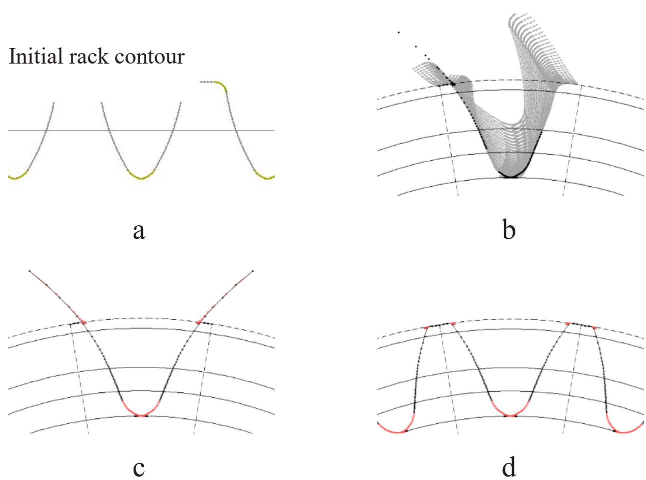


Figure 6. The shaping of the space between the teeth of the gear wheel with *m* = 5 and *z* = 20 by tool rack with a convex profile of the tooth flanks

In addition, at the first stage of shaping each wheel of a gear pair, the tool can be the same tool rail. The mating side surfaces of the teeth will have gears made in the same shaping sequence, if the number of shapes for one wheel is odd and for the other wheel is even.

The method of obtaining mating surfaces of the teeth of non-invasive gear wheels as the envelopes of the specified surfaces of the teeth of the tools in accordance with the proposed scheme of forming pairs of gear wheels suggests the following sequence of actions [48; 49]:

1. Sets the number of shaping stages for wheels 1 and 2.

2. In the frame associated with the tool rail in the first stage of shaping (or the instrumental gear wheel in subsequent stages of shaping), a set of coordinates and geometric characteristics of the points of the original tool profile are specified, and the parameters of the gear to be machined are also set.

3. On the basis of the algorithm for calculating the profile of the enveloping surfaces described in [41], the coordinates and the geometric characteristics of the profile points of the gear to be machined are calculated in the frame associated with the gear.

Points 2 and 3 are performed in a cycle for a given number of shaping stages for gear 1 and gear 2.

As a nonlinear profile of the side surface of the tooth of the slats, some section of one of the flat kinematic curves modeled earlier and described in [42; 44] can be considered.

6. Results of research of quality indicators

The interaction of two teeth, transmitting the load, is accompanied by a collapse of the surfaces of these teeth in the zone of contact. The surfaces of straight teeth can be approximately taken as the surfaces of round cylinders whose radii are equal to the radii of curvature of the corresponding surfaces at the points of tangency. When touching the surfaces of the teeth, which are cylindrical surfaces with varying radii of curvature, under the action of compressive forces, a contact area in the form of a narrow strip is formed, located along the line of contact of the teeth. The highest specific pressure (contact collapse stress) per unit length of the contact line will be in the middle of this strip directly on the contact line.

Element geometry of the teeth is the reduced radius of curvature ρ_r, which characterizes the interaction of the two teeth of the transmission wheels and is proportional to the module gearing m. Analyzing its magnitude at the points of contact, one can judge the width of the contact area and the magnitude of contact stresses at these points [50; 51].

For an external gearing, the reduced radius of curvature of the tooth profiles at the contact points, mm, is calculated by the formula:

$$\rho_r = \frac{\rho_1 \cdot \rho_2,}{\rho_1 + \rho_2}, \tag{2}$$

where ρ_1 and ρ_2 – the radii of curvature of the teeth profiles at the points of contact, mm.

To characterize the influence of the geometric shape of the teeth on the specific pressure regardless of the magnitude of the modulus, the specific pressure ratio is introduced as the ratio of the modulus to the reduced radius of curvature [52]:

$$\vartheta = \frac{m}{\rho_r}. \tag{3}$$

The coefficient ϑ does not depend on the module and, like the reduced radius of curvature ρ_r, does not refer to a separate wheel, but is a transmission characteristic. Since at different points of the tooth profiles ρ_1 and ρ_2 have different numerical values, the values of ρ_r (2) and ϑ (3) for different pairs of tangency points are also different.

An important characteristic of gearing is also the overlap ratio, which allows to evaluate the continuity and smoothness of the gearing in its work. For a cylindrical spur gear, the overlap ratio ε is defined as the ratio of the gearing arc of a gear wheel along any circle to the step size along the same circle. The transfer is continuous at $\varepsilon > 1$, and the greater this superiority, the higher its smoothness [52].

For a cylindrical gear, the angle of rotation of the gear from the position corresponding to the entry into gearing to the position corresponding to the disengagement of its working profile is called the angle of overlap of the wheel φ_{ov} [8; 53].

