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MATHEMATICAL WEAR MODEL FOR CONTACT OF RIGID RECTANGULAR PLANE PUNCH AND ELASTIC HALF-PLANE

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INTRODUCTION

Mathematical statement of contact problem with taking account of wear uses both equation of wear and contact boundary conditions. The experimental effect of exchanging of interior parameters of tribological system (intensity of wear, coefficient of friction, temperature of friction, microgeometry of surfaces etc.) delay is known as after-action effect [1]. Therefore, we should not ignore time interval, during which characteristics of contact surfaces which definited process of wear are being formed. This fact is described by wear inherited-aging model for contacting bodies

$$w(x,t) = \lambda(v)\int_{o}^{t} K_1(\tau)K_2(t-\tau)\,p(x,\tau)\,d\tau,$$

where $\lambda(v)$ – coefficient depended on frictional surfaces' sliding velocity; $p(x,\tau)$ – contact pressure; K_1 – ageing core; K_2 – adaptation core. We use this model for plane contact problem of rectangular punch and elastic half-plane consideration in stationary statement, therefore wear is representative by linear function with time

$$w(x,t) = (A+Bt)(p(x))^{\alpha}.$$

Introduction of new function “ageing“ or “forgetting“ $K_1(\tau)$ gives the opportunity to take account of difficult transformations and changes, which takes place in what is called “third body” [2]. “Third body” is a thin near surface layer with its physical, chemical and tribotechnical properties, which differ from properties of main material of contacting bodies. Let assume that after-action and existence of burnishing zones and established work regime effects are consequence of changes in the “third body”. Besides, we must take into account of wear processes delayed effect in new contact zones in the solution of contact problems with wear. In points x_s which are outside the initial elastic contact area $[-a_0\,,\ a_0]$ process of burnishing does not start at the moment $t=0$, but from the moment $t_s>0$, where t_s is a time when border of contact area reaches point x_s, thus $a(t_s)=x_s$. For these points a law of wear becomes following

$$w(x,t)=\lambda(v)\int_{t_s}^{t}K_1(\tau)\,K_2(t-\tau)\,p(x,\tau)\,d\tau\,.$$

Form of ageing function of must satisfy $K_1\to 0$ when $\tau\to\infty$, therefore tribological couple must "forget" about burnishing stage with time. Form of adaptation function K_2 must show that the fact of wear dependency from loading is characterized by after-action effect.

All in all, introduction of new ageing function $K_1(\tau)$ into inherited wear model allows describing wide experimental wear curves' class and taking into account delayed effect of burnishing process in new zones for problems with monotonically increasing contact area $[-a_0\,,\ a_0]$.

Mathematical statement of problem

The plane rigid punch with a plane base is pressed by the force P in the elastic half-plane by the time $\tau=0$ and under the punch stationary distribution of pressure is observed. As result, crushing of nonhomogeneouses of surfaces takes place. From the moment of the time $\tau=0$ punch moves along generatrix with constant relative velocity V_0 and erasing of half-plane take pace (Fig. 1). The surface of half-plane is unloaded out of the punch.

The integration of equations of theory of elasticity

$$\mu\Delta u+(\lambda+\mu)\,\partial\theta/\partial x=0\,, \tag{1}$$

$$\mu\Delta u+(\lambda+\mu)\,\partial\theta/\partial y=0\,, \tag{2}$$

with boundary conditions on surface $y=0$

$$v=f(x)+(k_1V_0\tau\,/\,H_B+k_2)\;\left|\sigma_y(x)\right|^{\alpha}, \tag{3}$$

$$\sigma_y = 0, \quad |x| > a \tag{4}$$

$$\tau_{xy} = 0, \quad |x| < \infty \tag{5}$$

are necessary for the solution to the problem, where Δ – Laplace's operator; $\theta = \partial u / \partial x + \partial u / \partial y$; u, v – components of displacement vector; λ, μ – Lame's coefficients; $f(x)$ – function of form of base of punch; a – half-width of the punch; H_B – firmness of half-plane material (Brinell scale); τ – time. The process of wear is descripted with parameters k_1, k_2, α, where $0 \le \alpha \le 1$, which were experimentally defined.