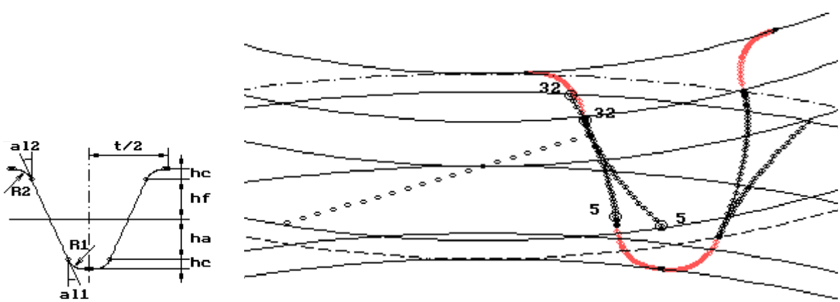
The overlap ratio can be represented as the ratio of the overlap angle of any of the transmission wheels to the angular pitch of the same wheel:

$$\varepsilon = \frac{\varphi_{ov} \cdot z}{2\pi}, \tag{4}$$

where z – the number of teeth of the wheel.

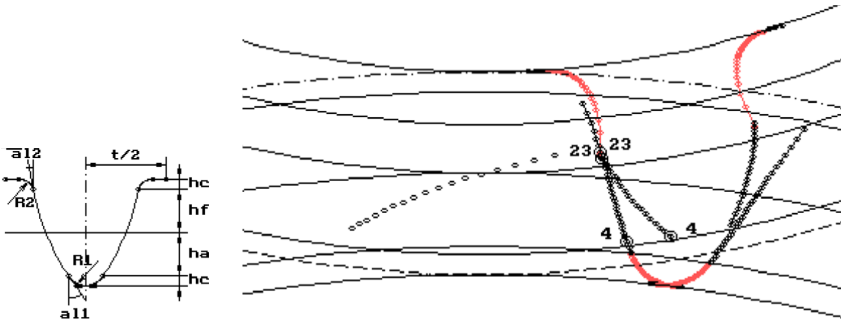
In accordance with the method described above for obtaining mating teeth surfaces, a Pascal program has been developed, the results of which are: calculating the coordinates and geometric characteristics of the points of the shaped profiles of gear wheels 1 and 2 that make up the gear pair (including the radii of curvature at the points of the tooth profiles); output on the computer screen images of the side surfaces of the teeth, their movements in the process of running with the selection of points at which at the considered moment of time the condition of contact of the profiles was fulfilled (their combination is the line of engagement). The program also allows you to: check the correctness of the mating of the side surfaces of the teeth in the gearing and determine the active parts of their profiles; calculate the length of the active sections of the tooth profiles, the angles of overlap of the gear wheels and the overlap ratio of the gear teeth; calculate the reduced radii of curvature and the pressure ratio at the points of contact of the tooth profiles.

Examples from the series of numerical experiments to analyze the quality indicators of gearing formed by slats with different profile of the side surfaces of the teeth [54] are presented in Figures 7-9. Examples from Figures 7-9 correspond to gearings of the side surfaces of the teeth of gears



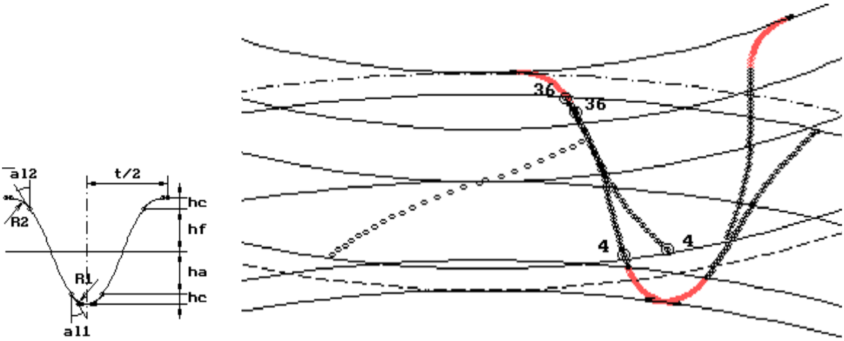
The length of the active portion of the tooth profile:
gear 1 – 8,663 mm; gear 2 – 8,309 mm.
Gearing overlap ratio: ε = 1,573.

Figure 7. The engagement of the profiles of the side surfaces of the teeth of gears with m = 5, z_1 = 20 and z_2 = 30, formed by tool rails with straight profiles



The length of the active portion of the tooth profile:
gear 1 – 6,619 mm; gear 2 – 5,137 mm.
Gearing overlap ratio: ε = 1,732.

Figure 8. The engagement of the profiles of the side surfaces of the teeth of gears with m = 5, z_1 = 20 and z_2 = 30, formed by tool rails with convex and concave profiles



The length of the active portion of the tooth profile:
gear 1 – 9,550 mm; gear 2 – 9,419 mm.
Gearing overlap ratio: ε = 1,268.

Figure 9. The engagement of the profiles of the side surfaces of the teeth of gears with m = 5, z_1=20 and z_2 = 30, formed by tool rails with convex-concave profiles

with a module m = 5 and the number of teeth z_1 = 20 and z_2 = 30, formed with tool rails 1 and 2 with straight, convex and concave, as well as convex concave profiles of the side surfaces of the teeth, respectively (the profiles of the tooth of the tool rails 1 and the hollow of the tool rails 2 are shown on the left). In the first case, the tooth profiles of the gears are involute.

Figures 7-9 show the positions of the profiles of the teeth of gears in the process of running, points are marked, at which the condition of touching the surfaces has been fulfilled at the current time. The combination of these points is a line of engagement. Selected active sections of the profiles of the side surfaces of the teeth. The numerical values of the lengths of the active sections of the profiles and the overlap ratio of the gearing are given. Graphs with the numerical values of the pressure ratio at the points of contact of the tooth profiles are shown in Figure 10.

As shown by the results of numerical experiments, non-involute gears can have large reduced radii of curvature (and consequently smaller pressure ratios) at the points of tangency of the profiles compared to involute gears with a slight increase or decrease in the gear overlap ratio.

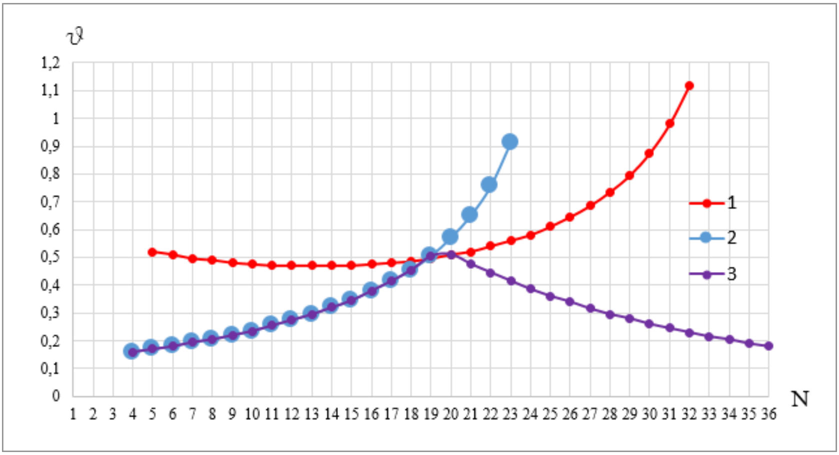


Figure 10. The pressure ratio at the points of contact of the teeth of gears with m = 5, z_1 = 20 and z_2 = 30, formed by pairs of tool rails with side tooth profiles: 1 – straight, 2 – convex and concave, 3 – convex-concave

In presented examples in gearing formed by rails 1 and 2 with convex and concave tooth profiles, as well as rails 1 and 2 with convex-concave tooth profiles, the pressure ratio at the points of contact of the profiles is less than 1.6 and 1.9 times than meshing, formed by slats with straight profiles. The overlap ratio (4) of gearing for them is 1.73 and 1.27, whereas for involute gearing it is equal to 1.57. The most preferable is the variant of the rails with convex and concave tooth profiles, which provides the best values of both quality indicators of the engagement.

7. Conclusions

Founded by Prof. B. A. Perepelitsa with a brief overview of presented here related publications, working systematically develops applied principles and virtual concretization tools for a scientific theory of multiparameter mappings of affine space in the application to shaping of complex curvilinear surfaces, including gearing, that makes it possible to synthesize novel gears with the desired performance.

Ways of further integration of development in modern engineering, in accordance with its achieved theoretical level and applied assimilation, should be primarily associated with the triad of continuously relevant organizational, technical and technological tasks of increasing the competitiveness of enterprises of the engineering complex with mechanical and laser shaping of constructional parts and forming tools for aircraft, transport, energetics and others branches of the economy. First, it is a virtual modeling of unitary objects and technical systems with new design solutions of increased functional efficiency, including using the developed structural approach to the mathematical description and visualization of kinematic curves [21; 44].

Secondly, it is the integration into the design of individual machining operations with varying degrees of certainty at the micro level kinematic-geometric schematics, including the prediction of the output microcutting parameters for optimizing the processing with abrasive tools at the macro level, by analogy with the developed adaptation of the general methodology that takes into account the functional specifics interference objects set forth in the works of the scientific school of prof. B. A. Perepelitsa [19; 23; 55; 56].

Thirdly, this is the use of development in the creation of applied mathematical software for renovation algorithms and the original integrated design of highly efficient multi-operational production processes and complexes of such processes of an industrial enterprise, including in the framework of the implementation of the concept of simulation modeling in accordance with the experience of development of the NTU «KhPI» in this direction [26; 57].