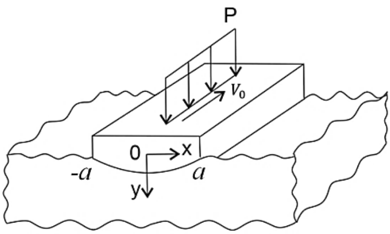

Fig. 1. Geometry of problem

Using Fourier's integral transformation [3]

$$\bar{u}(\xi, y) = \int_{-\infty}^{+\infty} u(x, y) e^{i\xi x} \, dx, \tag{6}$$

equations (1) and (2) can be represented in the space of transformants as follows:

$$\mu \frac{d^2 \bar{u}}{dy^2} - (\lambda + 2\mu)\xi^2 \bar{u} - i(\lambda + 2\mu)\xi \frac{\partial \bar{v}}{\partial y} = 0 \tag{7}$$

$$-i(\lambda + \mu)\xi \frac{d\bar{u}}{dy} + (\lambda + 2\mu)\frac{\partial^2 \bar{v}}{\partial y^2} - \mu \xi^2 \bar{u} = 0 . \tag{8}$$

Correlations (7)-(8) can be rewritten in the general form

$$a_1 \frac{d^2 \bar{u}}{dy^2} + b_1 \frac{d\bar{u}}{dy} + c_1 \bar{u} + d_1 \frac{d^2 \bar{v}}{dy^2} + e_1 \frac{d\bar{v}}{dy} + f_1 \bar{v} = 0 \tag{9}$$

$$a_2 \frac{d^2\bar{u}}{dy^2} + b_2 \frac{d\bar{u}}{dy} + c_2\bar{u} + d_2 \frac{d^2\bar{v}}{dy^2} + e_2 \frac{d\bar{v}}{dy} + f_2\bar{v} = 0 , \qquad (10)$$

where in our case

$$a_1 = \mu , \; b_1 = 0 , \; c_1 = -(\lambda + 2\mu)\xi^2 , \; d_1 = 0 , \; e_1 = -i(\lambda + \mu)\xi , \; f_1 = 0 ,$$
$$a_2 = 0 , \; b_2 = -i(\lambda + \mu)\xi , \; c_2 = 0 , \; d_2 = \lambda + 2\mu , \; e_2 = 0 , \; f_2 = -\mu\xi^2 .$$

We find solutions of (9) and (10) in the following form

$$\bar{u} = \left(d_1 \frac{d^2}{dy^2} + e_1 \frac{d}{dy} + f_1 \right) f = -i(\lambda + \mu)\xi \frac{df}{dy} \qquad (11)$$

$$\bar{v} = -\left(a_1 \frac{d^2}{dy^2} + b_1 \frac{d}{dy} + c_1 \right) f = -\left(\mu \frac{d^2}{dy^2} - (\lambda + \mu)\xi^2 \right) \qquad (12)$$

Then equation (9) is satisfied identically, and with (10) we have

$$\left(\mu \frac{d^2}{dy^2} - \xi^2 \right)^2 f = 0 , \qquad (13)$$

the solution of which has the following form for half-plane

$$f = (C_1 + C_2 y) e^{-|\xi| y} .$$

Then taking into account (11) and (12) we obtain

$$\bar{u} = -i(\lambda + \mu)\xi \left[-|\xi| C_1 + C_2 (1 - |\xi|) y \right] e^{-|\xi| y} \qquad (14)$$

$$\bar{v} = -i(\lambda + \mu)\xi \left[(\lambda + \mu)\xi^2 C_1 + 2\mu|\xi| C_2 + (\lambda + \mu)\xi^2 C_2 y \right] e^{-|\xi| y} , \qquad (15)$$

and according Hooke’s law

$$\bar{\sigma}_x = \lambda \frac{d\bar{v}}{dy} - i\xi(\lambda + 2\mu)\bar{u} , \; \bar{\sigma}_y = (\lambda + 2\mu)\frac{d\bar{v}}{dy} - i\xi\lambda\bar{u} , \; \bar{\tau}_{xy} = \mu \frac{d\bar{u}}{dy} - i\xi\mu\bar{v}$$