One of the current trends is the study of gears with a complex non-involute profile of the side surfaces of the teeth. This is due to the fact that involute transmissions have a number of drawbacks related to their quality indicators: large specific pressures on the side surfaces of the teeth due to their small radii of curvature and hence insufficient contact strength, small overlap ratio of the wheels and hence insufficient smoothness of engagement.

Cutting the teeth of gears can be made by copying or by the method of rounding (running). Agreeing with the innovative updates of the copying method in relation to some tasks characteristic of, for example, the mechanisms of autotractor transmissions, where the profiling of gear wheels with a tool-copier (for example, circular pulling [58]) allows to provide an increased radius of curvature of the transition curve at the base of the tooth, thereby reducing the stress concentration in its dangerous section and, accordingly, increasing the flexural strength of the tooth, the authors associate this study with the development of the capabilities of a more technologically advanced, kinematically accurate and productive method of rounding (running), which it is now and is represented in modern views on the history and future of the theory and practice of gearing [1].

A mathematical model for calculating the tooth profile of a gear-cutting tool for machining of non-involute gear wheels has been developed. As a profile of the rack tooth flank, a certain section of one of the modeled kinematic curves is considered.

A technique has been developed for obtaining the mating surfaces of the teeth of non-involute gears as envelopes of the specified surfaces of the teeth of the tools.

Based on the algorithm for calculating the profile of envelope surfaces, a method for geometric analysis of the process of shaping surfaces by rolling gear cutting tools has been developed.

Using computer graphics, the sequential shaping of the space between the teeth of non-involute gear wheels processed by rolling-in gear-cutting tools with a given tooth profile has been investigated.

A scheme is proposed for forming pairs of non-involute gears, from which a gearing can be composed. The analysis of the pressure ratio between the teeth of the gear transmission and the overlap ratio of the gear teeth when selecting the shape of the profiles of the teeth. Prerequisites have been created for choosing from the resulting geometric modeling of the curve field of such tooth profiles of tools, which would ensure the most rational combination of tooth profiles of the gears processed by them and the required quality indicators of gear teeth.

The presented development can be integrated with other models, algorithms, software products made by computer animation, constructive development and research of gears, shaping hopping tools for cutting teeth and gears for a given tool. For example, in [59], performed without considering the specifics of non-involute gears, the basis of computer simulation of the formation of a gear wheel in CAD systems (MathCAD and Maple) also laid the contour of the instrumental rail, and announced the corresponding algorithms and their implementation as programs written in AutoLISP in the AutoCAD environment, with input of the initial data through dialog boxes in the DCL (Dialog Control Language). From the standpoint of the development of the development presented here, the integration attractiveness of work [59] is associated with the implementation of solid modeling in it and, on this basis, along with the profiling of the toothed cutting tool, demonstrated by the study of cutting them teeth, obtaining the qualitative and quantitative characteristics of the process achieving optimal calculation and prognostic solutions, the prospect of which is particularly interesting in relation to many design and technological re- no involute gearing.

References:

1. Radzevich, S. P. (2018) *Theory of gearing: kinematics, geometry, and synthesis.* 2nd ed., Boca Raton, Florida, CRC Press.

2. Gofman, H. I. (1886) *The gear theory, generalized and developed by analysis* [*Teorija zaceplenij, obobshhennaja i razvitaja putem analiza*]. Odessa, Novorossiysk Univ. (in Russian)

3. Pavlov, A. I. (2005) *Modern theory of gearing* [*Sovremennaja teorija zubchatyh zaceplenij*]. Kharkiv, Kharkiv Nat. Automobile&Road Univ. (in Russian)

4. Dudas, I. (2000) *The theory and practice of worn gear drives*. London, Penton Press.

5. Perepelitsa, B., I. Dudas, and N. V. Zubkova (2004) Unification of operators and elements of space mapping in 3D modeling of cutting tools [Unifikacija operatorov i jelementov otobrazhenij prostranstva pri 3D-modelirovanii rezhushhih instrumentov]. *Science and education: Collection of works (to the 40th anniversary of the cooperation of the National Technical University "Kharkiv Polytechnic Institute" and the*

University of Miskolc) [*Nauka i osvita: Zbirnyk naukovykh prats (do 40-richchia spivpratsi Natsionalnoho tekhnichnoho universytetu "Kharkivskyi politekhnichnyi instytut" ta Mishkoltskoho universytetu)*]. Kharkiv, NTU "KPI", 169–174. (in Russian)

6. Baxter, M. L. (1961) Basic geometry and tooth contact of hypoid gears. *Industrial Mathematics*, 11(2): 19–42.

7. Lashnev, S. I. (1971) *Generation of gear tooth flanks by means of rack-type and worm-type cutting tools* [*Formoobrazovanie zubchatyh detalej reechnymi i chervjachnymi instrumentami*]. Moscow: Mashinostroenie. (in Russian)