we obtain

$$\bar{\sigma}_x = -\mu\xi^2\left[-2(\lambda+\mu)|\xi|C_1 + 2(2\lambda+\mu)C_2 - 2(\lambda+\mu)|\xi|C_2 y\right]e^{-|\xi|y} \quad (16)$$

$$\bar{\sigma}_y = -\mu\xi^2\left[2(\lambda+\mu)|\xi|C_1 + 2\mu C_2 + 2(\lambda+\mu)|\xi|C_2 y\right]e^{-|\xi|y} \quad (17)$$

$$\bar{\tau}_{xy} = -i\xi\mu\left[2(\lambda+\mu)\xi^2 C_1 - 2\lambda|\xi|C_2 + 2(\lambda+\mu)\xi^2 C_2 y\right]e^{-|\xi|y} \quad (18)$$

When the boundary condition (5) is satisfied and the constant C_2 is determined as

$$C_2 = (\lambda+\mu)|\xi|C_1/\lambda, \quad (19)$$

then boundary conditions (3)-(4) are satisfied, and we obtain the following dual integral equations using (19)

$$\frac{1}{2\pi}\int_{-\infty}^{+\infty}(\lambda+\mu)(\lambda+2\mu)C_1\xi^2 e^{-i\xi x}/\lambda d\xi =$$

$$= f(x) + (k_1 V_0 \tau / H_B + k_2)\left|\sigma_y(x)\right|^{\alpha}, \ |x| \le a \quad (20)$$

$$-\frac{\mu}{\pi}\int_{-\infty}^{+\infty}(\lambda+\mu)^2 C_1\xi^2|\xi|e^{-i\xi x}/\lambda d\xi = 0, \ |x| > a \quad (21)$$

Solution of a system of dual integral equations

In the domain we have boundary condition

$$-\frac{\mu}{\pi}\int_{-\infty}^{+\infty}(\lambda+\mu)^2 C_1\xi^2|\xi|e^{-i\xi x}/\lambda d\xi = \sigma_y(x)H(a-|x|), \quad (22)$$

where $H(x)$ – Heaviside step function. By means of the Fourier transformations we determine

$$C_1 = \frac{\lambda}{2\mu(\lambda+\mu)^2\xi^2|\xi|}\int_{-a}^{+a}\sigma_y(x)e^{i\xi x}dx \quad (23)$$

By representing the unknown contact stresses $\sigma_y(x)$ in the form of a Fourier's series

$$\sigma_y(x) = \sum_{n=-N}^{N} a_n e^{i\pi nx/a} \tag{24}$$

we obtain

$$\int_{-a}^{+a} \sigma_y(x) e^{i\xi x}\, dx = 2\sum_{n=-N}^{N} a_n \frac{\sin(\xi a + \pi n)}{\xi + \pi n / a} \tag{25}$$

and

$$C_1 = \frac{\lambda}{\mu(\lambda+\mu)^2 \xi^2 |\xi|} \sum_{n=-N}^{N} a_n \frac{\sin(\xi a + \pi n)}{\xi + \pi n / a} \tag{26}$$

After substituting (26) in the integral equation (20) we obtain the following relation

$$\frac{\lambda + 2\mu}{\mu(\lambda+\mu)} \sum_{n=-N}^{N} a_n \int_{-\infty}^{+\infty} \frac{\sin(\xi a + \pi n)}{|\xi|(\xi + \pi n / a)} e^{-i\xi x}\, d\xi =$$
$$= -2\pi\left(f(x) + (k_1 V_0 \tau / H_B + k_2)\left|\sigma_y(x)\right|^{\alpha}\right) \tag{27}$$

After substituting (24) in the relation (27) we obtain the following finally relation

$$\frac{\lambda + 2\mu}{\mu(\lambda+\mu)} \sum_{n=-N}^{N} a_n \int_{-\infty}^{+\infty} \frac{\sin(\xi a + \pi n)}{|\xi|(\xi + \pi n / a)} e^{-i\xi x}\, d\xi +$$
$$+2\pi\left(k_1 V_0 \tau / H_B + k_2\right)\left(\sum_{n=-N}^{N} a_n e^{i\pi nx/a}\right)^{\alpha} = -2\pi f(x) \tag{28}$$