8. Litvin, F. L. (1968) *Theory of gearing* [*Teorija zubchatyh zaceplenij*]. 2nd ed. Moscow: Nauka. (in Russian)

9. Lyukshin, V. S. (1967) *Theory of screw surfaces in the design of cutting tools* [*Teorija vintovyh poverhnostej v proektirovanii rezhushhih instrumentov*]. Moscow: Mashinostroenie. (in Russian)

10. Grechishnikov, V. A., G. I. Kirsanov, and A. V. Kataev (1984) *Automated design of metal-cutting tools* [*Avtomatizirovannoe proektirovanie metallorezhushhego instrumenta*]. Moscow: Moscow Machine Tool Inst. (in Russian)

11. Druzhinsky, I. A. (1985) *Complex surfaces: mathematical description and technological support* [*Slozhnye poverhnosti: matematicheskoe opisanie i tehnologicheskoe obespechenie*]. Leningrad: Mashinostroenie. (in Russian)

12. Radzevich, S. P. (1991) *Shaping complex surfaces on CNC machines* [*Formoobrazovanie slozhnyh poverhnostej na stankah s ChPU*]. Kiev: Vishcha shkola. (in Russian)

13. Litvin, F. L. (1997) *Development of Gear Technology and Theory of Gearing*: US NASA Reference Publication 1406. Cleveland, Ohio, Lewis Research Center.

14. Grechishnikov, V. A., A. R. Maslov, Yu. M. Solomentsev, and A. G. Shirtladze (2000) *Instrumental provision of automated production* [*Instrumental'noe obespechenie avtomatizirovannogo proizvodstva*]. Moscow: Stankin. (in Russian)

15. Perepelitsa, B. A. (1974) On geometric shapes as affine varieties [O geometricheskih figurah kak affinnyh mnogoobrazijah]. *Applied geometry and engineering graphics* [*Prikladnaja geometrija i inzhenernaja grafika*], 1974/24: 47–48. (in Russian)

16. Perepelitsa, B. A. (1976) Cutters as affine manifolds [Rezhushhie instrumenty kak affinnye mnogoobrazija]. *Cutting and tooling* [*Rezanie i instrument*], 1976/16: 36–40. (in Russian)

17. Perepelitsa, B. A. (1981) *Mapping of affine space in theory of surface forming by cutting* [*Otobrazhenija affinnogo prostranstva v teorii formoobrazovanija poverhnostej rezaniem*]. Kharkiv: Vyshcha shkola. (in Russian)

18. Perepelitsa, B. A. (1985) *Automated profiling of cutting tools (theory and algorithms)* [*Avtomatizirovannoe profilirovanie rezhushhih instrumentov (teorija i algoritmy)*]. Kharkiv: KhPI. (in Russian)

19. Development of the theory of three-dimensional (3D) geometric modeling of cutting tools and the processes of shaping surfaces based on multiparameter mappings of affine space [*Razrabotka teorii trehmernogo (3D) geometricheskogo modelirovanija rezhushhih instrumentov i processov formoobrazovanija poverhnostej na osnove mnogoparametricheskih otobrazhenij affinnogo prostranstva*]: Research report No. 0202U002517 (2001). Under the guidance of B. Perepelitsa. Kharkiv: NTU "KhPI". (in Russian)

20. Perepelitsa, B. A. (2006) Systems of parameters of space mappings in the annex to 3D modeling [Sistemy parametrov otobrazhenij prostranstva v prilozhenii k 3D-modelirovaniju]. *Modern technologies in mechanical engineering* [*Suchasni tekhnolohii u mashynobuduvanni*], 2006/1: 402–408. (in Russian)

21. Creation of theory of unified multiparametric information database for CAD/CAM systems of gearing, tools and gear processing [*Sozdanie teorii unificirovannoj mnogoparametricheskoj informacionnoj bazy dlja CAD/CAM sistem zubchatyh zaceplenij, instrumentov i processov zuboobrabotki*]. Research report No. 0210U001273 (2009). Under the guidance of B. Perepelitsa & V. Dobroskok. Kharkiv: NTU "KhPI". (in Russian)

22. Rodin, P. R. (1990) *Fundamentals of cutting tools design* [*Osnovy proektirovanija rezhushhih instrumentov*]. Kiev: Vishcha Shkola. (in Russian)

23. 3D-modeling tools, forming and allowance removal in cutting [*3D-modelirovanie instrumentov, formoobrazovanija i shema pripuska pri obrabotke rezaniem*] (2001) Ed. by P. R. Rodin. Kharkiv, NTU "KhPI". (in Russian)

24. Krivosheya, A. V. (2001) Schematic representation of methods for synthesizing of shape-producing gearing systems [Shematicheskoe predstavlenie sposobov sinteza formoobrazujushhe-proizvodjashhih sistem zubchatyh zaceplenij]. *Bulletin of KhSPU* [*Vestnik KhGPU*], 2001/6: 129–139. (in Russian)

25. Krivosheya, A. V., M. G. Storchak, Yu. M. Danilchenko, and V. V. Krivosheya (2006) The generalized classification of kinematic schemes for shaping gear wheels [Obobshhennaja klassifikacija kinematicheskih shem formoobrazovanija zubchatyh koles]. *High technology in mechanical engineering* [*Vysoki tekhnolohii v mashynobuduvanni*], 2006/2(13): 402–408. (in Russian)

26. Gutsalenko, Yu. G., A. V. Krivosheya, and A. V. Fadeev (2009) Working out means of CAD/CAM system infotainment for gearing design and production [Razrabotka sredstv informacionnoj podderzhki CAD/CAM sistem dlja proektirovanija i proizvodstva zubchatyh peredach]. *Physical and computer technologies: Proceedings of the 15th international scientific and technical conference* [*Fizicheskie i komp'juternye tehnologii: Trudy 15-j mezhdunarodnoj nauchno-tehnicheskoj konferencii*]. Kharkiv: KhNPK "FED", 234–240. (in Russian)

27. Krivosheya, A. V., Yu. M. Danilchenko, M. G. Storchak, D. T. Babichev, V. E. Melnik, V. I. Frantsuzov, Yu. G. Gutsalenko, and T. E. Tretyak (2014) On the problem of classification of kinematic schemes and mathematical models for the shaping of gears [K voprosu klassifikacii kinematicheskih shem i matematicheskih modelej formoobrazovanija zubchatyh peredach]. *Bulletin of NTU "KhPI"* [*Visnyk NTU "KhPI"*], 2014/31(1074): 75–84. (in Russian)

28. B. A. Perepelitsa, P. R. Rodin, A. V. Krivosheya, and Yu. G. Gutsalenko (2006) Multiparametric mapping of space in theory of tooth gear forming [Mnogoparametricheskie otobrazhenija prostranstva v teorii formoobrazovanija zubchatyh koles]. *Cutting and tool in technological systems* [*Rezanie i instrument v tehnologicheskih sistemah*], 2006/71: 103–106. (in Russian)

29. P. R. Rodin, G. A. Linkin, and V. N. Tatarenko (1976) *Processing of shaped surfaces on machine tools with numerical program control* [*Obrabotka fasonnyh poverhnostej na stankah s chislovym programmnym upravleniem*]. Kiev: Technika. (in Russian)

30. Perepelitsa, B., Yu. Gutsalenko, and N. Krukova (2008) Methodology of data calculation for shaping of objects with curvilinear intricate profile. *Fiability & Durability* [Targy Jiu, Romania, Academica Brancusi], 2008/1: 69–72.

31. Kovalyukh, V. R. (1982) Two-parameter gearing with equal-high-broad teeth and gashes [Dvuhparametricheskie kolesa s ravnovysokoshirokimi zub'jami i vpadinami]. *Mechanical Engineering* [*Mashinostroenie*], 1982/1: 43–45. (in Russian)

32. Kovalyukh, V. R., and R. V. Kovalyukh (1988) A new method of controlling the speed of rotation of a gear drive and the synthesis of gear variators on its basis [Novyj sposob regulirovanija skorosti vrashhenija zubchatoj peredachi i sintez na ego osnove zubchatyh variatorov]. *Theory of mechanisms and machines* [*Teorija mehanizmov i mashin*], 1988/44: 107–112. (in Russian)

33. Yu. G. Gutsalenko, A. L. Mironenko, T. E. Tretyak, N. V. Kryukova, and N. V. Zubkova (2011) Tooling design and development of shaping technology of bevel gears of double-link variators. *XXV microCAD International Scientific Conference 31 March – 1 April 2011, University of Miskolc (Hungary), Section L: Production Engineering and Manufacturing Systems*: 73–77.

34. Yu. G. Gutsalenko, A. L. Mironenko, and T. E. Tretyak (2011) Equidistant tooth generation on noncylindrical surfaces for two-parameter gearing. *Fiability & Durability* [Targy Jiu, Romania, Academica Brancusi], 2011/2(8): 67–72.

35. T. Tretyak, Yu. Gutsalenko, and A. Mironenko (2012) Modeling of running cutters for shaping of improved noninvolute tooth gears. *Fiability & Durability* [Targy Jiu, Romania, Academica Brancusi], 2012/1(9): 164–169.