Using points collocation method [4] in $x = x_j = -a + a(j-1)/N$, $\left(j = \overline{1,\ 2N+1}\right)$, for (28), we obtain a system of non-linear algebraic equations for finding the unknown coefficients a_n, $\left(n = \overline{1,\ 2N+1}\right)$:

$$\vec{z}\,\|A\| + C\left(\vec{z}\,\|D\|\right)^{\alpha} = \vec{b}\,, \tag{29}$$

where

$$a_{k,j} = \frac{\lambda + 2\mu}{\mu(\lambda + \mu)} \int_{-\infty}^{+\infty} \frac{\sin(\xi a + \pi n)}{|\xi|(\xi + \pi k / a)} e^{-i\xi x} \, d\xi, \qquad d_{k,j} = e^{i\pi k x_j / a},$$

$$b_j = -2\pi f(x_j), \ \vec{z} = (a_{-N}, a_{-N+1}, \dots, a_{N-1}, a_N) = (z_1, z_2, \dots, z_{2N+1})$$

$$C = 2\pi(k_1 V_0 \tau / H_B + k_2), \ \left(k, j = \overline{1, \ 2N+1}\right) \tag{30}$$

We can reduce non-linear system (29) to a system of linear algebraic equations in the cases $\alpha = 1$ and $\alpha = 0$ in the following forms

$$\vec{z}_1 \|A\| + C(\|D\|) = \vec{b} \tag{31}$$

$$\vec{z}_0 \|A\| = \vec{b} - \vec{c} \tag{32}$$

These limited cases ($\alpha = 0$, $\alpha = 1$) are the most interesting because they give the opportunity to compute the largest and smallest wear. If $0 < \alpha < 1$, the solution of the problem (27) will be between solutions (31) and (32).

Using iterative method [4], we obtain the system in the following form

$$\vec{z} = \vec{g}(\vec{z}) \ \text{or} \ z_j = g_j(z_1, z_2, \dots, z_k), \quad (k = 2N+1), \tag{33}$$

where

$$g_j(z_1, z_2, \dots, z_k) = -\frac{1}{a_{j,j}} \left\{ \sum_{m=j+1}^{j-1} a_{j,m} z_m + \sum_{m=1}^{k} a_{j,m} z_m + \right.$$
$$\left. + C \left(\sum_{m=1}^{k} a_{j,m} z_m \right)^{\alpha} \right\} + b_j \tag{34}$$

We make iteration by formula

$$\vec{z}^{(n+1)} = \vec{g}\left(\vec{z}^{(n)}\right), \qquad (35)$$

or in other forms

$$z_j^{(n+1)} = g_j\left(z_1^{(n)}, \ldots, z_k^{(n)}\right), \qquad (\, j = \overline{1, k}\,) \qquad (36)$$

or

$$\begin{cases} z_1^{(n+1)} = g_1\left(z_1^{(n)}, \ldots, z_k^{(n)}\right) \\ z_2^{(n+1)} = g_2\left(z_1^{(n)}, \ldots, z_k^{(n)}\right) \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ z_k^{(n+1)} = g_k\left(z_1^{(n)}, \ldots, z_k^{(n)}\right) \end{cases} \qquad (37)$$

For zero approximation we choice mean value between solutions of problems (31) and (32)

$$\vec{z}^{(0)} = \left(\vec{z}_0 + \vec{z}_1\right)/2\,. \qquad (38)$$

Computation of integrals

It is necessary to compute the following integrals in the system (29) for determining the coefficients $a_{k,j}$

$$I = \int\limits_{-\infty}^{+\infty} \frac{\sin\left(\xi a + \pi k\right)}{|\xi|\left(\xi + \pi k / a\right)} e^{-i\xi x_j}\, d\xi, \qquad (\, k = \overline{1,\ 2N+1}\,) \qquad (39)$$