36. N. E. Ternyuk, V. R. Kovaliukh, and Yu. V. Nasedkin (1981) Ways to machining of spherical gears [Sposoby obrabotki sfericheskih zubchatyh koles]. *Modern problems of cutting with tools from superhard materials: Abstracts of reports at the All-Union Scientific and Technical Conference* [*Sovremennye problemy rezanija instrumentami iz sverhtverdyh materialov: Tezisy dokladov Vsesojuznoj nauchno-tehnicheskoj konferencii*]: 46–49. (in Russian)

37. R. V. Kovaliukh, M. A. Podrigalo, E. B. Kondusova, A. L. Mironenko, T. E. Tretyak (1996) Kinematic schemes of forming of spherical gears with equidistant teeth lines [Kinematicheskie shemy formoobrazovanija sfericheskih zubchatyh koles s jekvidistantnymi linijami zub'ev]. *Cutting and tooling in technological systems* [*Rezanie i instrument v tehnologicheskih sistemah*], 1996/50: 105–109. (in Russian)

38. Tretyak, T., and A. Mironenko (2009) Influence of parameters of bevel gear forming with a constant normal pitch on the quality indicators of two-parameter gears. *Cutting and tooling in technological systems* [*Rezanie i instrument v tehnologicheskih sistemah*], 2009/76: 194–197. (in Russian)

39. Development of design and technological bases and technical solutions of diamond tools of the increased functionality in realization of electrophysicochemical methods of grinding [Rozrobka konstruktsiino-tekhnolohichnykh osnov i tekhnichnykh rishen almaznykh instrumentiv pidvyshchenoi funktsionalnosti v realizatsii elektrofizykokhimichnykh metodiv shlifuvannia]: Research report No. 0219U003639 (2018). Under the guidance of A. Grabchenko. Kharkiv: NTU "KhPI". (in Ukrainian)

40. A. V. Krivosheya, V. E. Melnik, V. V. Vozny, S. V. Ryabchenko, V. N. Bychikhin, and T. E. Tretyak (2016) Finishing of spur gears with elas-

tic diamond worm hones [Finishnaja obrabotka cilindricheskih zubchatyh koles almaznymi jelastichnymi chervjachnymi honami]. *Rock-cutting and metal-working tools – technique and technology of its manufacture and application* [*Porodorazrushajushhij i metalloobrabatyvajushhij instrument – tehnika i tehnologija ego izgotovlenija i primenenija*], 2016/19: 444–450. (in Russian)

41. E. B. Kondusova, T. E. Tretyak, and A. V. Krivosheya (1996) Algorithm for calculating the profile of enveloping surfaces for rolling tools and parts [Algoritm rascheta profilja ogibajushhih poverhnostej dlja obkatochnyh instrumentov i detalej]. *Proc. of the Fifth Int. Conf. "New Leading-Edge Technologies in Machinebuilding", Rybachie, Ukraine, Sept. 18-21 1996* [*Trudy Pjatoj mezhdunarodnoj konferencii "Novye tehnologii v mashinostroenii", Rybach'e, 18-21 sentjabrja 1996 g*.]: 140–141. (in Russian)

42. A. V. Krivosheya, T. E. Tretyak, and E. B. Kondusova (2001) Structural approach to the mathematical description of kinematic curves [Strukturnyj podhod k matematicheskomu opisaniju kinematicheskih krivyh]. *Cutting and tooling in technological systems* [*Rezanie i instrument v tehnologicheskih sistemah*], 2001/59: 129–134. (in Russian)

43. N. V. Zubkova, E. B. Kondusova, and T. E. Tretyak (2001) Matrix descriptions of the tangency condition and kinematic angles during cutting [Matrichnye opisanija uslovija kasanija i kinematicheskih uglov pri obrabotke rezaniem]. *Cutting and tooling in technological systems* [*Rezanie i instrument v tehnologicheskih sistemah*], 2001/59: 106–110. (in Russian)

44. T. Tretyak, A. Mironenko, Yu. Gutsalenko, N. Krukova, and S. Mironenko (2018) Structural approach to the mathematical description and computer visualization of plane kinematic curves for the display of gears. *Fiability & Durability* [Targy Jiu, Romania, Academica Brancusi], 2018/1(21): 7–11.

45. T. Tretyak, Yu. Gutsalenko, A. Shelkovoy, A. Mironenko, and S. Mironenko (2019) Mathematical modeling of the profile of a gear cutting rolling tool for machining of non-involute gear wheels. *Fiability & Durability* [Targy Jiu, Romania, Academica Brancusi], 2019/2(24): 5–11.