For the computation these integrals first we calculate integral which is derivative of integral (39) with variable x:

$$I^{'} = -i \int\limits_{-\infty}^{+\infty} sign\,\xi \frac{\sin\left(\xi a + \pi k\right)}{|\xi|\left(\xi + \pi k / a\right)} e^{-i\xi x_j}\, d\xi, \quad (\, k = \overline{1,\ 2N+1}\,) \qquad (40)$$

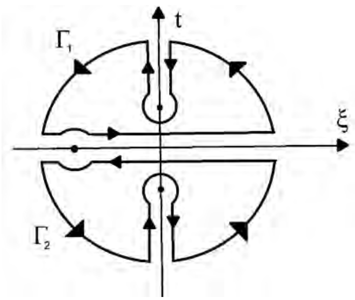

Fig. 2. Contours of integration

By representing integral (40) in the form

$$I' = \frac{1}{2}\left((-1)^{k-1} I_1 + (-1)^k I_2\right), \tag{41}$$

where

$$I_1 = \int_{-\infty}^{+\infty} \frac{sign\,\xi}{\xi + \pi k / a} e^{-i\xi(a - x_j)}\, d\xi, \qquad (k = \overline{1,\ 2N+1}) \tag{42}$$

$$I_2 = \int_{-\infty}^{+\infty} \frac{sign\,\xi}{\xi + \pi k / a} e^{-i\xi(a + x_j)}\, d\xi, \qquad (k = \overline{1,\ 2N+1}) \tag{43}$$

we use Cauchy theorem about residuals on the contour Γ_1 for I_1 and Γ_2 for I_2 (Fig. 2) and obtain

$$I_1 = \frac{2}{a}\int_0^{\infty} \frac{e^{-(a - x_j)t}}{t - i\pi k / a}\, dt + (-1)^k \pi i \frac{e^{i\pi k x_j / a}}{a} \tag{44}$$

$$I_2 = -\frac{2}{a}\int_0^{\infty} \frac{e^{-(a + x_j)t}}{t + i\pi k / a}\, dt + (-1)^k \pi i \frac{e^{i\pi k x_j / a}}{a} \tag{45}$$

According [5]:

$$\int_0^\infty \frac{e^{-(a-x_j)t}}{t-i\pi k/a}dt = -Ei\left(i\pi k\left(a-x_j\right)/a\right)e^{-i\pi k(a-x_j)/a} \tag{46}$$

$$\int_0^\infty \frac{e^{-(a+x_j)t}}{t+i\pi k/a}dt = Ei\left(-i\pi k\left(a+x_j\right)/a\right)e^{i\pi k(a+x_j)/a} \tag{47}$$

After substituting expressions (46) and (47) in (41) we obtain

$$I^{'} = (-1)^k\left(Ei\left(i\pi k\left(a-x_j\right)/a\right)e^{-i\pi k(a-x_j)/a} + Ei\left(i\pi k\left(a+x_j\right)/a\right)e^{i\pi k(a+x_j)/a}\right).$$

We use Filon's method [6] for computing I during integration with x of oscillatory functions (46)-(47). This method aims at consideration the quadrature of the integral

$$\int_a^b y(p)\,dp\,, \tag{48}$$

in which $y(p)$ is a numerically specified function. The error incurred in evaluating (48) with an n-th order scheme is proportional to $y^{(n)}(z)$ for some z between a and b. Such a scheme assumes the existence of the Taylor expansions

$$y(p)=\sum_{k=o}^{n-1}\frac{y^{(k)}(a)(p-a)^k}{k!}+\sum_{k=o}^{n-1}\frac{y^{(n)}(z)(p-a)^n}{z!}$$

or an equivalent polynomial expansion and integrates it term by term. Different quadrature formulae are obtained depending on what method is used to estimate the derivatives $y^{(k)}(a)$ from the numerical values of y. The maximum possible error E_n is then

$$E_n(y,a,b)=\sup\left[y^{(n)}\right](b-a)^{n+1}/(n+1)!$$

in which $\sup\left\lfloor y^{(n)} \right\rfloor$ is the maximum of the function $y^{(n)}$ on the closed interval $[a,b]$. An estimate of the relative error RE_n can then be obtained by dividing E_n by $\sup[y](b-a)$, which is an upper bound for the integral itself:

$$RE_n(y,a,b) = \frac{\sup\left[y^{(n)}\right](b-a)^n}{\sup[y](n+1)!}$$

Then if the maximum allowable relative error is r the step size should be less than

$$(b-a)_{\max} = \left\{r \ \sup[y](n+1)! \big/ \sup\left[y^{(n)}\right]\right\}^{1/n} \qquad (49)$$