46. Yu. N. Sukhorukov, and R. I. Evstatiev (1983) *Tools for machining gears using the free rolling method* [*Instrumenty dlja obrabotki zubchatyh koles metodom svobodnogo obkata*]. Kiev: Technika. (in Russian)

47. S. I. Lashnev, A. N. Borisov, and S. G. Emelyanov (1997) *Geometric theory of the formation of surfaces by cutting tools* [*Geometricheskaja teorija formoobrazovanija poverhnostej rezhushhimi instrumentami*]. Kursk: Kursk State Tech. Univ. (in Russian)

48. Krivosheya, A. V. (1997) Features of the joint geometric design of gears and gear-cutting tools [Osobennosti sovmestnogo geometricheskogo proektirovanija zubchatyh koljos i zuboobrabatyvajushhih instrumentov]. *Proc. of the Int. conf. "High technology in engineering: development trends, management, marketing"* [*Trudy Mezhdunarodnoj konferencii "Vysokie tehnologii v mashinostroenii: tendencii razvitija, menedzhment, marketing"*]. Kharkiv: KhSPU, 291. (in Russian)

49. Krivosheya, A. V. (1995) Multiparameter mapping structure, generalizing machine and working gear links [Struktura mnogoparametricheskogo otobrazhenija, obobshhajushhaja stanochnye i rabochie zubchatye zaceplenija]. *Proc.*

of the Int. conf. "High technology in engineering: modeling, optimization, diagnostics" [*Trudy Mezhdunarodnoj konferencii "Vysokie tehnologii v mashinostroenii: modelirovanie, optimizacija, diagnostika"*]. Kharkiv: KhSPU, 71. (in Russian)

50. Wulgakov, E. B. (1974) *Gears with improved properties. Generalized theory and design* [*Zubchatye peredachi s uluchshennymi svojstvami. Obobshhennaja teorija i proektirovanie*]. Moscow: Mashinostroenie. (in Russian)

51. M. D. Genkin, M. A. Ryzhov, and N. M. Ryzhov (1981) *Improving the reliability of heavily loaded gears* [*Povyshenie nadezhnosti tjazhelonagruzhennyh zubchatyh peredach*]. Moscow: Mashinostroenie. (in Russian)

52. Gavrilenko, V. A. (1962) *Gears in mechanical engineering* [Zubchatye peredachi v mashinostroenii]. Moscow: Mashgiz. (in Russian)

53. E. G. Ginzburg, N. F. Golovanov, N. B. Firun, and N. T. Khalebskiy (1980) *Gears: Handbook* [*Zubchatye peredachi: Spravochnik*]. Leningrad: Mashinostroenie. (in Russian)

54. T. E. Tretyak, A. N. Shelkovoy, Yu. G. Gutsalenko, and S. A. Mironenko (2019) Analysis of qualitaty indicators of non-evolvent gearing [Analiz kachestvennyh pokazatelej nejevol'ventnyh zubchatyh peredach]. *Cutting and tooling in technological systems* [*Rezanie i instrument v tehnologicheskih sistemah*], 2019/90: 72–84. (in Russian)

55. B. A. Perepelitsa, Yu. G. Gutsalenko, and N. V. Zubkova (2004) Simulation methodology of cutting process and calculation of output indices on the base of multiparametric mappings [Metodika modelirovanija processov rezanija i raschet vyhodnyh pokazatelej na osnove mnogoparametricheskih otobrazhenij]. *Bulletin of NTU "KhPI"* [*Visnyk NTU "KhPI"*], 2004/44: 155–160. (in Russian)

56. Yu. Gutsalenko, A. Mironenko, A. Shelkovoi, T. Tretyak, S. Mironenko (2018) Some generalizations and representations of the possibilities of multi-parametric mapping of affin space in an appendix to the curvolinear forming and transmission movement. *Fiability & Durability* [Targy Jiu, Romania, Academica Brancusi], 2018/2(22): 5–12.

57. O. M. Shelkovoi, V. A. Fadeev, Yu. G. Gutsalenko, and E. A. Kostygov (2008) Experience of application of simulation modeling in solving the problem of modernization of the production units in the aviation industry [Dosvid zastosuvannia imitatsiinoho modeliuvannia pry vyrishenni zadachi modernizatsii vyrobnychykh pidrozdiliv aviatsiinoi promyslovosti]. *Aerospace and technology* [*Aviatsiino-kosmichna tekhnika i tekhnolohiia*], 2008/7: 181–188. (in Ukrainian)

58. V. M. Vinogradov, A. A. Cherepakhin (2009) The effect of gear cutting on the bending strength of the teeth of the cylindrical wheels of cars and tractors [Vlijanie metoda zubonarezanija na izgibnuju prochnost' zub'ev cilindricheskih koles avtomobilej i traktorov]. *Transactions of MSTU "MAMI"* [*Izvestija MGTU "MAMI"*], 2009/2(8): 136–141. (in Russian)

59. Lyashkov, A. A. (2012) Modeling the formation of gears by centroid bending method [Modelirovanie formoobrazovanija zubchatyh koles metodom centroidnogo ogibanija]. *Transactions of Transsib* [*Izvestija Transsiba*], 2012/2(10): 109–116. (in Russian)