A problem occurs when $y = f \exp(sp)$ for then if s is large $y^{(n)} \approx y\, s^n$. Thus $(b-a)_{\max} = s^{-1}\left[r(n+1)!\right]^{1/n}$ so the number of steps necessary for the complete contour is proportional to s. Filon's method addresses this problem by expanding f as a polynomial instead of y. If the Taylor series expansion for f about the point $p=a$ is substituted into

$$\int_a^b f(p) \exp(sp)\, dp \qquad (50)$$

and integrated term by term the result is

$$\int_a^b f(p)\exp(sp)\,dp = \sum_{k=0}^{n-1} \frac{f^{(k)}(a)}{k!} \int_a^b (p-a)^k \exp(sp)\,dp + $$
$$+\int_a^b f^{(n)}(z) \frac{\exp(sp)(p-a)^n}{n!}\,dp\ .$$

The maximum possible error is then given by

$$E_n\left[f\exp(sp),a,b\right] = \sup\left[f^{(n)}\exp(sp)\right](b-a)^{n+1}\big/(n+1)!$$

and an estimate of the relative error is

$$RE_n\left[f\exp(sp), a,\ b\right] = \frac{\sup\left[f^{(n)}\right](b-a)^n}{\sup[f](n+1)!}.$$

Since the relative error is independed of *s*, so too is the step size $(b-a)$ necessary to obtain a given accuracy. In fact the formula for step size, given a maximum allowable relative error *r*, is just equation (49) with *f* substituted for y.

Point out that different Filon's method quadrature formulae are obtained by using different methods to estimate the derivatives $f^{(k)}(a)$ from the numerical values of f. Thus for every quadrature formula for integrals of the form (48) there is an analogous Filon's method quadrature formulae for integrals of the form (50) [7].

Results

Numerical computations were carried out for the following parameters (system SI): material of half-plane – aluminum ($H_B = 11.3$, $\lambda = 5.6\times10^{10}$, $\mu = 2.6\times10^{10}$), $V_0 = 0.25$, $a = 0.25$, $k_1 = 10^{-10}$, $k_2 = 10^{-11}$, $f(x) = const = 0.01$, $N = 23$.

Distribution of displacements v and contact stresses $\sigma_y^* = 2\sigma_y a / P$ is given in Fig. 3 for $\tau = 0$ and $\tau = 100$, curve 1 corresponds to $\alpha = 0$, curve 2 corresponds to $\alpha = 1$. Value of force P is computing by formulae

$$P = -\int_{-a}^{+a} \sigma_y(x)\,dx = -\int_{-a}^{+a} \sum_{k=-N}^{N} z_{N+k+1} \exp\frac{i\pi kx}{a} = -2az_{N+1} \qquad (51)$$

It should be noted that, for the anti-plane problem, the boundary conditions on the tangential stresses $\tau_{yz} = f_{\mathrm{T}}\sigma_y$ must be satisfied (normal stresses σ_y are defined by formula (24), f_{T} – coefficient of friction).

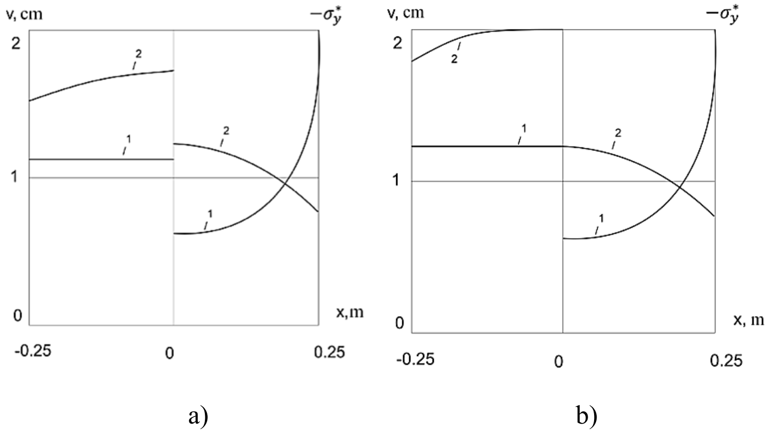

Fig. 3. Distribution of displacements and stresses for $\tau = 0$ (a) and $\tau = 100$ (b)

CONCLUSIONS

First contact problems with wear were considered in [8]. Mathematical models of wear were presented in [9]. Statement of two-dimensional contact problem with heat release in the process of friction was given in [10]. We investigate plane problem of elastic half-plane wear under the base of moving along generatrix with constant velocity rectangular rigid punch. Half-plane is considered in independent plane and anti-plane deformation conditions. We investigate process of wear, described by boundary condition including empirical parameter α, where $0 \le \alpha \le 1$. These limited cases ($\alpha = 0$, $\alpha = 1$) are the most interesting because they give the opportunity to compute the largest and the smallest wear. In the case $\alpha = 1$ vertical displacements increase due to the interaction time, it shows half-plane material abrasion increasing (curves 2 on Fig. 3). In the case $\alpha = 0$ we obtain invariability of vertical displacements (curves 1 on Fig. 3). Results obtained for the limited case $\alpha = 0$, $\tau = 0$ coincide with the one known in literature [7].

Further mathematical statement of plane contact problem with heat generation and wear account of friction is planned. Let plane punch of the height H with a plane base is pressed by the force P in the elastic half-plane and moves along generatrix with the constant relative velocity V_0. The heat exchange between the side surfaces of the punch and the external medium occurs according to Newton’s law with the coefficient of heat exchange γ_a. The convective heat exchange with external medium through the unloaded surface is realized with the coefficient of heat exchange γ_0. The upper end of the punch carries out a heat exchange with external medium with the coefficient of heat exchange γ_H. The heat contact between bodies is non-ideal. The direction of the heat fluxes into the interactive bodies has been generated due to the action of frictional forces and wear process. The coefficient of heat flux redistribution is not considered in the formulation of the boundary conditions, which is insignificant. The temperature of the external medium presumably equals zero without restriction of generality. Investigation methods of contact problem solution which are similar to solved problem are planned. The existence of half-plane detaching zones from punch is predicted. This effect we obtained in the problem about thermoelastic contact of half-plane and rectangular punch with heat generation account of friction without wear [11]. In this case the solution of heat conduction problem for the punch must be obtained, and boundary conditions for area with full contact and for the detaching zones punch must be formulated. Besides, methods of solution building for problems with non-perfect contact are not investigated well.

SUMMARY

Investigation of thermal stresses and wear in the contact couple is an important problem for many engineering researches. The steady problem of thermoelasticity currently is sufficiently investigated. However, taking into account the actual operating conditions, in particular wear, leads to complication of statement and mathematical problem modeling. This is due to mathematical difficulties that arise in the solution of dual integral equations. A method of constructing solutions of contact problems with wear is developed. The plane contact problem of elastic half-plane wear by a rigid punch has been considered. The punch moves along generatrix with constant velocity. Arising thermal effects are neglected because the problem is investigated in stationary statement. In this case the crumpling of the nonhomogeneities of the surfaces and abrasion of half-plane take place. The surface of half-plane is unloaded out of the punch. The solution for problem of theory of elasticity is constructed by means of Fourier integral transformation. Contact stresses are found in Fourier series which coefficients satisfy the dual integral equations. It leads to the system of nonlinear algebraic equations for unknown coefficients by a method of collocations. Cauchy theorem about residuals is used for

computing integrals. This system is reduced to linear system in the partial most interesting cases for computing of largest and smallest wear. The iterative scheme is considered for investigation of other nonlinear cases, for initial approximation the mean value of boundary cases is exploited. Filon's method for computing oscillatory integrals is used. The evolutions of contact stresses, wear and abrasion in the time are given. For both last cases increase or invariability of vertical displacements correspondently is obtained. In the boundary cases coincidence of results with known is obtained.

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